

Handling Blur

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Introduction and motivation

Blur is an inevitable unwanted phenomenon, which is present in all digital images. It results in smoothing high-frequency details, which makes the image analysis difficult. Heavy blur may degrade the image so seriously, that neither automatic analysis nor visual interpretation of the content are possible. If we did not have proper tools for processing and analyzing blurred images, many unique images would become useless. Two major approaches to handling blurred images exist. They are more complementary rather than concurrent; each of them is appropriate for different tasks and employs different mathematical methods and algorithms.

Image restoration is one of the oldest areas of image processing. It appeared as early as in 1960's and 1970's in the work of the pioneers A. Rosenfeld, H. Andrews, B. Hunt, and others. In the last ten years, this area has received new impulses and has undergone a quick development. We have been witnesses of the appearance of multichannel techniques, blind techniques, and superresolution enhancement resolved by means of variational calculus in very high-dimensional spaces. A common point of all these methods is that they suppress or even remove the blur from the input image and produce an image of a high visual quality. However, image restoration methods are often ill-posed, ill-conditioned, and time consuming.

On the contrary, *blur-invariant* approach, proposed originally in 1995, works directly with the blurred data without any preprocessing. Blurred image is described by features, which are invariant with respect to convolution with some group of kernels. Image analysis is then performed in the feature space. This approach is suitable for object recognition, template matching, and other tasks where we want to recognize/localize objects rather than to restore the complete image. The mathematics behind it is based on projection operators and moment invariants.

Tutorial scope

In this tutorial, we will focus on both approaches. We start with **blur modeling** and analyzing potential sources of blur in real images. In the **image restoration** part of the tutorial we review traditional as well as modern deconvolution techniques, including blind deconvolution, space variant deconvolution, and multichannel deconvolution. The next section covers **invariants to image blurring**. The flowchart of both approaches is visualized in Fig. 1. The tutorial will be completed

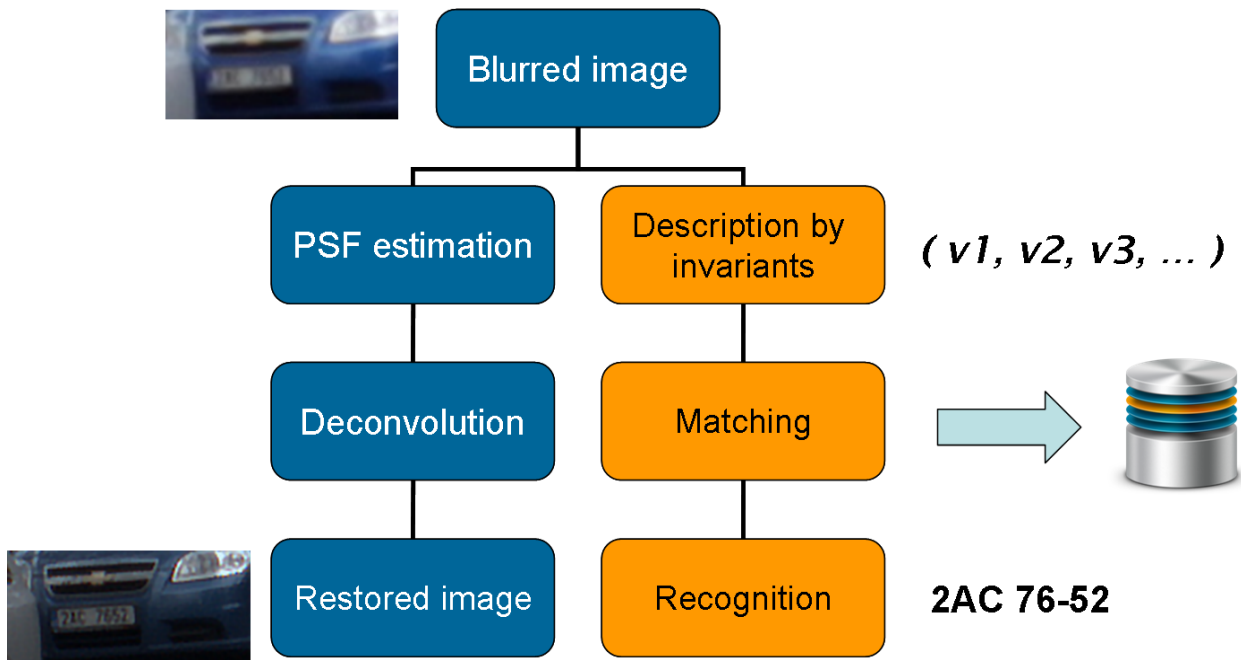


Figure 1: The flowchart of image restoration (left) and of the direct recognition by blur invariants (right).

with numerous **demonstrations and practical examples**. The tutorial originates from the 20-years speakers' experience in image restoration, deconvolution, invariants, and related fields. Four basic blocks of the tutorial are sketched below.

Blur modeling

- Relation between the true latent image $u(x, y)$ and the degraded observed image $g(x, y)$:

$$g = Hu + n,$$

where H is the degradation operator and n is additive noise.

- The most common type of degradation – blur – modeled as space-variant convolution

$$[Hu](x, y) = \int u(s, t)h(x - s, y - t, s, t)dsdt,$$

where $h(s, t, x, y)$ is called a space-variant convolution kernel (image of a point source at location (x, y)).

- Space-invariant blur model – h fixed in the image space – modeled as standard convolution

$$[Hu](x, y) = h * u = \int u(s, t)h(x - s, y - t)dsdt.$$

- Common examples of blurs: out-of-focus, motion, camera shake, or turbulence; see in Fig. 2.
- Blind versus non-blind methods



Figure 2: Examples of blurs: (left) three blurs caused by out-of-focus lens with 7-blade diaphragm and different focal length and aperture size; (middle) three blurs caused by camera motion during exposure; (right) two blurs caused by atmospheric turbulence.

Image restoration

- Traditional non-blind approaches: Wiener filter, constrained optimization, role of image priors
- Single-channel blind methods: Maximum a Posteriori method, Marginalization and the Variational Bayesian strategy
- Multichannel deconvolution: a better-posed problem of multiple blurred observations
- Superresolution: beyond camera resolution
- Space-variant case: parametric models, patch-based approaches, open challenges

Invariants to image blurring

- The notion of blur invariance
- Projection operators on kernel subspaces.
- Blur invariants in frequency domain. The notion of the *primordial image*. Particular cases for centrosymmetric, radial, N -fold symmetric, dihedral, and Gaussian blur.
- Blur invariants in image domain as recurrent functions of image moments.
- Numerical experiments on the recognition power and stability.

Applications

Numerous practical applications of image restoration as well as of blur invariants will be presented during the tutorial. We show the use in remote sensing, astronomy, security, forensic imaging, and biomedical imaging. We will also demonstrate the application in consumer photography, implemented in a smartphone.

The invariants to image blurring have found successful applications in face recognition on out-of-focused photographs, in normalizing blurred images into the canonical forms, in template-to-scene matching of satellite images, in blurred digit and character recognition, in registration of images obtained by digital subtraction angiography, and in focus/defocus quantitative measurement. Many of these applications will be presented in the tutorial.

Required prior knowledge

There is no specific required knowledge of the tutorial participants except standard undergraduate courses of image processing and pattern recognition. The tutorial is self-contained.

Target audience and time allocation

The target audience of the tutorial are

- Researchers from all application areas who need to analyze blurred images
- Software professionals, industry researchers, and application developers of Computer Vision or Image Processing software.
- Graduate students of computer science, artificial intelligence, image analysis, pattern recognition, and related areas.

Estimated audience – 40 to 50 participants.

Time allocation – half day (four hours including a 30-minute break).

Supplementary reading

The tutorial is not based on any single book or paper. For the attendees interested to learn more on this subject, we recommend the following monographs as the main references:

- Campisi P. and Egiazarian K., ed.: *Blind Image Deconvolution: Theory and Application*, CRC Press, 2007
- Milanfar P., ed.: *Super-resolution imaging*, CRC Press, 2010
- Rajagopalan A. N. and Chellappa R., ed.: *Motion Deblurring: Algorithms and Systems*, Cambridge University Press, 2014
- Flusser J., Suk T., Zitová B. : *2D and 3D Image Analysis by Moments*, Wiley, to appear in November 2016
- Flusser J., Suk T., Zitová B. : *Moments and Moment Invariants in Pattern Recognition*, Wiley, 2009.

Other references to each tutorial section will be provided to the audience during the tutorial.

Speaker bios

Jan Flusser received the M.Sc. degree in mathematical engineering from the Czech Technical University, Prague, Czech Republic in 1985 and the Ph.D. degree in computer science from the Czechoslovak Academy of Sciences in 1990. Since 1985 he has been with the Institute of Information Theory and Automation, Academy of Sciences of the Czech Republic, Prague. In 1995-2007 he was holding the position of a head of Department of Image Processing. In 2007 he was appointed the Director of the Institute. Since 1991 he has been also affiliated with the Faculty of Mathematics and Physics, Charles University, Prague and with the Czech Technical University, Prague (full professorship in 2004), where he gives undergraduate and graduate courses on Digital Image Processing and Pattern Recognition and specialized graduate course on Invariants and wavelets. He has research and teaching experience from many universities and institutions worldwide.

Jan Flusser has a 25-years experience in basic and applied research on the field of image analysis, pattern recognition, and related fields. He has been involved in applications in remote sensing, medicine, and astronomy. He has authored and coauthored more than 200 research publications in these areas. He has presented more than 20 tutorials and invited/keynote talks at international conferences (ICIP'05, ICIP'07, EUSIPCO'07, CVPR'08, FUSION'08, SPPRA'09, SCIA'09, ICIP'09,

SPPRA'10, COMPSTAT'06, WIO'06, DICTA'07, AIA'14, and others). Some of his journal papers became classical and are frequently cited (Google Scholar reports more than 10 000 citations of J. Flusser's publications).

J. Flusser has received several national and international scientific awards and prizes (Scopus 1000 Award, Felber Medal, Czech Science Foundation Award, The Czech Academy of Sciences Prize, and several best paper awards). His book *Moments and Moment Invariants in Pattern Recognition*, Wiley, 2009, has become the world-wide textbook and the main reference on the field of moment-based image analysis.

Filip Sroubek received the MS degree in computer science from the Czech Technical University, Prague, Czech Republic in 1998 and the PhD degree in computer science from Charles University, Prague, Czech Republic in 2003. From 2004 to 2006, he was on a postdoctoral position in the Instituto de Optica, CSIC, Madrid, Spain. In 2010 and 2011, he was the Fulbright Visiting Scholar at the University of California, Santa Cruz. He is currently with the Institute of Information Theory and Automation, the Czech Academy of Sciences, as the vice-head of the image processing department, and gives a graduate course on variational methods in image processing at the Czech Technical University and Charles University.

His research covers all aspects of image processing, in particular, image restoration (denoising, blind deconvolution, super-resolution) and image fusion (multimodal, multifocus). He is an author of 8 book chapters and over 60 journal and conference papers. In addition, he co-authored several tutorials at major international conferences (ICIP'05, EUSIPCO'07, CVPR'08, ICCV'15) and was a keynote speaker at SPIE-IS&T'15 and ICIIP'13. He is a co-inventor of two patents.

His scientific achievements were awarded by several national prizes – the Josef Hlavka Student Prize, the Otto Wichterle Premium of the Czech Academy of Sciences for excellent young scientists, and the Czech Science Foundation Award.

Barbara Zitova received the M.Sc. degree in computer science and the Ph.D. degree in software systems from Charles University, Prague, Czech Republic, in 1995 and 2000, respectively. Since 1995, she has been with the Institute of Information Theory and Automation, Czech Academy of Sciences. Since 2008, she has been the Head of the Department of Image Processing. She gives undergraduate and graduate courses on digital image processing and wavelets in image processing with the Czech Technical University and Charles University. Her research interests include geometric invariants, image enhancement, image registration and image fusion, and image processing applications in cultural heritage and medical imaging.

She has authored/co-authored over 60 research publications in these areas, including monographs *Moments and Moment Invariants in Pattern Recognition* (Wiley, 2009), *2D and 3D Image Analysis by Moments* (Wiley, 2016), and tutorials at major conferences (ICIP'05, ICIP'07, EUSIPCO'07, CVPR'08, ICIP'09). Some of her journal papers became classical and are frequently cited (Google Scholar reports more than 7 000 citations of B. Zitova's publications).

She has received several awards - the Josef Hlavka Student Prize, the Otto Wichterle Premium of the Czech Academy of Sciences for excellent young scientists, Czech Science Foundation Award, The Czech Academy of Sciences Prize, several best paper awards, and the SCOPUS 1000 Award for more than 1000 citations of a single paper in 2010.

Selected speakers' publications relevant to the tutorial

Books

- Flusser J., Suk T., Zitová B. : *Moments and Moment Invariants in Pattern Recognition*, Wiley & Sons Ltd., 2009, 317 pp., ISBN 978-0-470-69987-4, http://zoi.utia.cas.cz/moment_invariants
- Flusser J., Suk T., Zitová B. : *Moments and Moment Invariants in Pattern Recognition (Chinese Edition)*, Univ. of Science and Technology of China Press, 2014 (in Chinese)
- Flusser J., Suk T., Zitová B. : *2D and 3D Image Analysis by Moments*, Wiley & Sons Ltd., in print, to appear in early 2016, 700 pp.

Journal papers

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- Matúš F., Flusser J. : "Image Representations via a Finite Radon Transform", *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 15, pp. 996-1006, 1993
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- Suk T., Flusser J. : "Projective Moment Invariants", *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 26, pp. 1364-1367, 2004
- Šroubek F., Flusser J. : "Multichannel Blind Deconvolution of Spatially Misaligned Images", *IEEE Trans. Image Processing*, vol. 14, pp. 874-883, 2005
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