Handling Blur

Jan Flusser, Filip Sroubek, Barbara Zitova

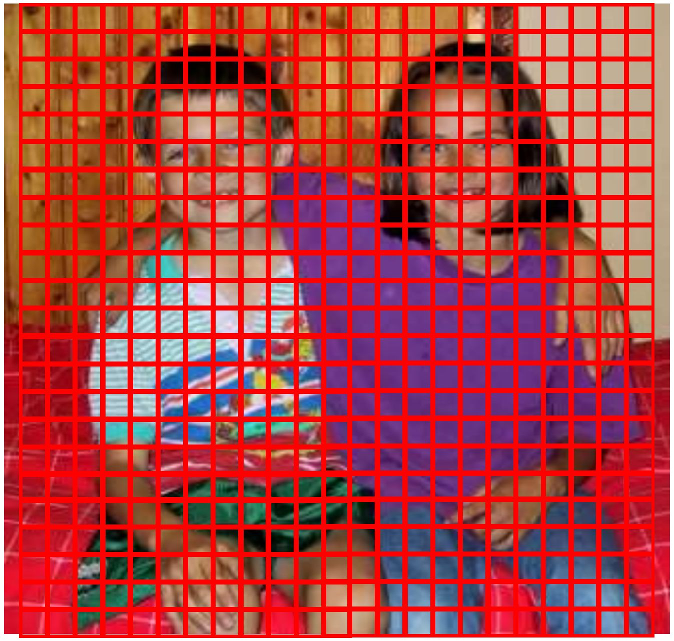
Institute of Information Theory and Automation

{flusser,sroubekf,zitova}@utia.cas.cz
www.utia.cas.cz

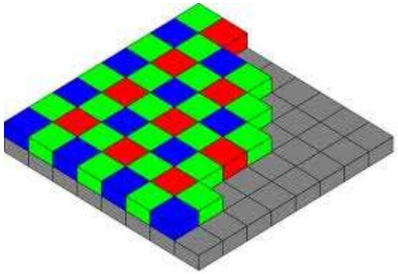
Institute of Information Theory and Automation
Czech Academy of Sciences

Department of Image Processing
Pod vodarenskou vezi 4, 182 08 Prague 8

What is a digital image?



$$\begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$



Acquired image is a degraded version of the original scene

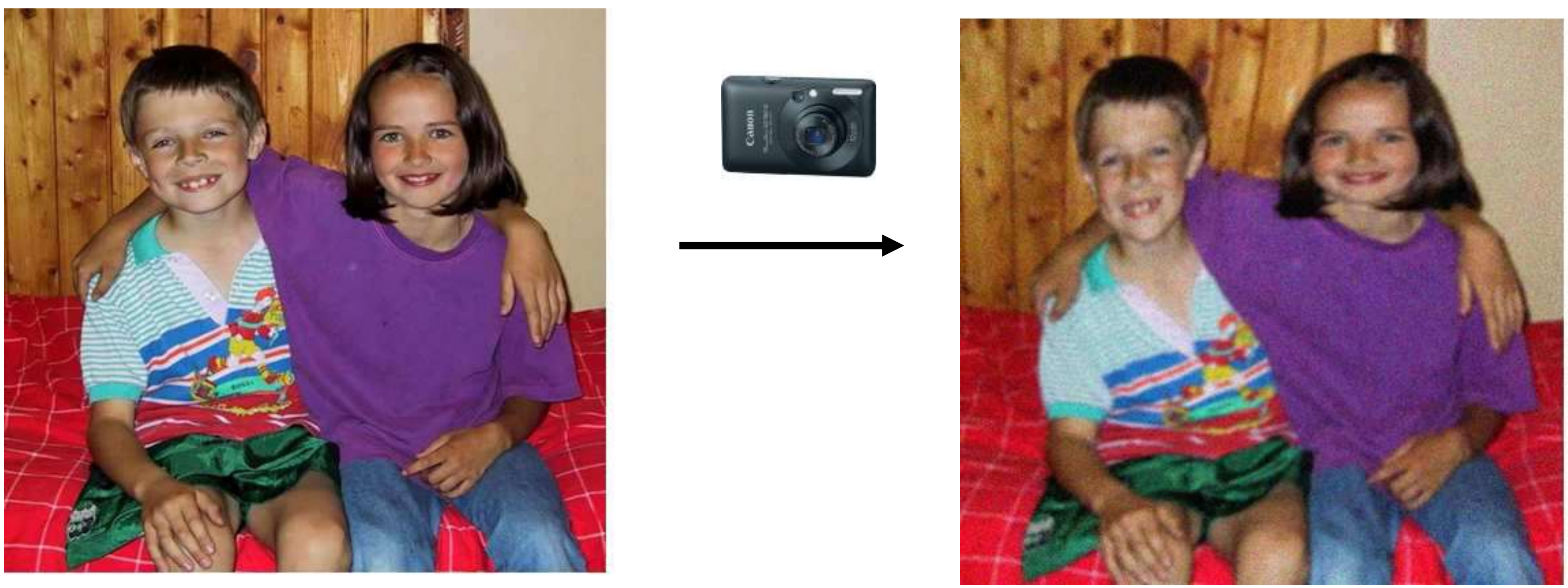
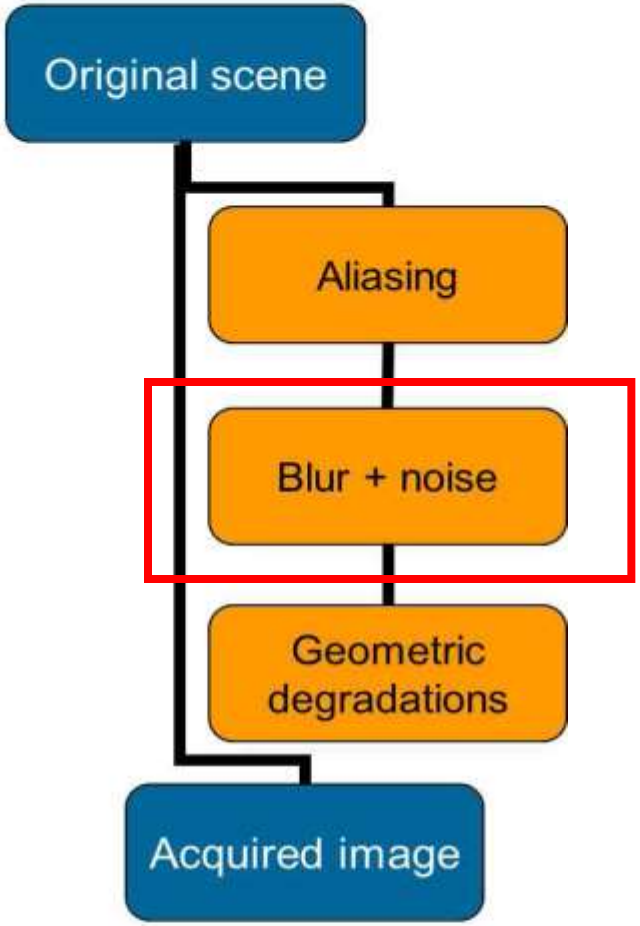
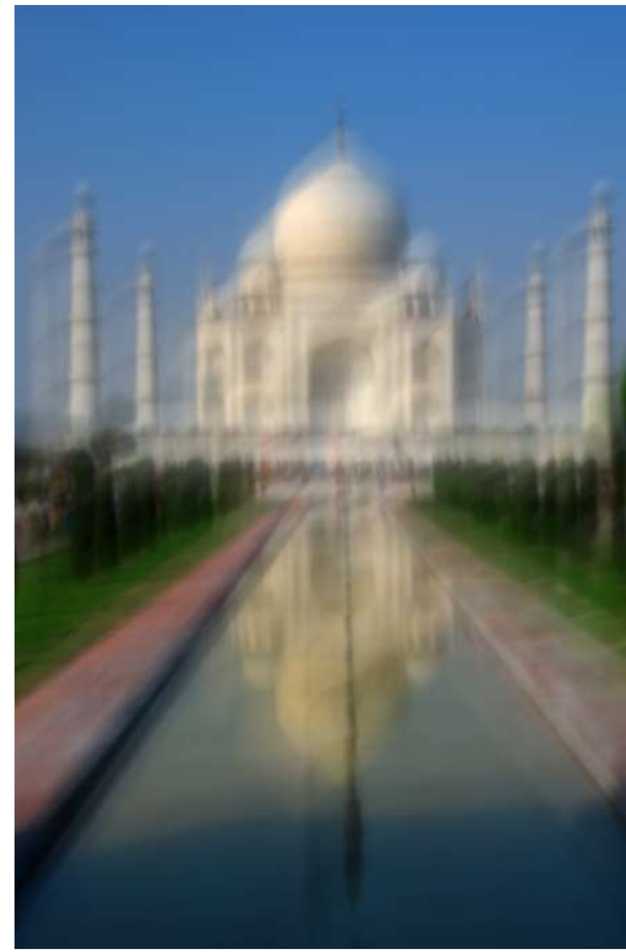


Image degradations



Blur examples



Typical blur sources

Camera shake/motion

Typical blur sources

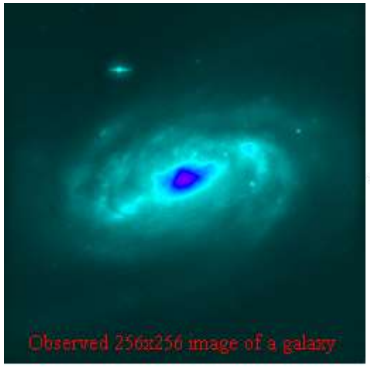
Scene motion

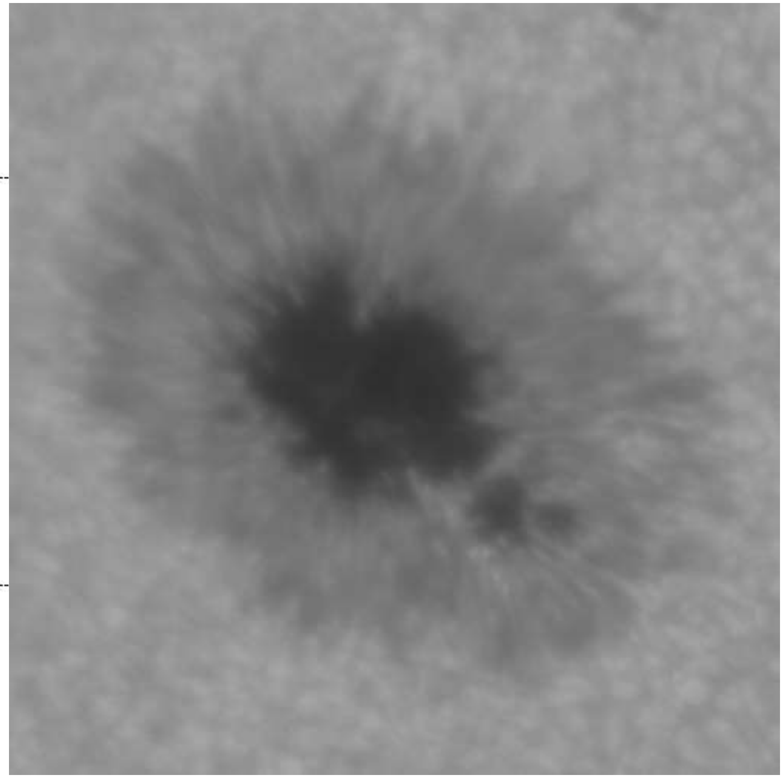
Typical blur sources

Out-of-focus

Typical blur sources

Atmospheric turbulence





Typical blur sources

Medium turbulence

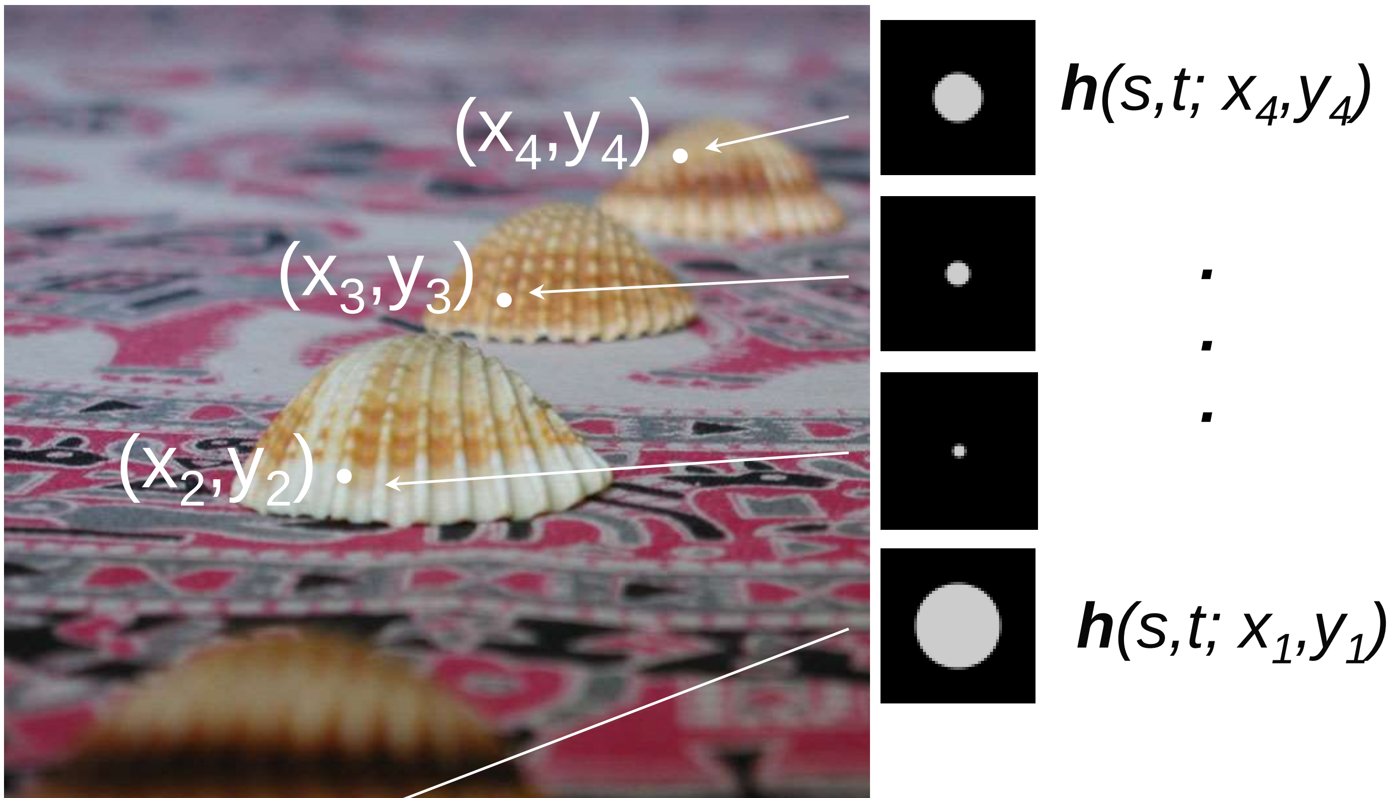
Typical blur sources

Medium turbulence

The topic of the tutorial

How can we extract the information, that we are looking for, from a blurred image?

General space-variant blur model



$$\mathbf{z}(x,y) = \int_{\Omega} \mathbf{u}(x-s, y-t)\mathbf{h}(s,t; x-s, y-t)\, dsdt + \mathbf{n}(x,y)$$

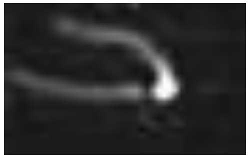
Simplified space-invariant blur model

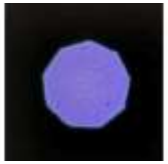
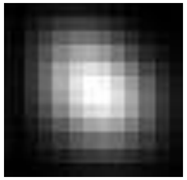
Flat scene

Constant motion

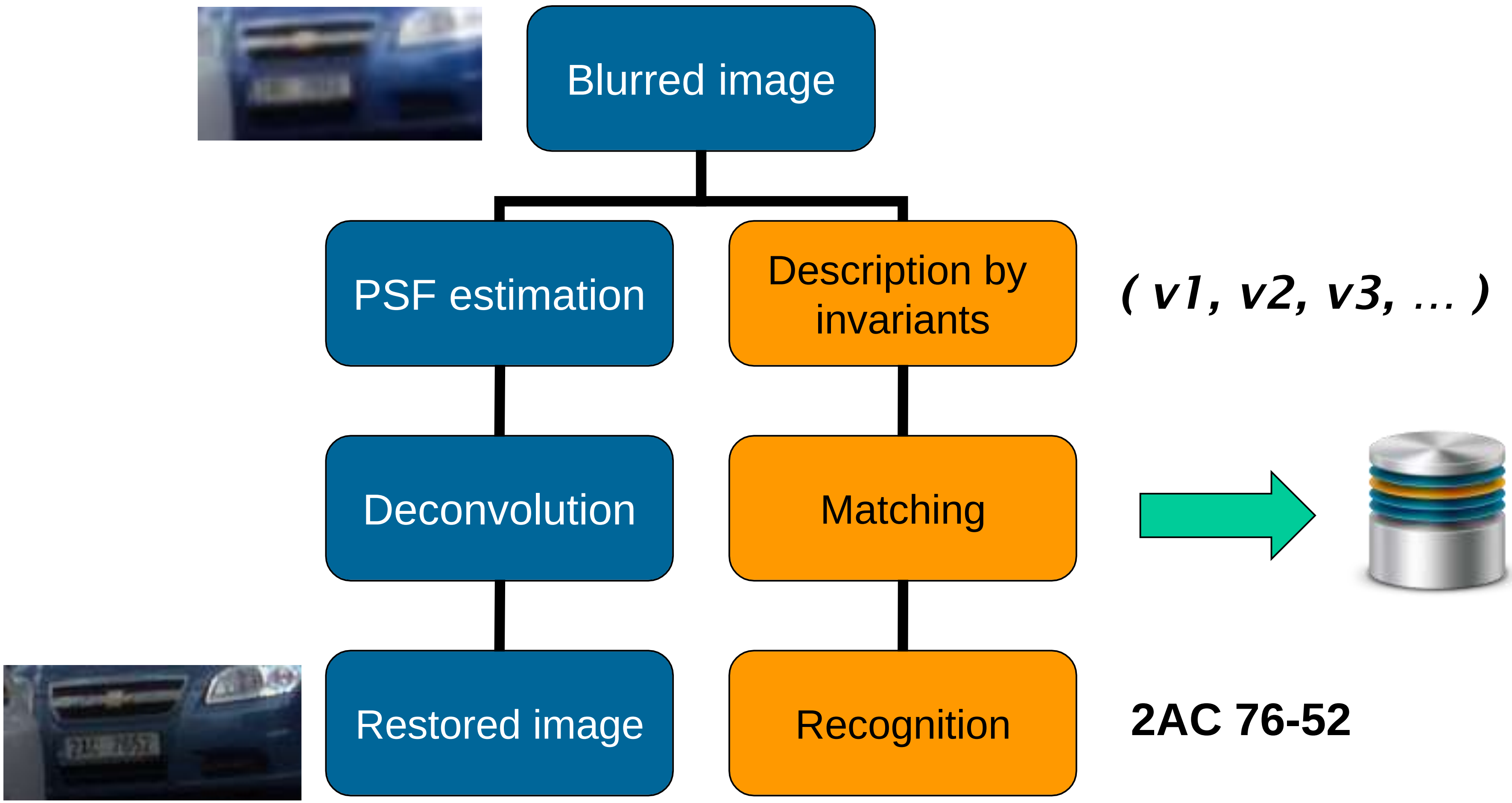
$$z(x) = (h * u)(x) + n(x)$$

h(x,y) is a PSF of the camera



Two approaches to the blur handling



Historical remark

Image restoration has been a very traditional area of image processing

A. Rosenfeld: *Picture Processing by Computer*, Academic Press, 1969

2016 - over 1 000 000 search results found at Google Scholar

Why is image restoration difficult?

- The problem is ill-conditioned and/or ill-posed

$$z(x) = (h * u)(x) + n(x)$$

- Too many unknowns
- Even if the PSF is known, the noise makes the task ill-conditioned and requires a special care (regularization)

Why is image restoration difficult?

- Even if we had a perfect deconvolution algorithm and no noise was present, there would be still a solution ambiguity

$$z(x) = ((h_1 * h_2 * \cdots * h_L) * u)(x)$$

- We need to incorporate our preferences/expectations/priors into the restoration algorithms

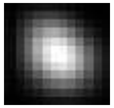
Traditional image restoration flowchart

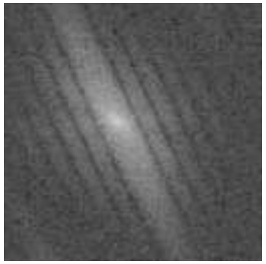
Blurred image

Traditional image restoration flowchart

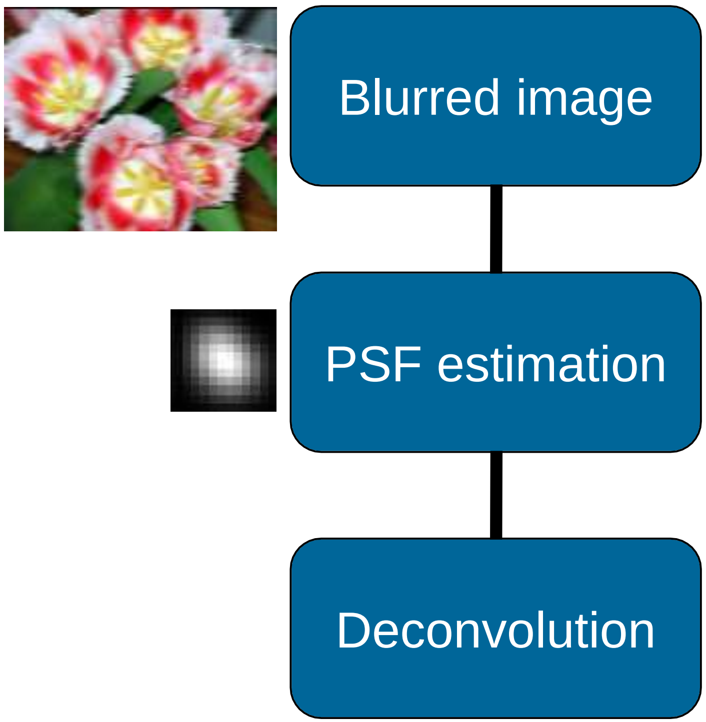
Blurred image

PSF estimation





Traditional image restoration flowchart



Traditional image restoration flowchart

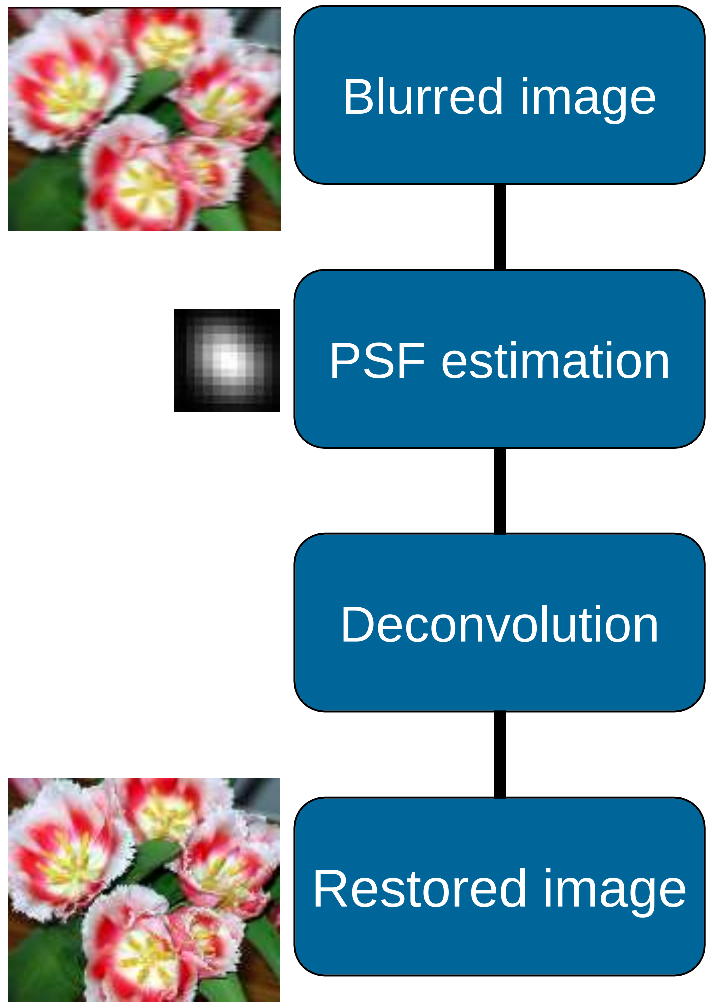
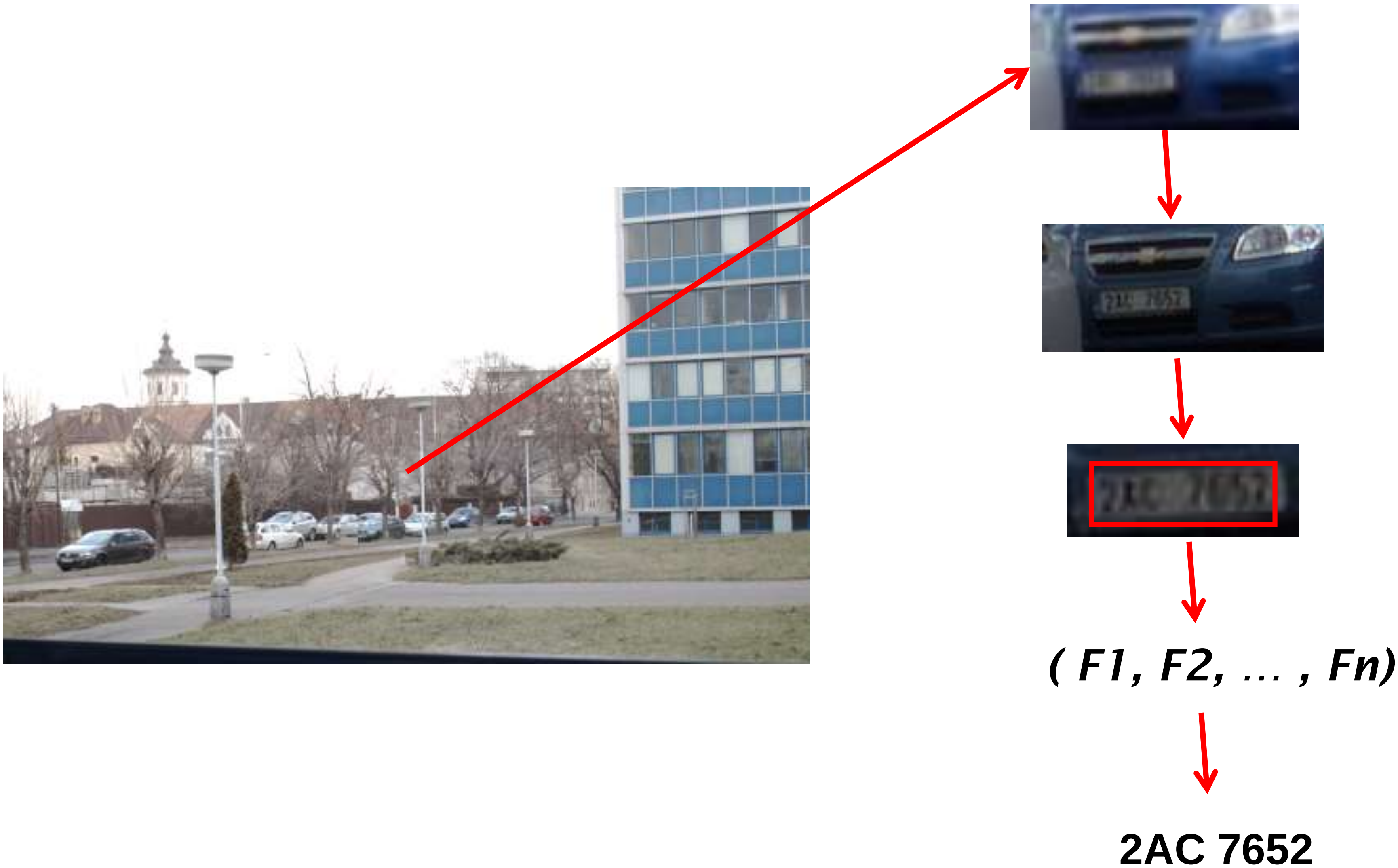


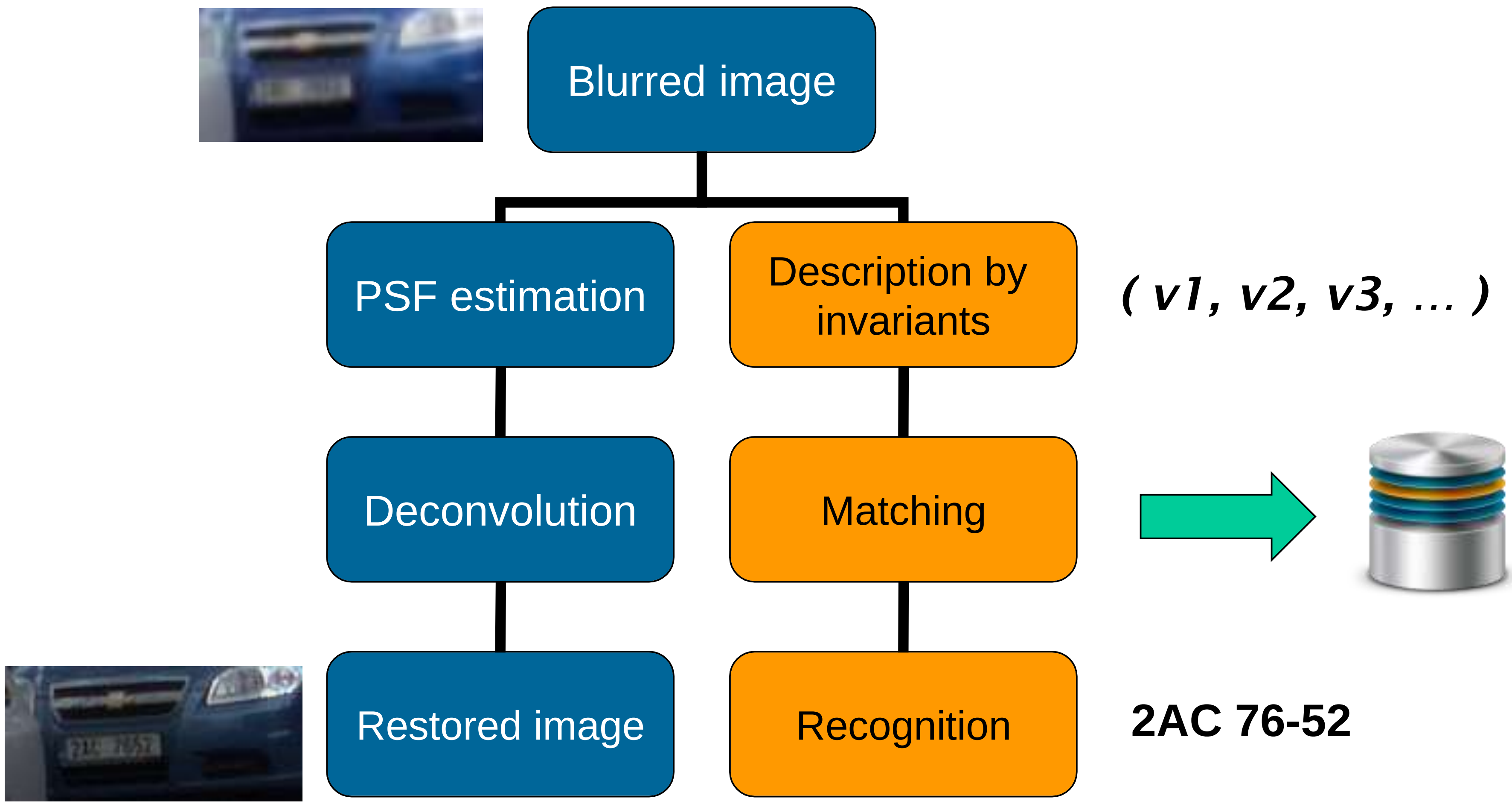
Image restoration categories

- **From a single image (single-channel)**
  - PSF is completely known
  - PSF is of a known parametric shape
  - PSF is constant and unknown
  - PSF is variable and unknown
- **From multiple images (multi-channel)**

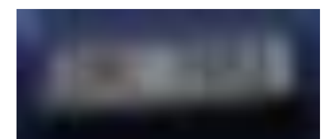
Analysis with restoration



Two approaches to the blur handling



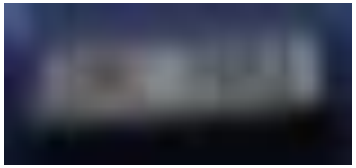
Analysis without restoration



(F1, F2, ... , Fn)

2AC 7652

Invariants to convolution



$$g(x,y) = (f * h)(x,y)$$

$$I(f * h) = I(f)$$

for any admissible *h*

Projection operators

$\mathcal{I}$ – the image space
$\mathcal{S} \subset \mathcal{I}$ – the PSF space closed under convolution
P : projection operator $\mathcal{I} \to \mathcal{S}$, $P^2 = P$

$\mathcal{I} = \mathcal{S} \oplus \mathcal{A}$, i.e. any f can be expressed as $f = Pf + f_A$
Assuming that $\mathcal{A}$ is also closed under convolution, we have

$$P(f * h) = Pf * h$$

Fundamental theorem of blur invariants

$$I(f) \equiv \frac{\mathcal{F}(f)}{\mathcal{F}(Pf)}$$

$$I(f*h) = \frac{\mathcal{F}(f*h)}{\mathcal{F}(P(f*h))} = \frac{F \cdot H}{\mathcal{F}(Pf) \cdot H} = I(f)$$

Invariants to convolution: Demo video

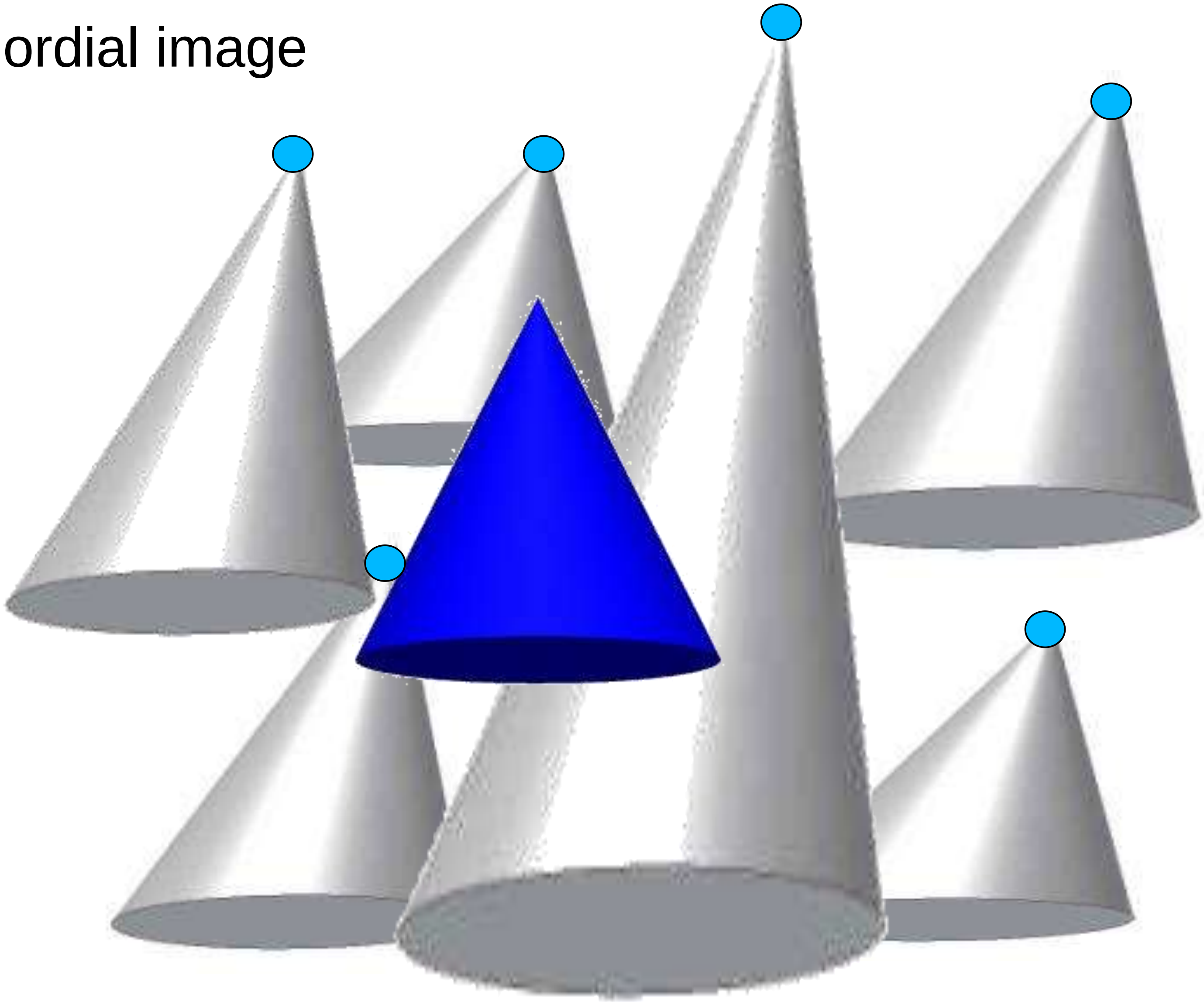
The meaning of the FTBI

$$I(f) \equiv \frac{\mathcal{F}(f)}{\mathcal{F}(Pf)}$$

- *I(f)* can be understood as a “deconvolved” image *f* by *Pf* in Fourier domain
- IFT of *I(f)* is a “primordial image”
- Each primordial image defines a semi-orbit, which is a blur-equivalent class

Semi-orbits in the image space

- Primordial image



Discrimination power of *I(f)*

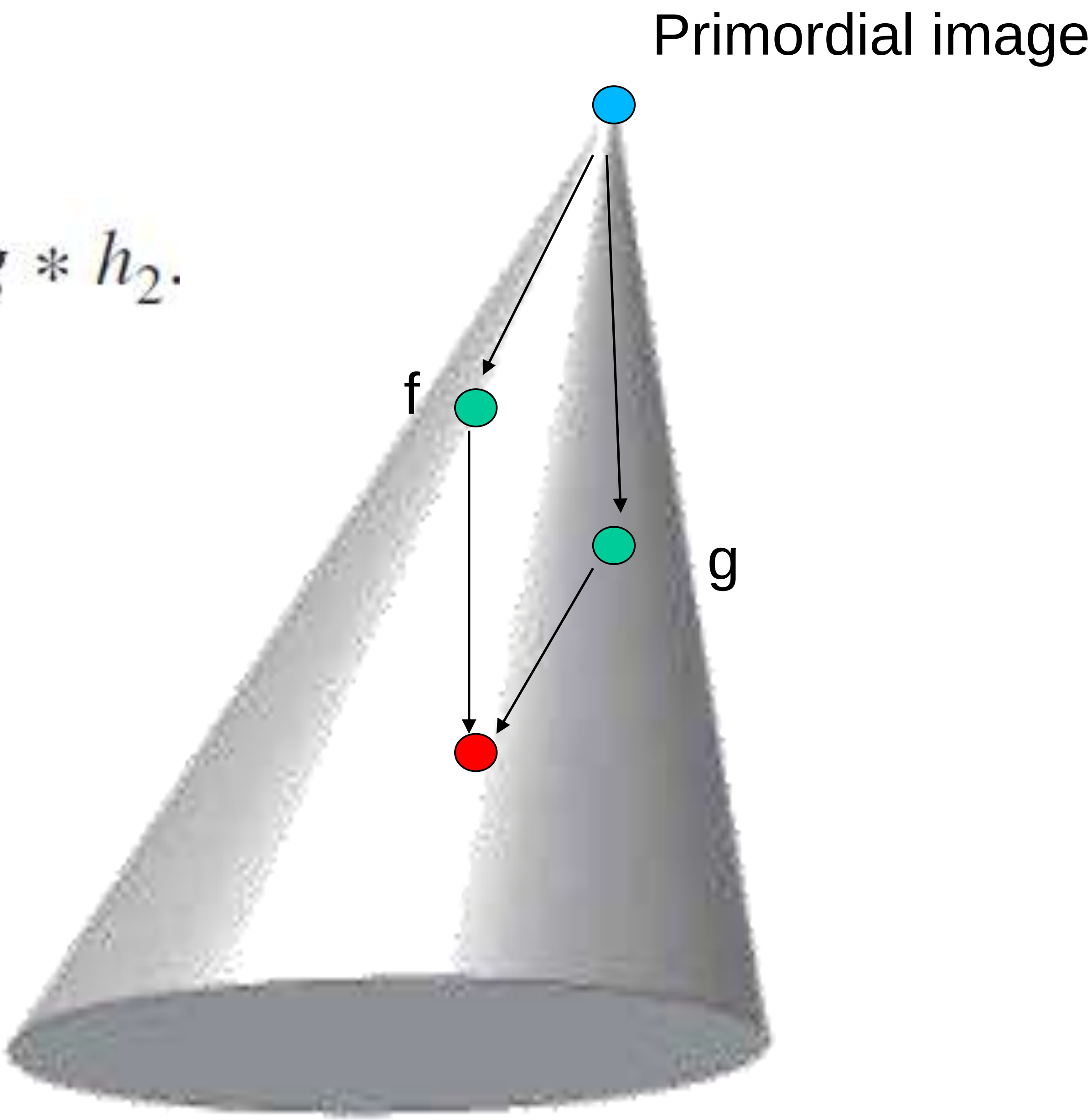
$$I(f) \equiv \frac{\mathcal{F}(f)}{\mathcal{F}(Pf)}$$

Theorem 6.2 Let $f, g \in \mathcal{I}$ such that $I(f) = I(g)$ almost everywhere. Then there exist functions $h_1, h_2 \in S$ such that

$$f * h_1 = g * h_2.$$

- *I(f)* can never distinguish *f* and *g* with the same primordial image, but can do so in all other cases
- In particular, we cannot distinguish any two elements of *S*

$$f * h_1 = g * h_2.$$



Applicability of the FTBI

- What PSF spaces *S* are closed under convolution?
- For which *S* can we construct *P* ?
- Are there any *S* and *P* such that the invariants given by FTBI are non-trivial? If yes, do they correspond with real-life PSF's?
- Can we calculate the invariants directly in the image domain rather than in the FT domain?

The answers in the last part of the tutorial.

Thank you very much.

The end of the first part