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Prof. Ing. Jan Flusser, DrSc.

Digitální zpracování obrazu
Lecture 1

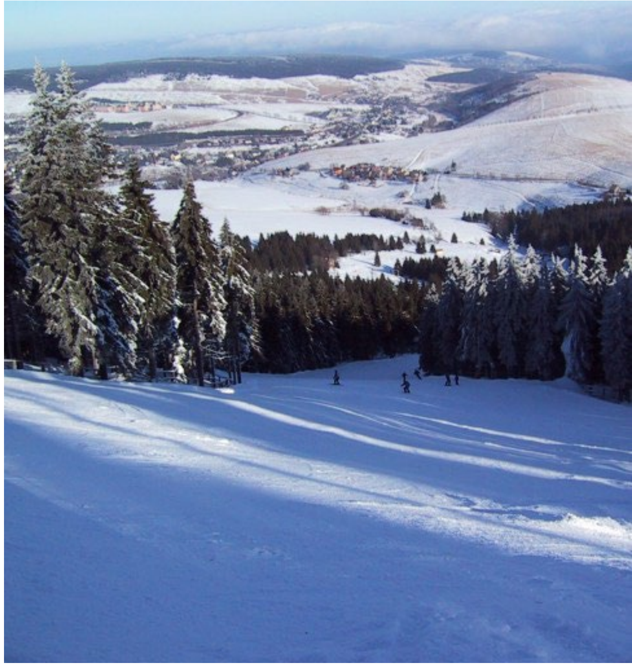
Důležité informace

- Rozsah: ZS, 3+0, Zk
- http://zoi.utia.cz/teaching

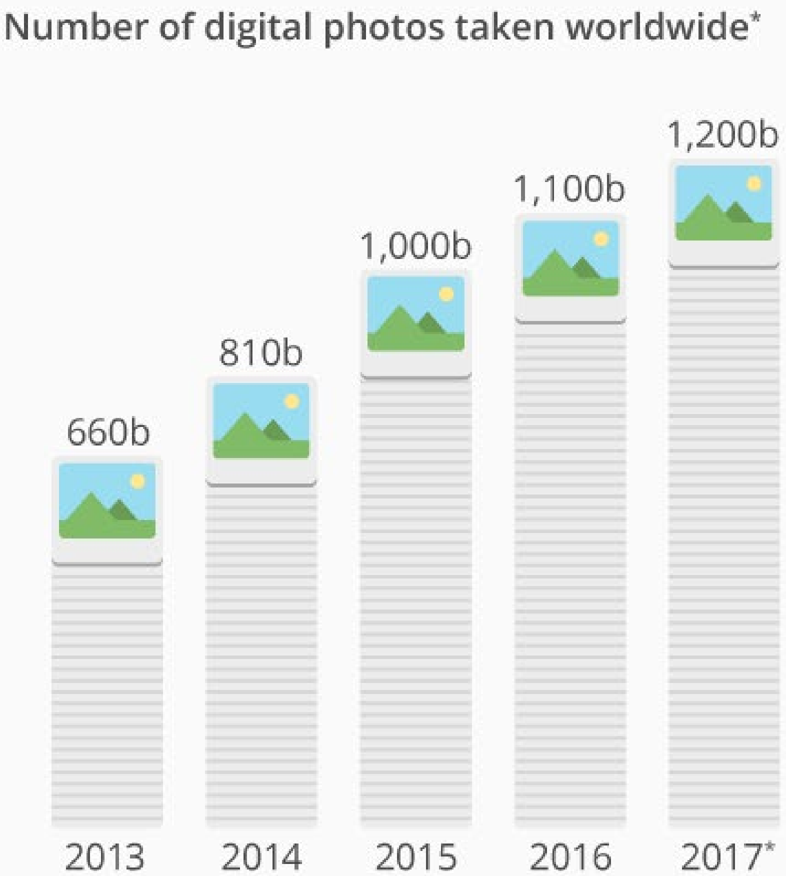
Navazující předměty

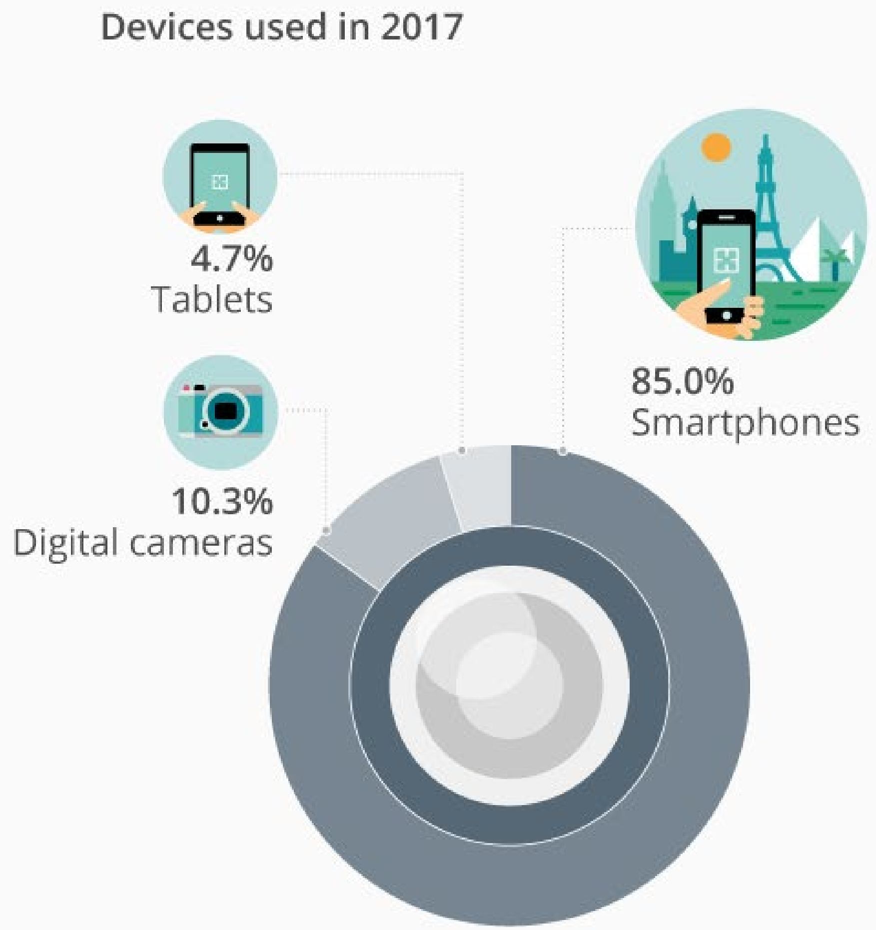
- **NPGR013 (J. Flusser, B. Zitová)**
- **NPGR022 (J. Flusser, B. Zitová)**
- **NPGR029 (F. Šroubek, J. Flusser)**
- **NPGR032 (A. Novozámský, B. Zitová)**

Digitálních fotografií je všude plno ...



Smartphones Cause Photography Boom

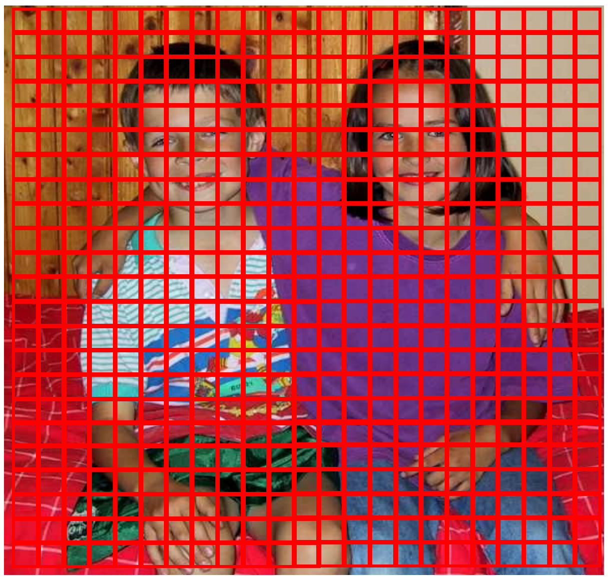




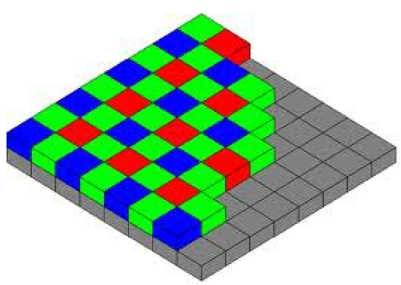
* estimates

Source: InfoTrends via Bitkom

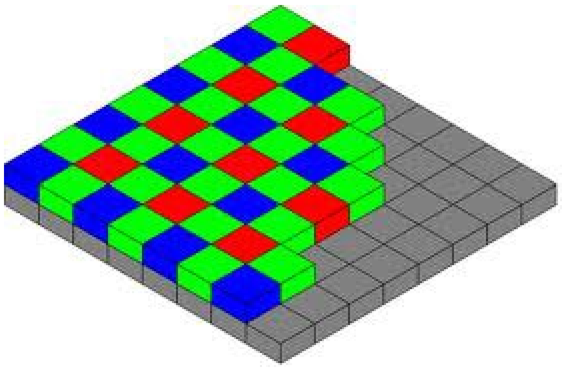
Jak vzniknou?

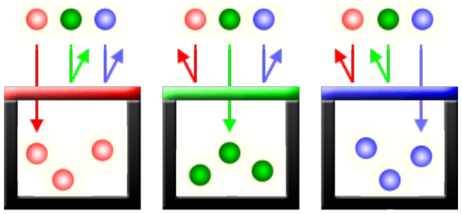


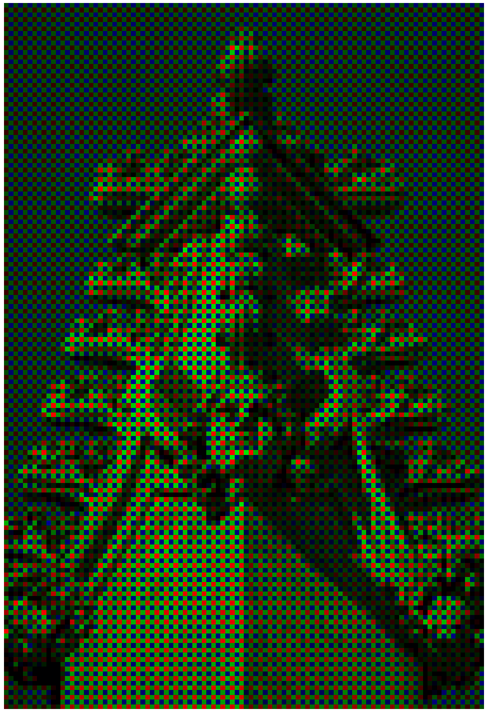
$$\begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$



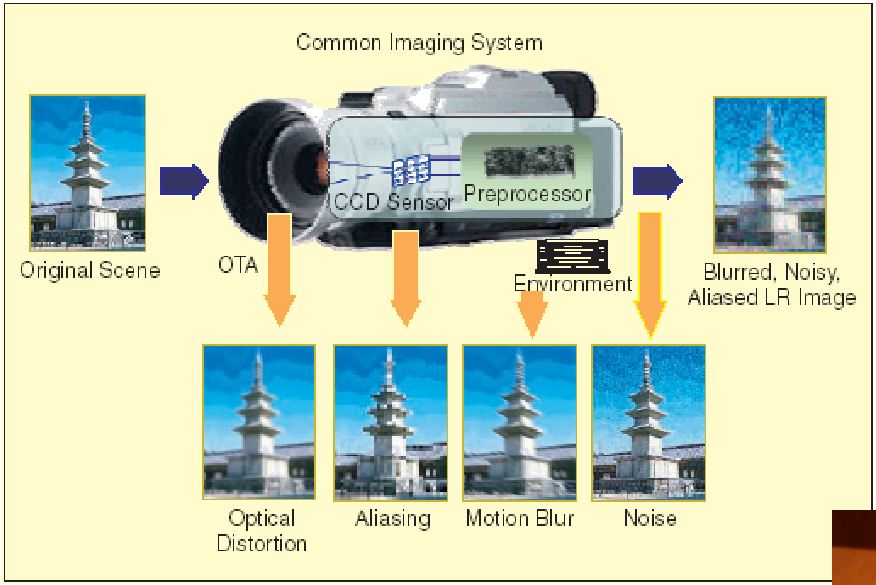
Jak se tam dostanou barvy?





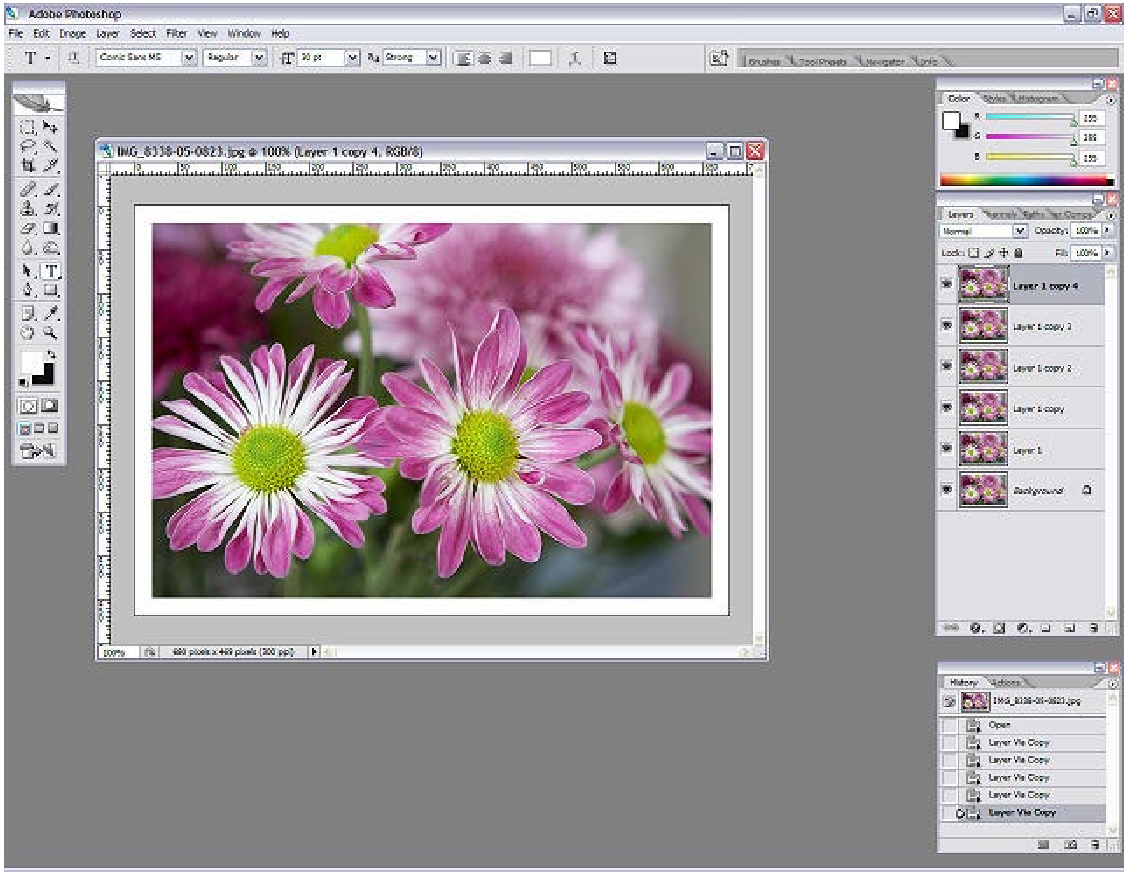


Fotografie nebývají ideální



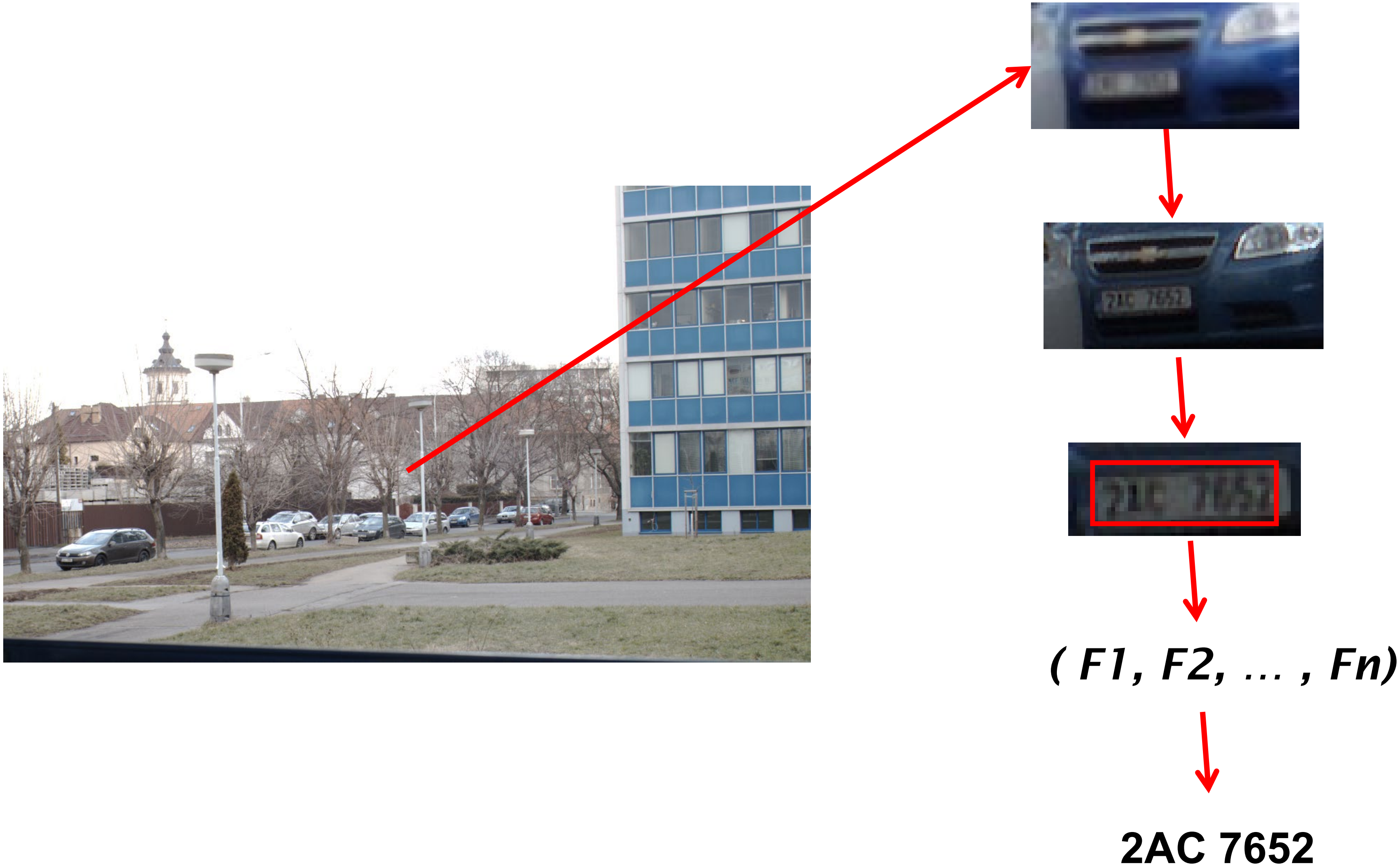
Jak vylepšit nekvalitní foto?

Všichni známe Photoshop ...



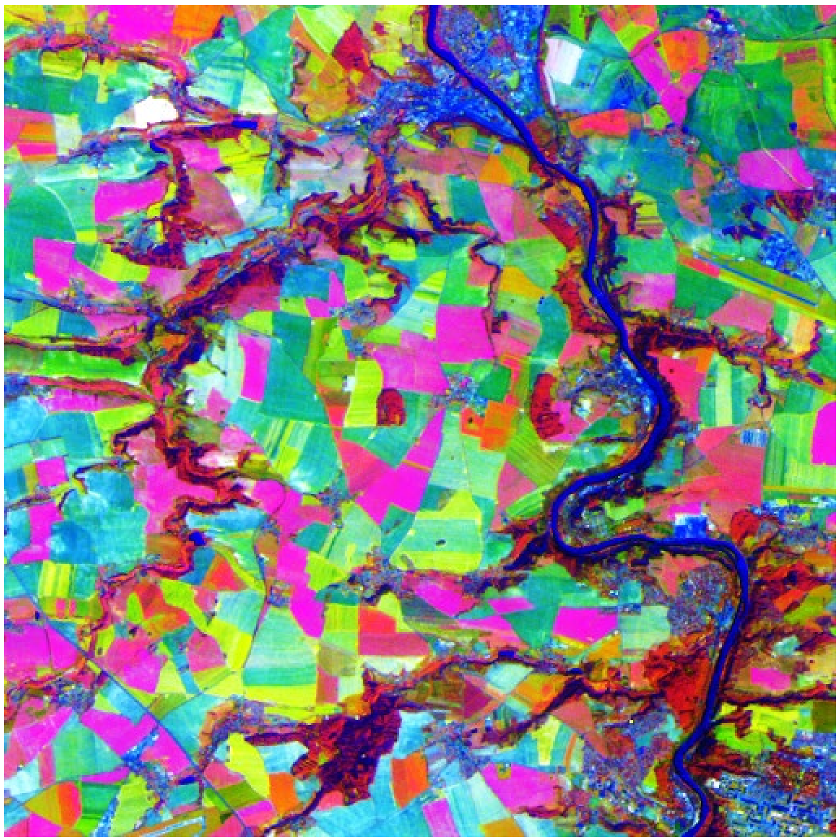
... na jednoduché úpravy stačí

Kde Photoshop nestačí

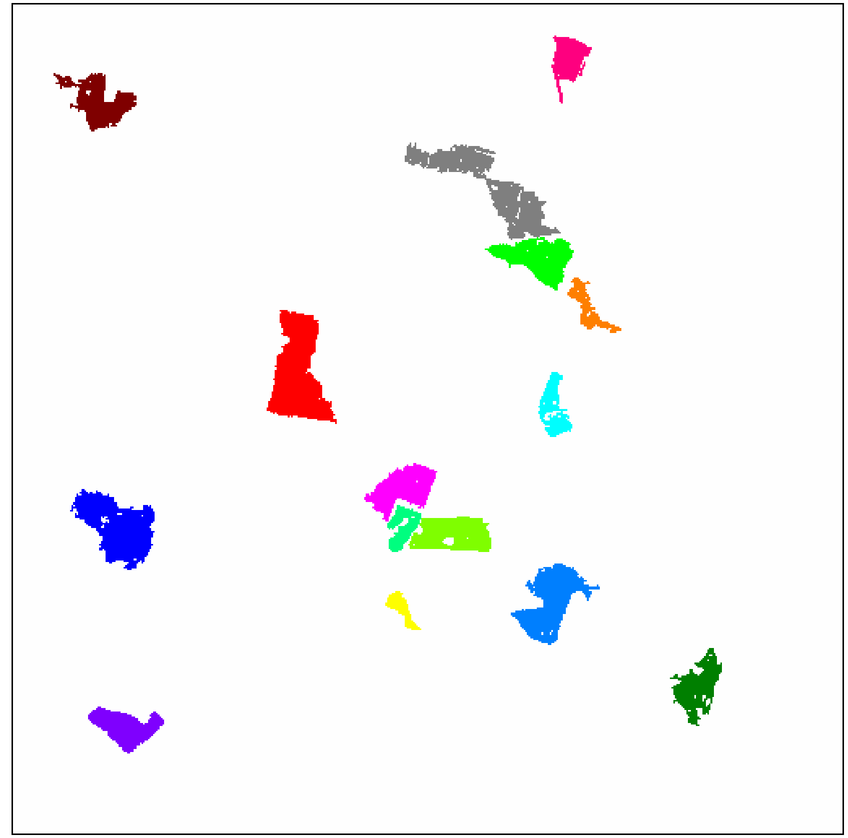


Jak odstranit třes ruky?

Segmentace

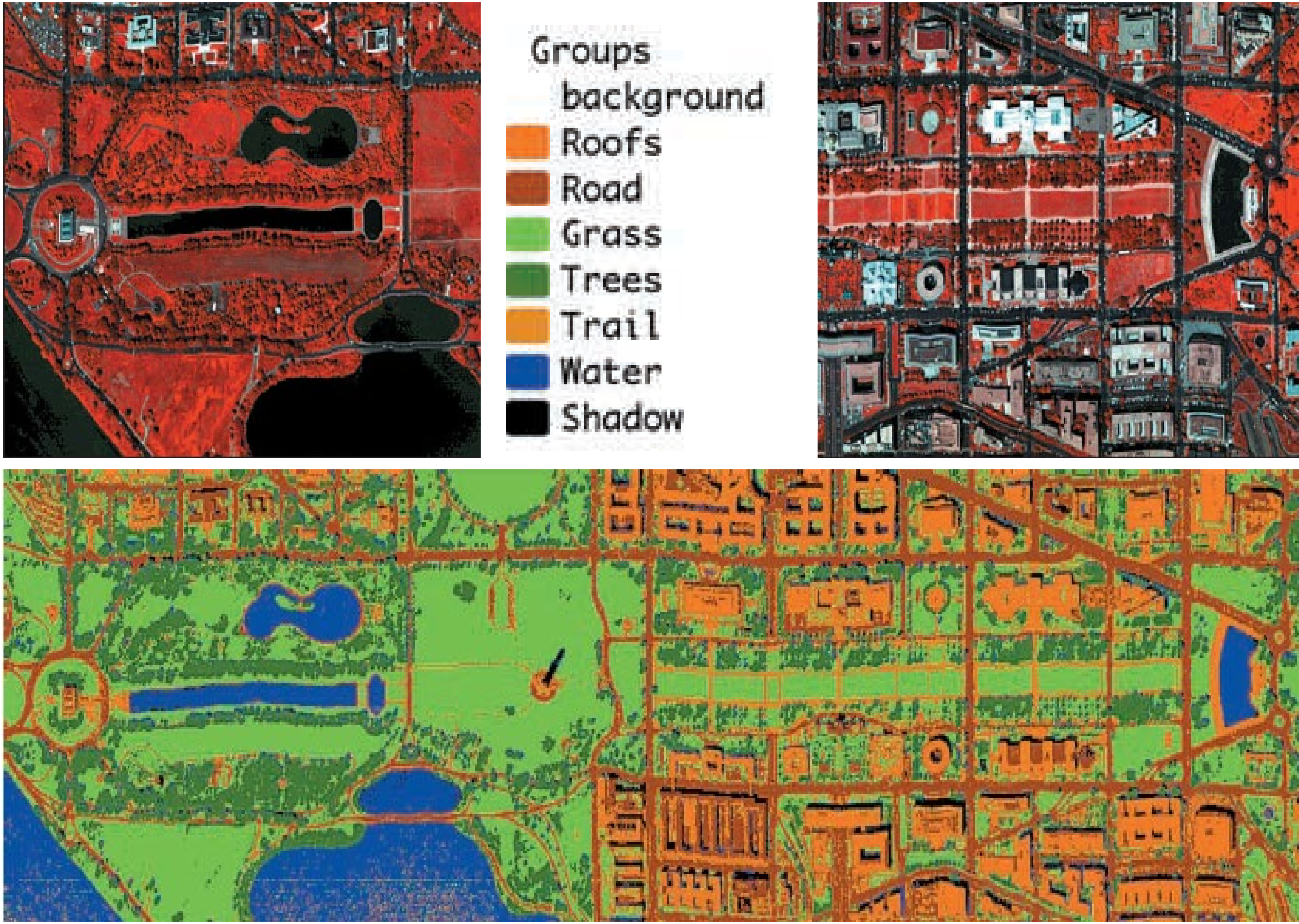


Originál



Detekce objektů

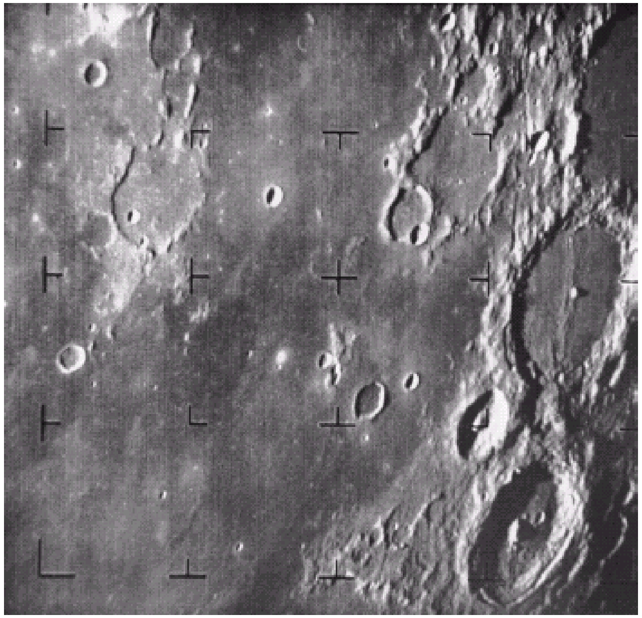
Úplná


Groups
background
Roofs
Road
Grass
Trees
Trail
Water
Shadow

Historie

1921

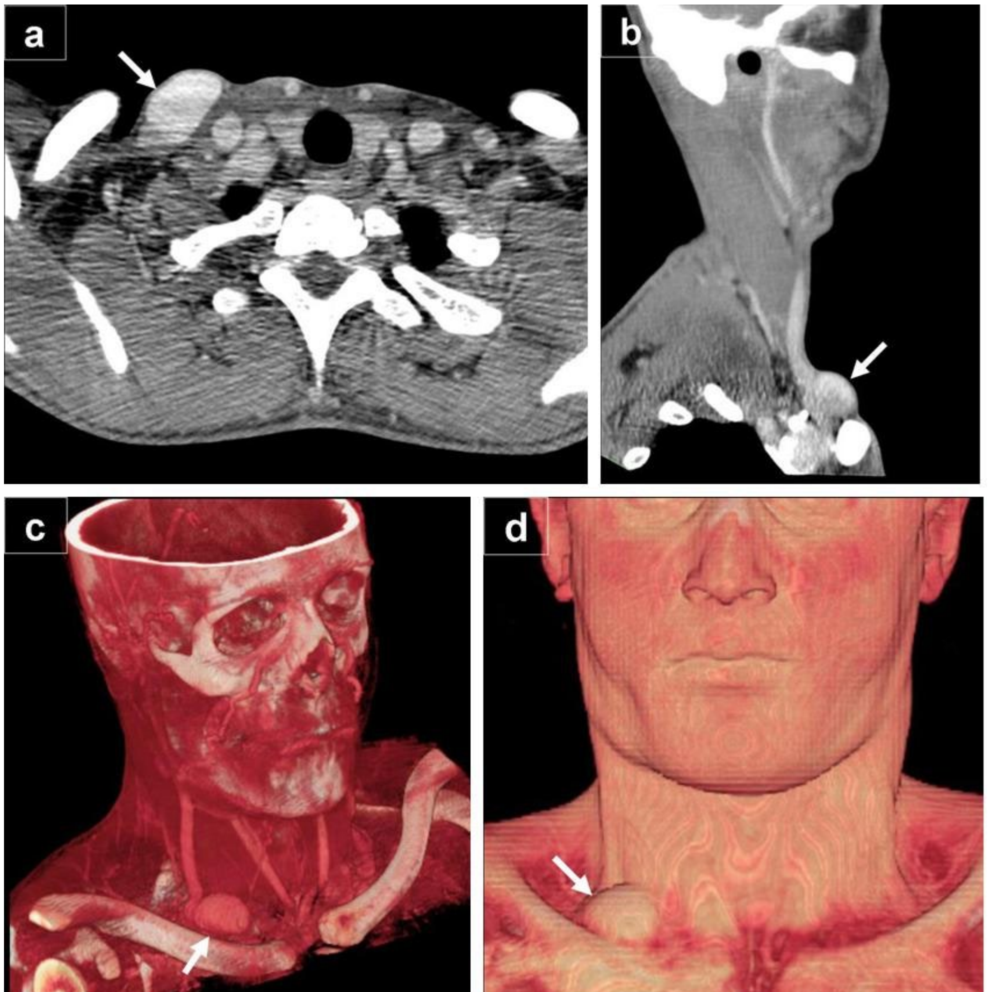
1929



1964

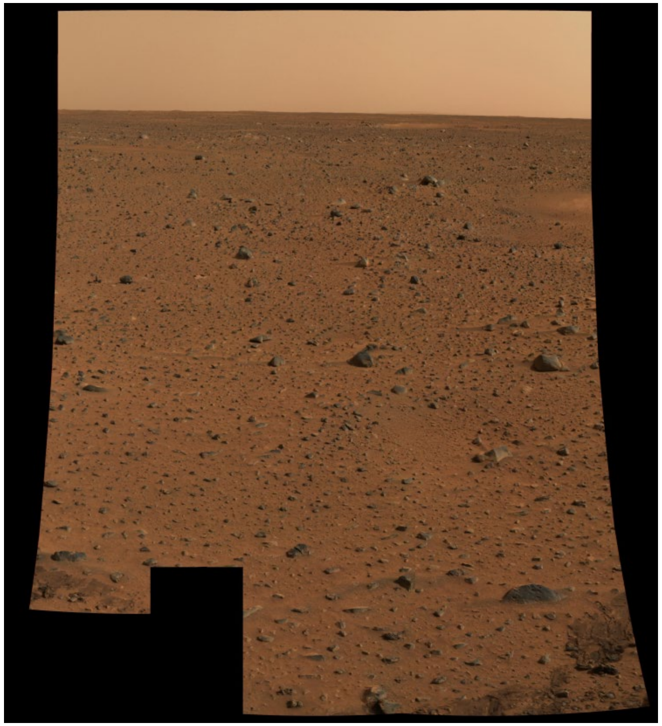
Aplikační oblasti

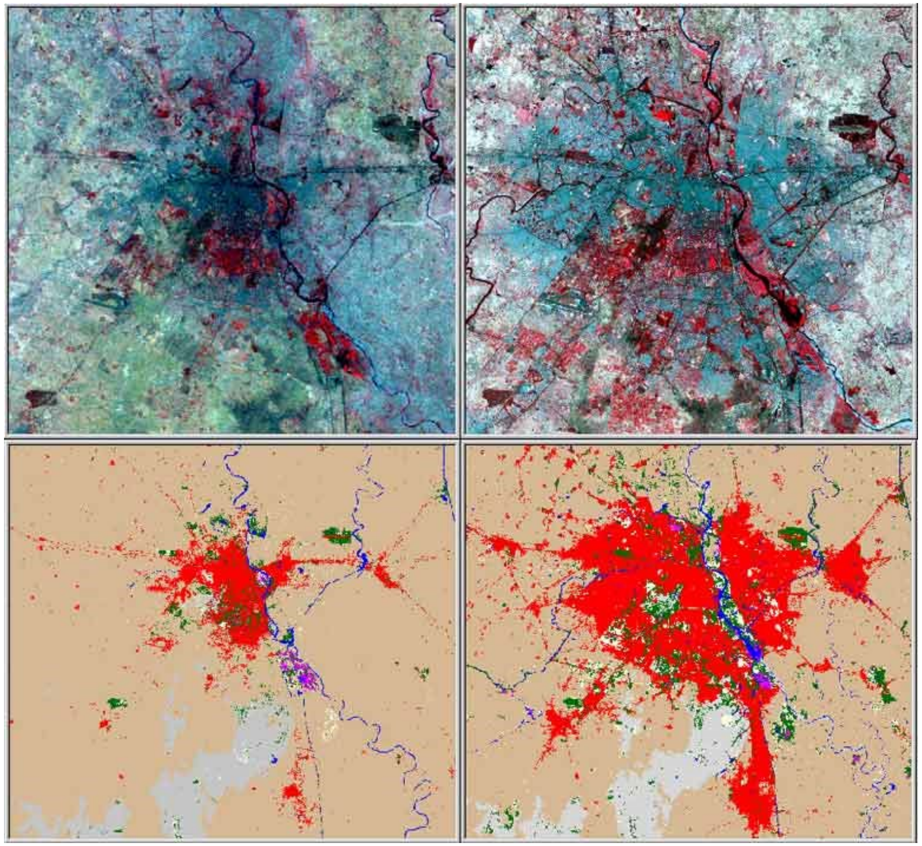
- medicína



Aplikační oblasti

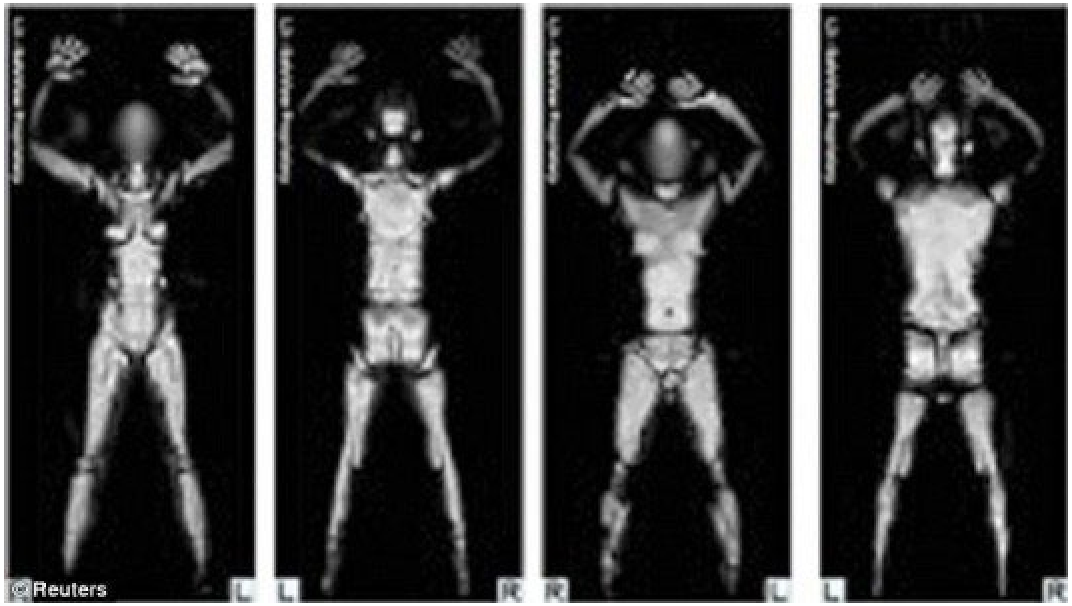
- Dálkový průzkum





Aplikační oblasti

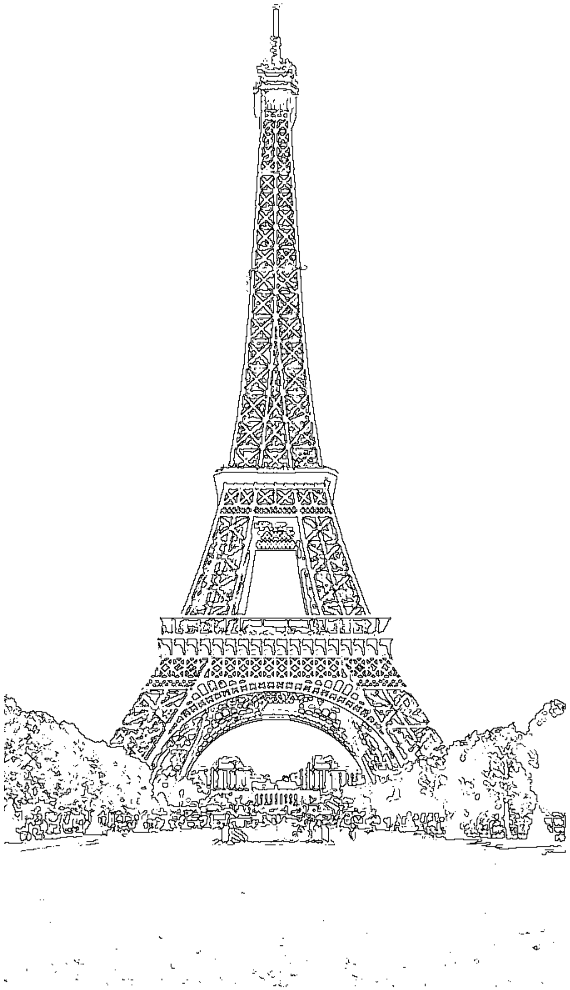
- Bezpečnost, doprava



Aplikační oblasti

- Kulturní dědictví

Image Proc. x Comput. Graphics



Digital Image Processing

- Digitization (sampling + quantizing)
- Preprocessing (contrast and brightness changes, denoising, sharpening, …)
- Image analysis (object detection and recognition, scene understanding)
- Image coding (compression)

Mathematical background

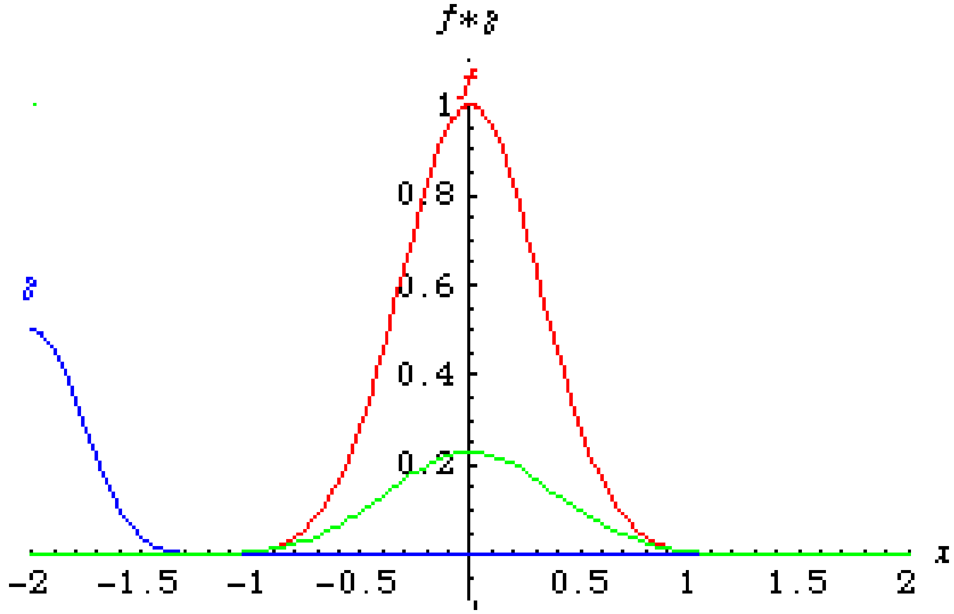
- Convolution
- Fourier Transform

Convolution

- Definition in continuous domain
- Properties
- Delta function
- Discrete convolution, boundary effect

Convolution

$$* : L_1 \times L_1 \to L_1 \qquad (f * g)(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt$$



Basic properties

$$f * g = g * f$$

$$f * (g * h) = (f * g) * h$$

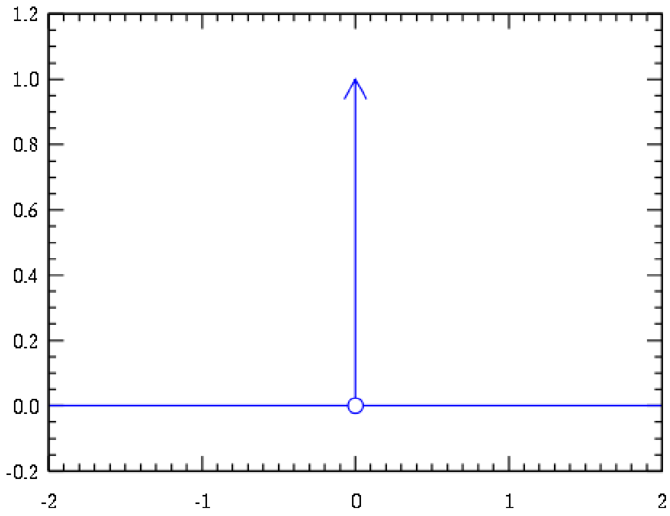
$$a(f * g) = (af) * g = f * (ag)$$

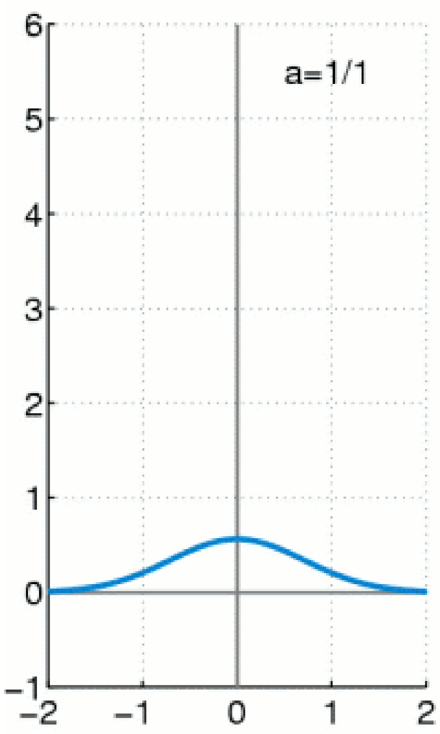
$$f * (g + h) = (f * g) + (f * h)$$

Dirac delta-function

$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int \delta(x)\, \mathrm{d}x = 1$$





Delta-function properties

$$\delta(ax) = \frac{\delta(x)}{|a|}$$

$$f * \delta = f$$

$$(f * \delta)(a) = \int_{-\infty}^{\infty} f(x)\delta(a - x)dx = f(a)$$

$$(f * \delta)(x - a) = f(x - a)$$

Cross-correlation

$$(f \star g)(x) = \int_{-\infty}^{\infty} f(t)g(t-x)dt$$

$$f \star g \neq g \star f$$

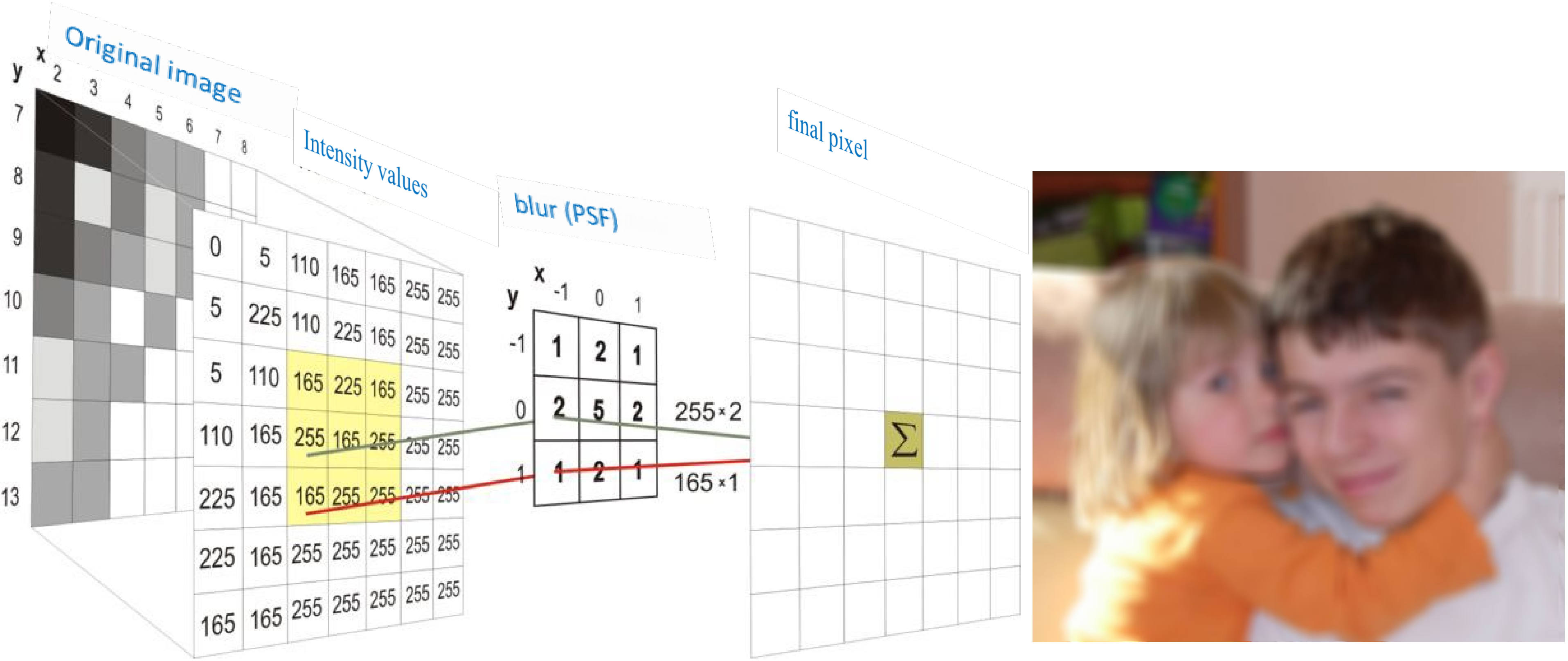
Convolution in 2D

- Definition $* : L_1 \times L_1 \rightarrow L_1$

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s, t) g(x - s, y - t) ds dt$$

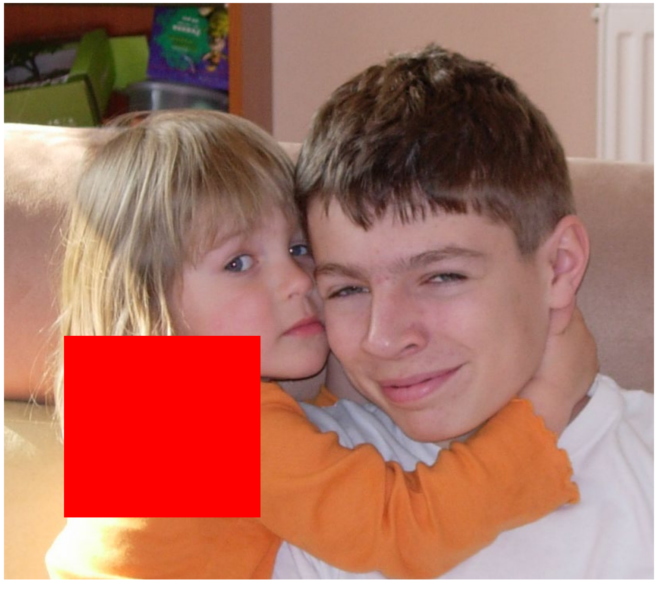
Discrete convolution

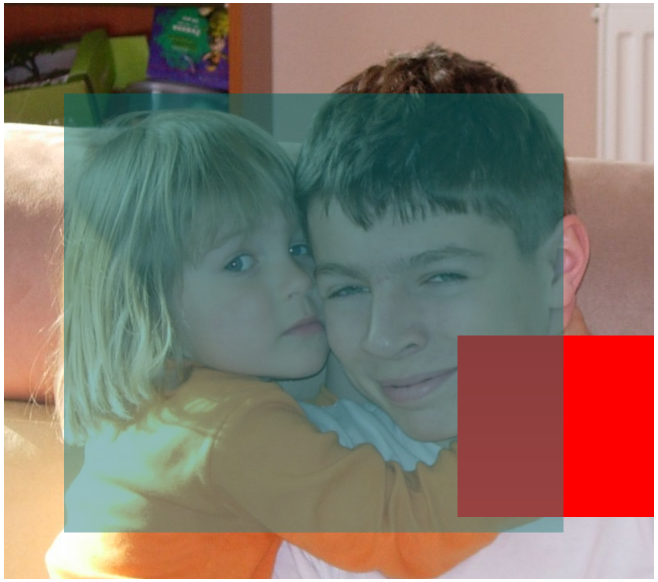
$$(f * g)(n) = \sum_{m=-\infty}^{\infty} f[m] \cdot g[n-m]$$



Boundary effect

Convolution mask





Type
“valid”

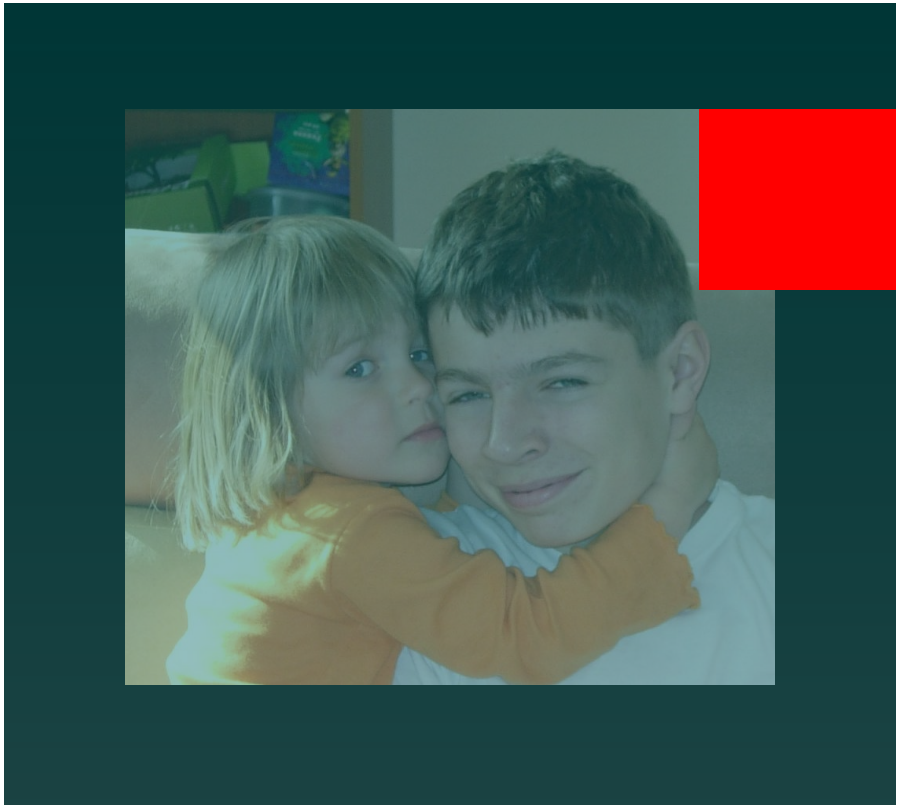
Type
“same”

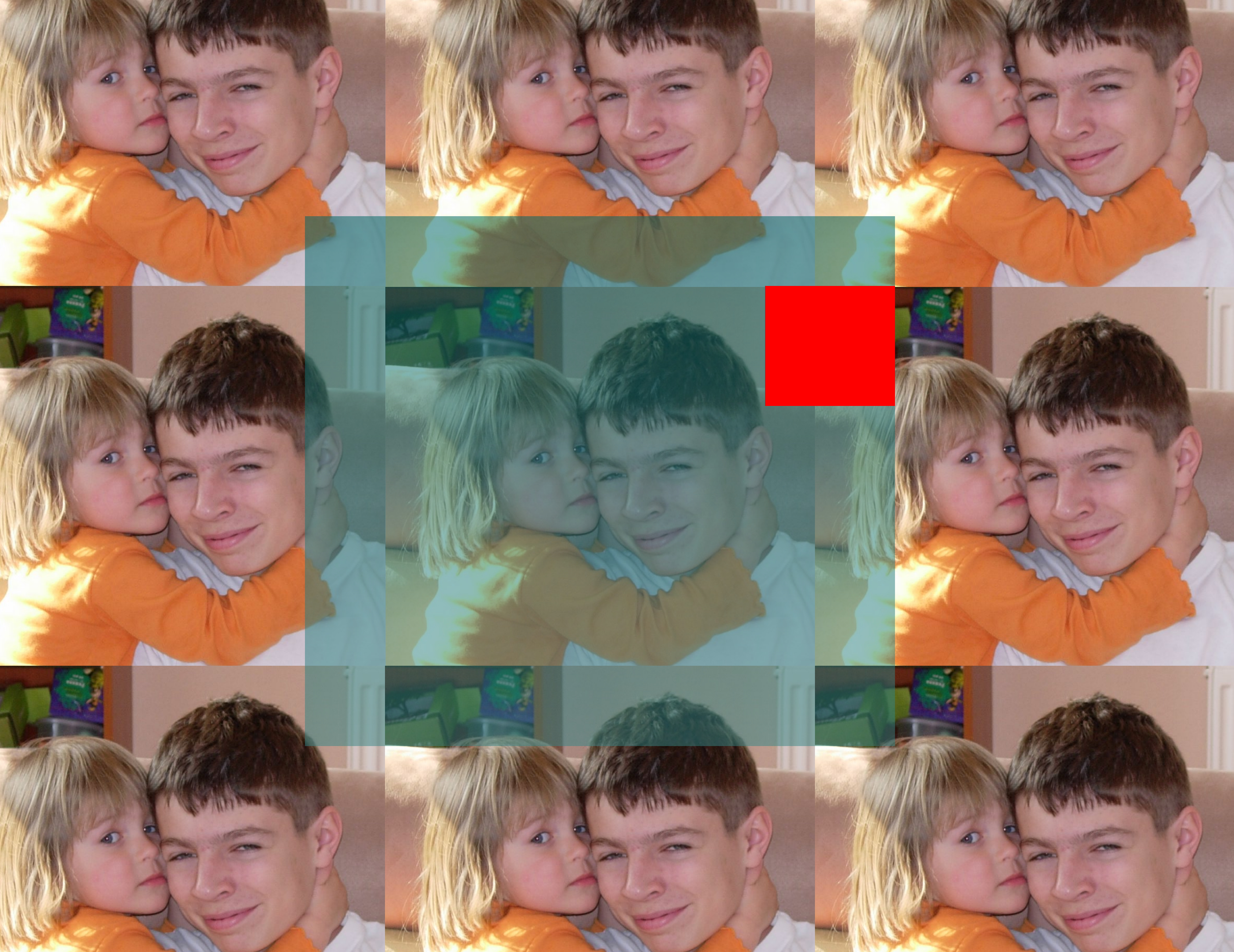
Type

“full”

Boundary effect treatment

- Zero padding
- Mirror extension
- Periodic extension





Computing complexity

- From definition – O(N^2.M^2)
- For a separable kernel (rank = 1) – O(N^2.M)

$$g(x,y) = g_1(x) \cdot g_2(y)$$

$$(f*g)(x,y) = \int_{-\infty}^{\infty} g_1(x-s)\left(\int_{-\infty}^{\infty} f(s,t)g_2(y-t)dt\right)ds$$

- Hardware acceleration (DSP, graphic cards, ...)

Fourier transform

Recalling Fourier series

$\{\phi_k\}$ – ON basis of a Hilbert space $\mathcal{H}$, $\quad f \in \mathcal{H}$

$c_k = (f, \phi_k)$ – Fourier coefficients

$$f = \sum_{i=1}^{\infty} c_k \phi_k$$

Fourier series in $L_2(-\pi, \pi)$

$$\{\phi_k\} = \{1, \sin nx, \cos mx\}$$

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx \, \mathrm{d}x \qquad b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx \, \mathrm{d}x$$

Fourier transform

$$\{\phi_u(x)\} = \{e^{-2\pi i(ux)}\}$$

$$F(u) = \int_{\mathrm{R}} f(x) e^{-2\pi iux} \mathrm{d}x$$

$$f(x) = \int_{\mathrm{R}} F(u) e^{2\pi iux} \mathrm{d}u$$

Properties of the FT

If $f \in L_1$ then FT exists

Generally $F \notin L_1$

FT is "one-to-one" mapping: $f \longleftrightarrow F$

$$F = \mathcal{R}(F) + i\mathcal{I}(F) = |F| \cdot e^{i \cdot ph(F)}$$

Properties of the FT

- **linearity**

$$\mathcal{F}[af(x) + bg(x)] = a\mathcal{F}[f(x)] + b\mathcal{F}[g(x)] = aF(k) + bG(k).$$

- **convolution** — convolution theorem

$$\mathcal{F}[f]\mathcal{F}[g] = \mathcal{F}[f * g]$$

- **shift** — shift theorem

$$\mathcal{F}_x[f(x - x_0)](k) = e^{-2\pi i k x_0} F(k).$$

- **rotation**

$$\mathcal{F}(R(f)) = R(\mathcal{F}(f))$$

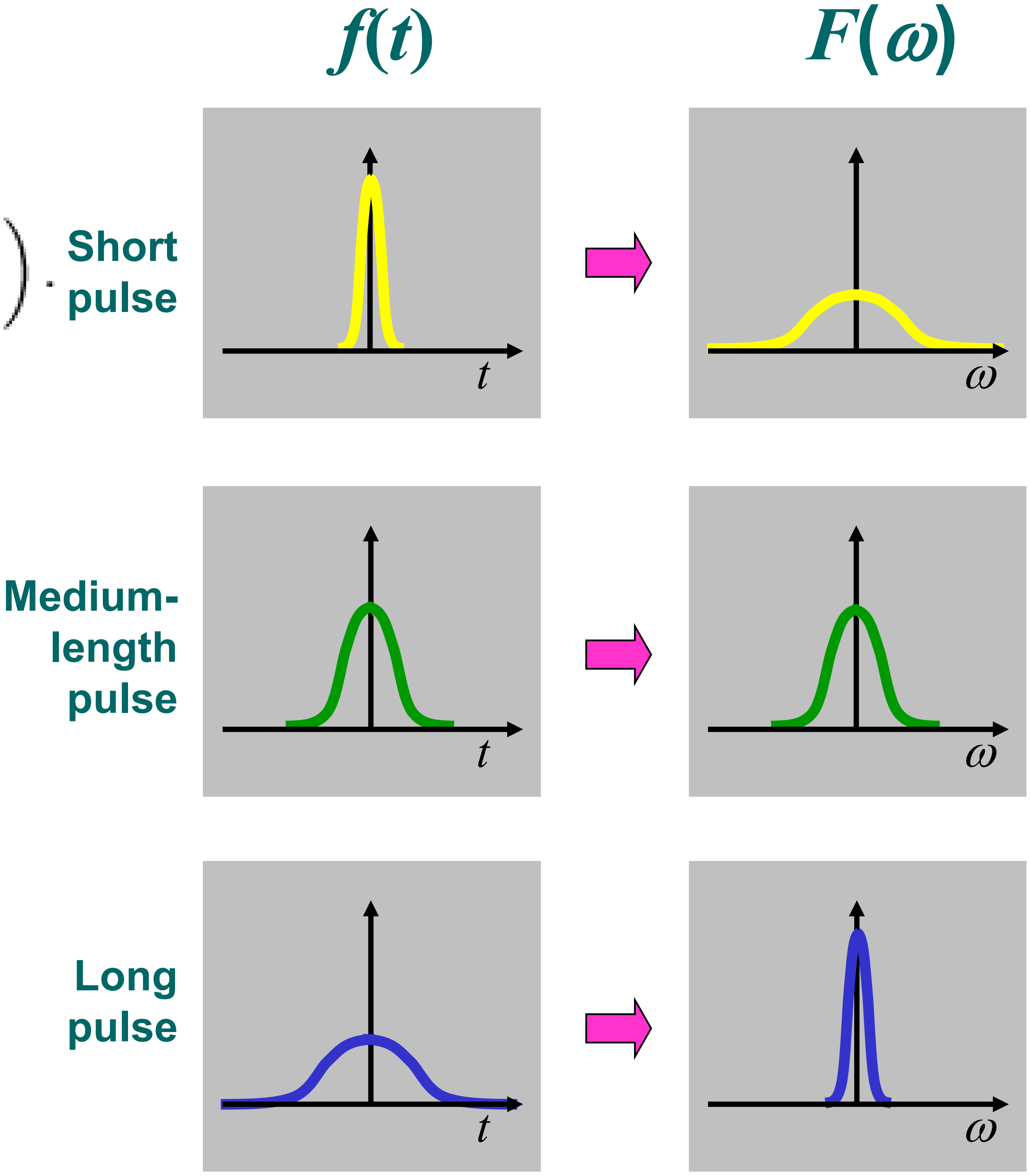
- **scaling** — similarity theorem

$$\mathcal{F}_x[f(ax)](k) = |a|^{-1} F\left(\frac{k}{a}\right).$$

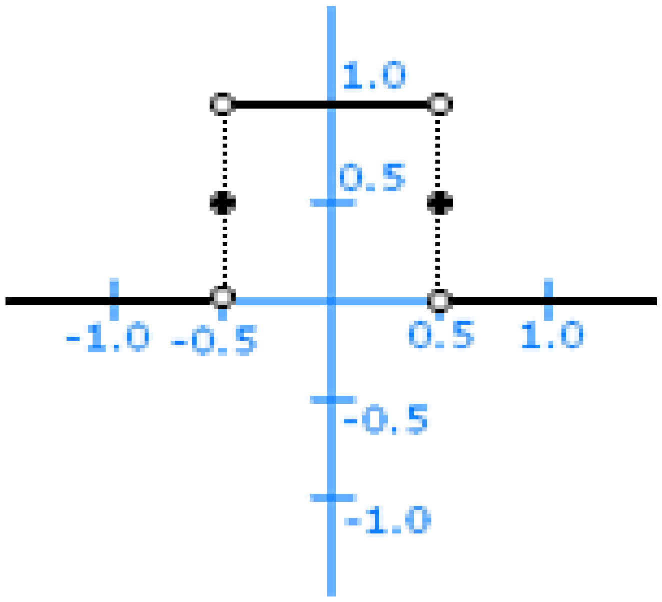
scaling

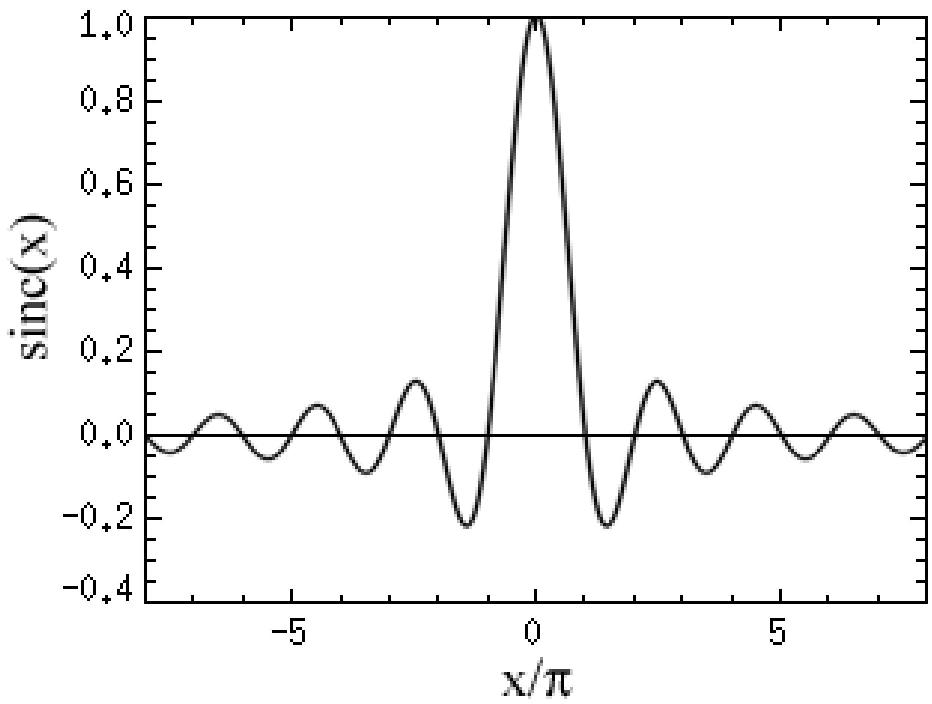
similarity theorem

$$\mathcal{F}_x[f(ax)](k) = |a|^{-1} F\left(\frac{k}{a}\right).$$

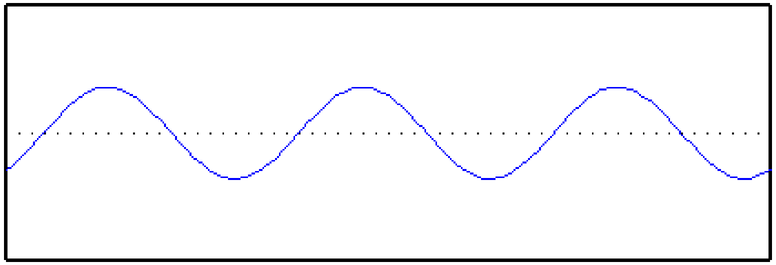


Rectangular pulse





Harmonic wave

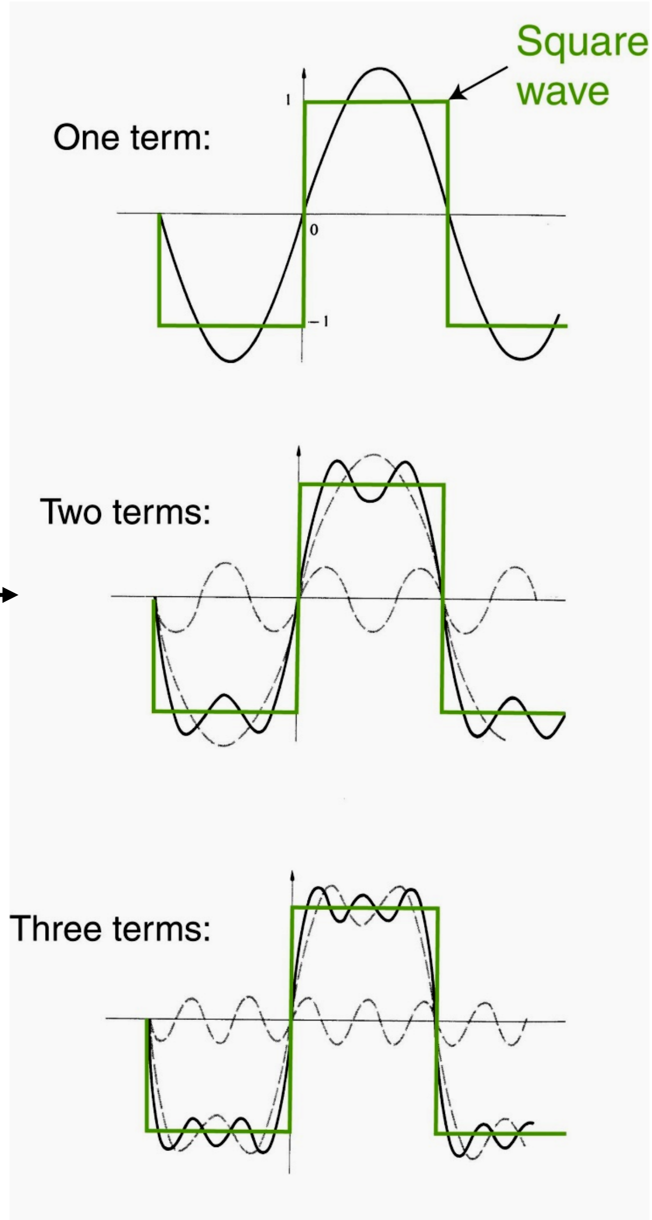


FT is a delta-function properly shifted

Discrete Fourier Transform

$$F_n \equiv \sum_{k=0}^{N-1} f_k e^{-2\pi i n k/N}.$$

$$f_k = \frac{1}{N} \sum_{n=0}^{N-1} F_n e^{2\pi i n k/N}.$$



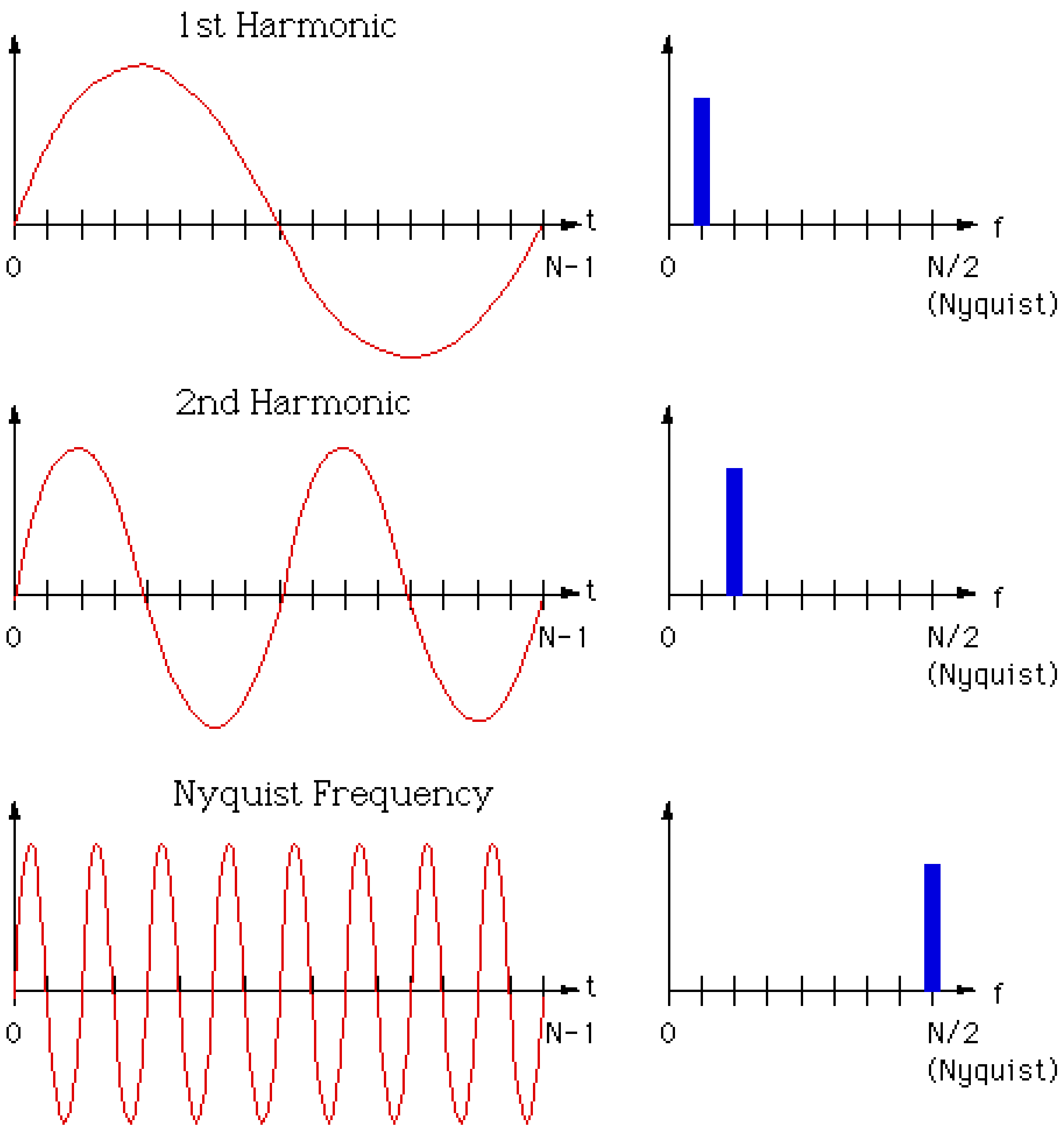
sin(*wt*)

sin(3wt)

sin(5wt)

Discrete Fourier Transform

- What are the frequencies in DFT?
- Which frequency is the lowest/highest one?
- Formally DFT exists for any *n* → periodicity. Only *N* samples are independent.


1st Harmonic
t
0
N-1
f
0
N/2
(Nyquist)
2nd Harmonic
t
0
N-1
f
0
N/2
(Nyquist)
Nyquist Frequency
t
0
N-1
f
0
N/2
(Nyquist)

DFT calculation

$$F_n \equiv \sum_{k=0}^{N-1} f_k e^{-2\pi i n k/N}$$

Directly - *O(N^2)*

FFT - O(N logN)

(Cooley and Tookey 1965, Gauss 1866)

Compare to the complexity of convolution.

2D Fourier transform

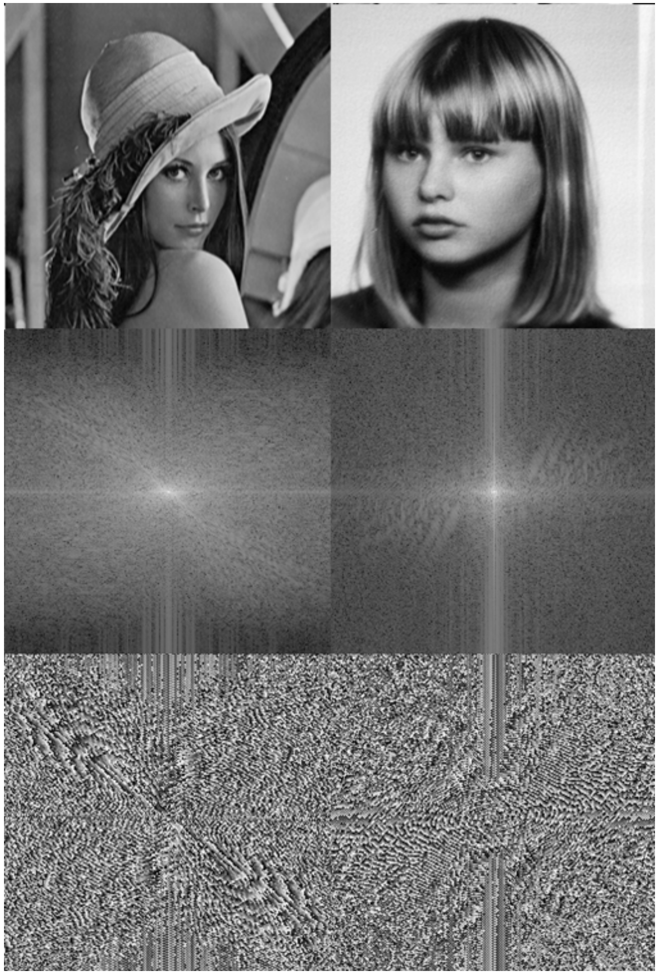
$$\{\phi_{uv}(x,y)\} = \{e^{-2\pi i(ux+vy)}\}$$

$$F(u,v) = \iint_{\mathrm{R}^2} f(x,y)e^{-2\pi i(ux+vy)}\mathrm{d}x\mathrm{d}y$$

$$f(x,y) = \iint_{\mathrm{R}^2} F(u,v)e^{2\pi i(ux+vy)}\mathrm{d}u\mathrm{d}v$$

2D Fourier transform is separable

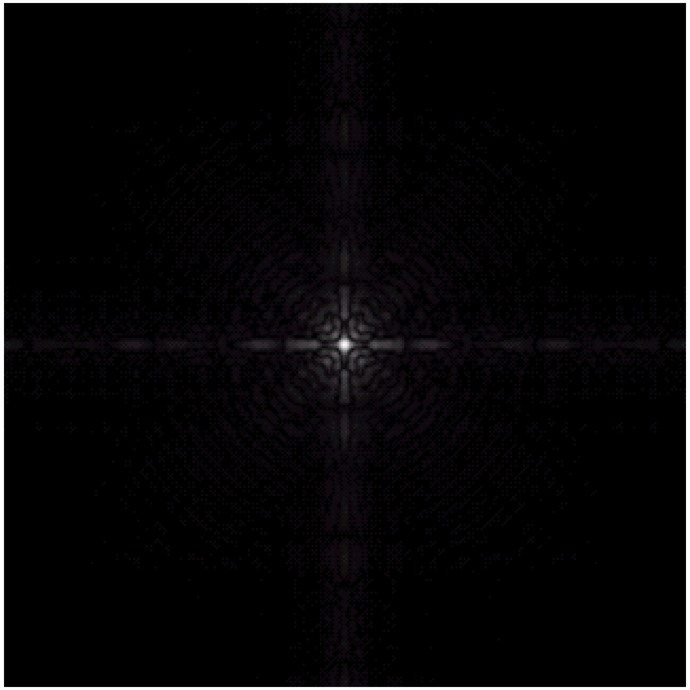
Visualisation



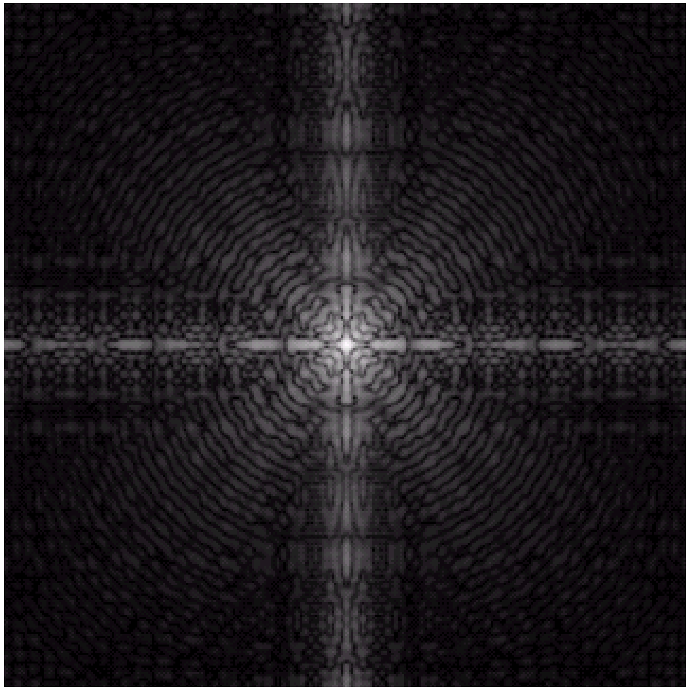
Amplitude

Phase

Visualisation

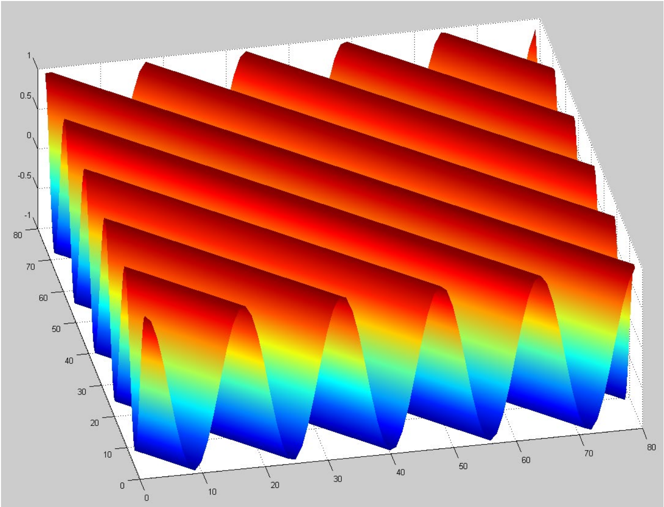


ampl



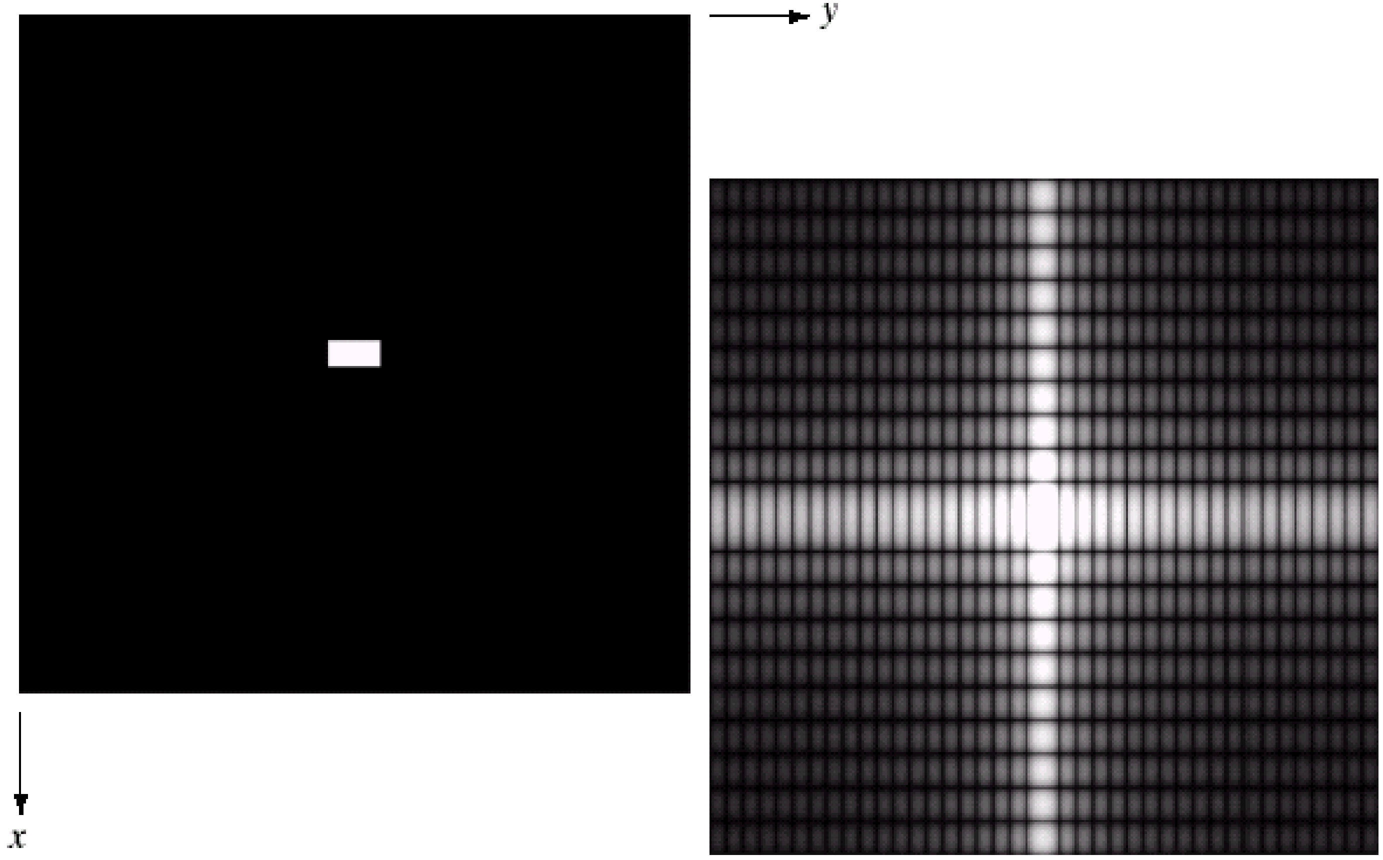
log(ampl + 1)

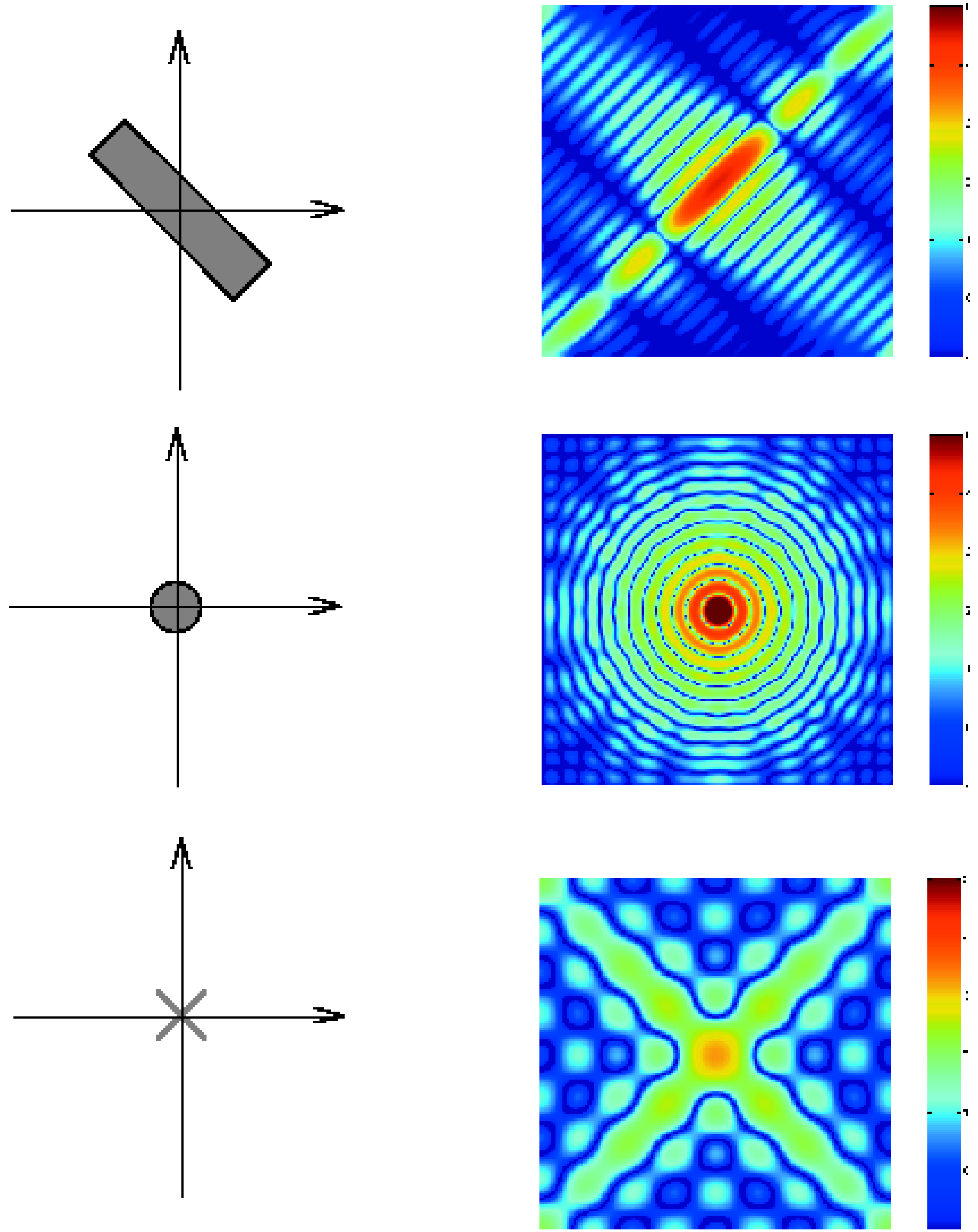
Basis functions of 2D FT

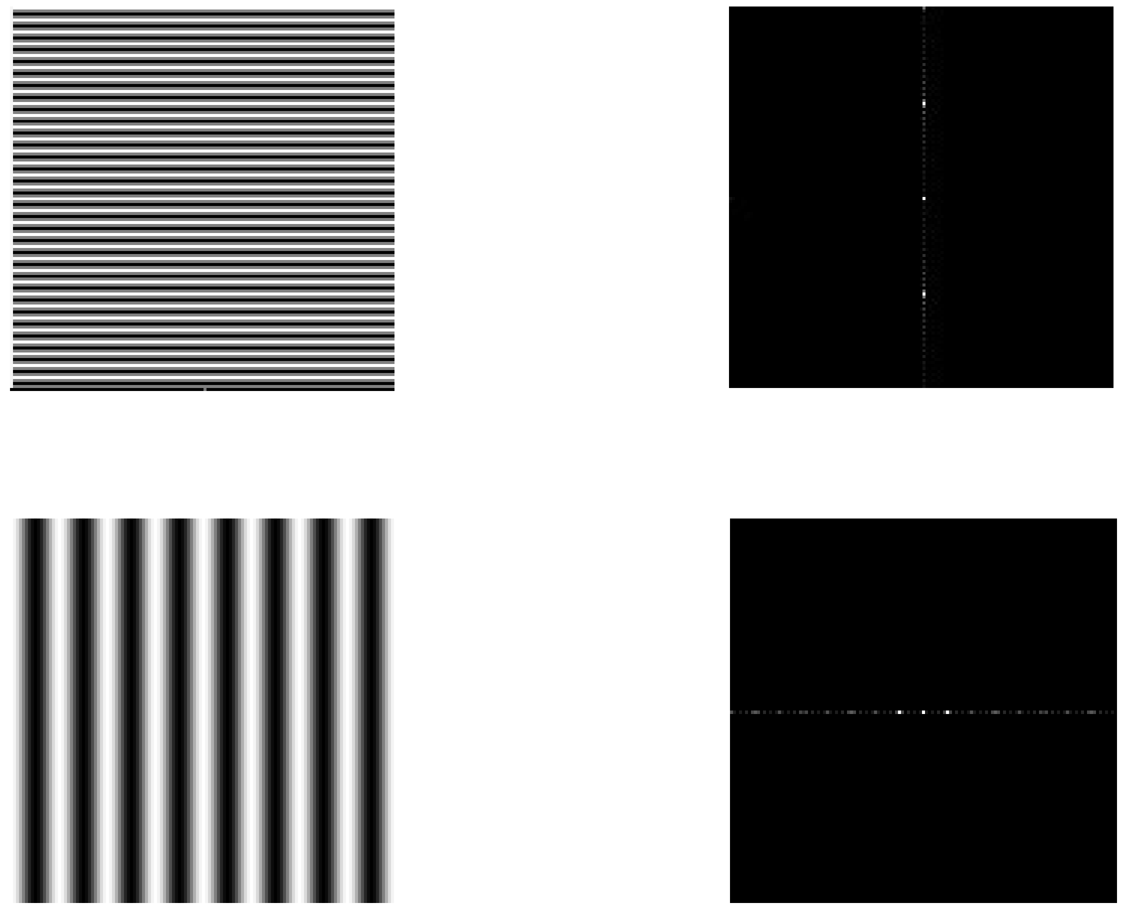


Real part, u=v

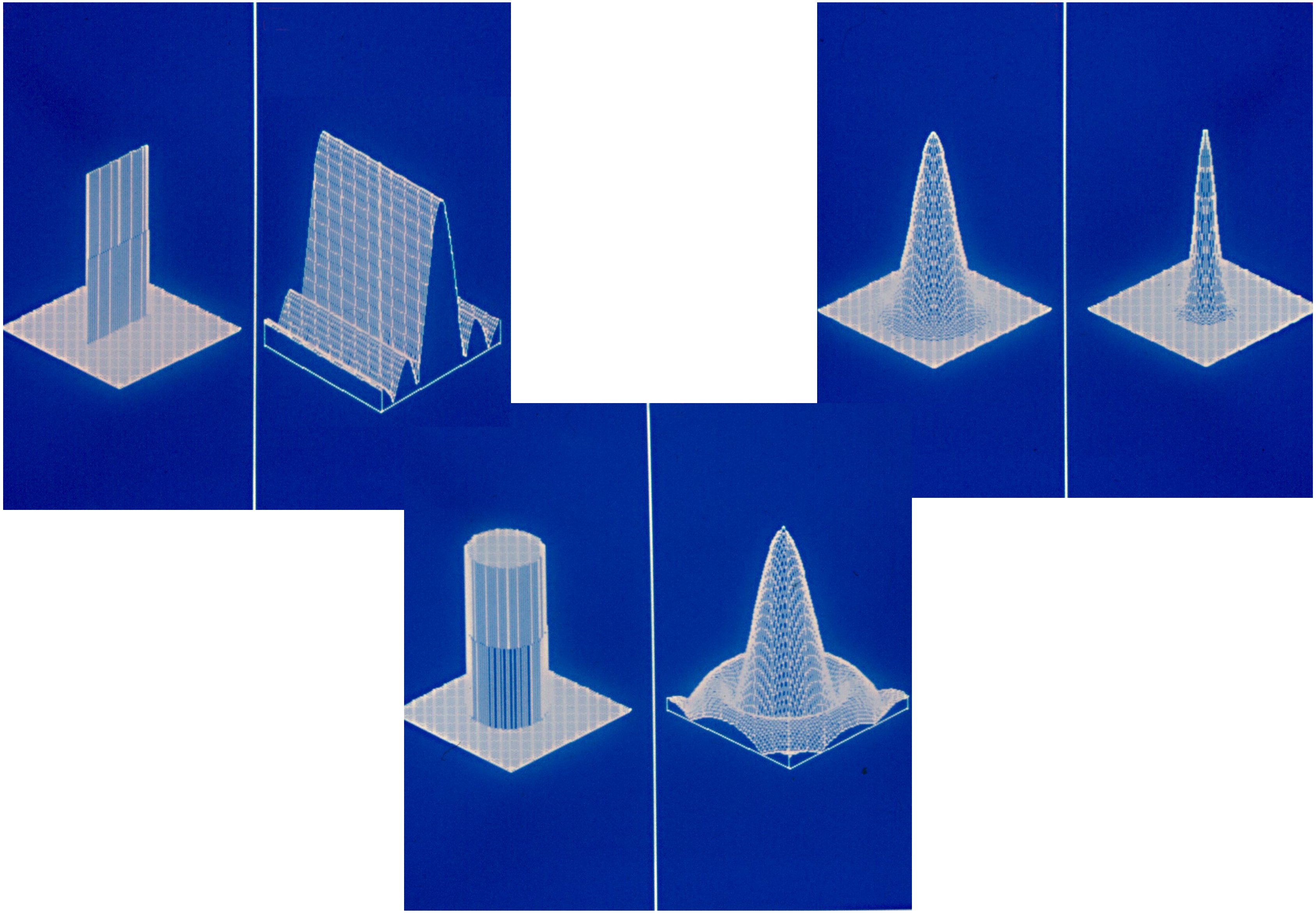
Rectangular pulse in 2D







Other important functions

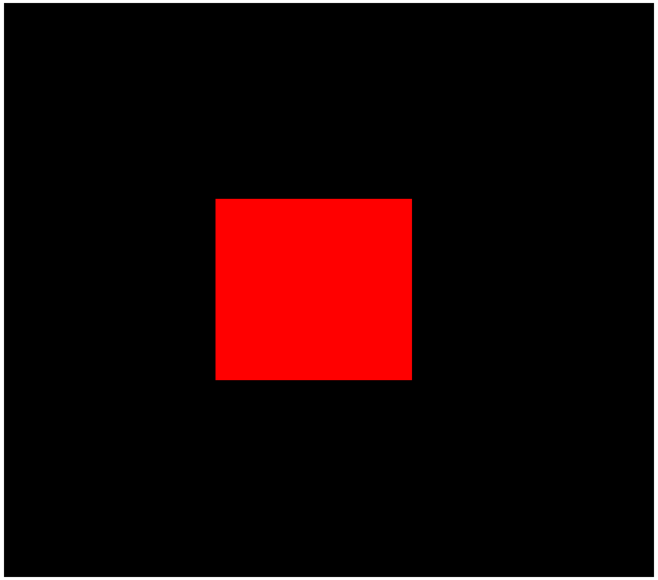


Discrete convolution theorem

Convolution mask

... holds for periodic convolution only!

Discrete convolution via FT



Zero padding to the same size

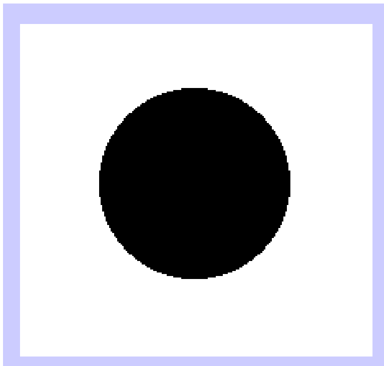
DFT calculation of both

Multiplication of the spectra

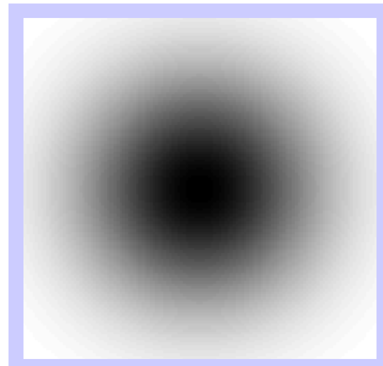
Inverse DFT

Filtering in the Fourier domain

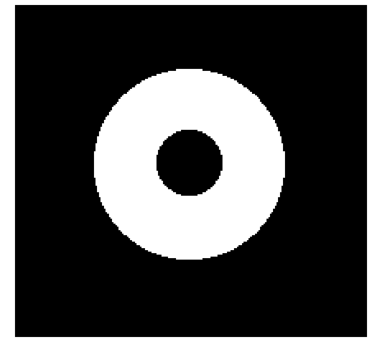
$$\mathcal{F}[f * g] = \mathcal{F}[f]\mathcal{F}[g]$$



high pass

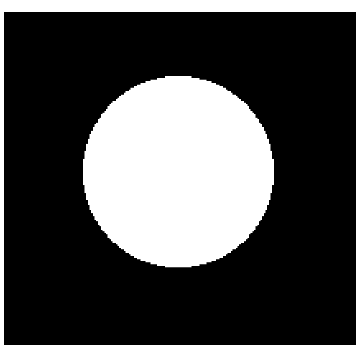


Gaussian high pass

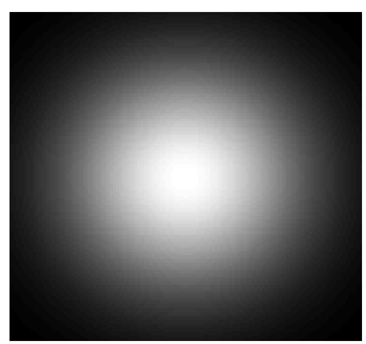


band pass

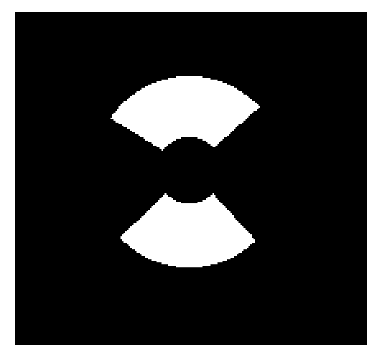
low pass

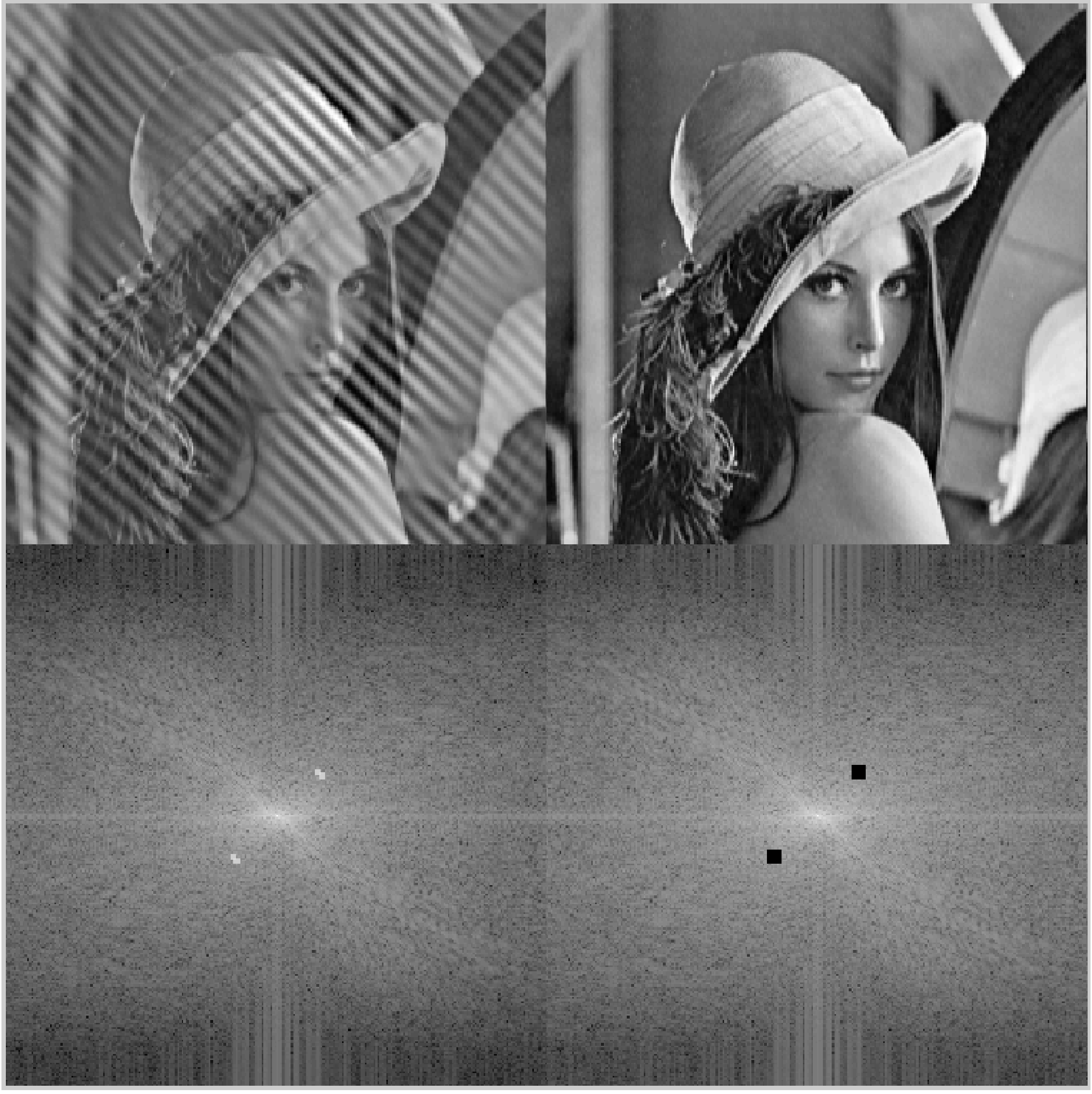


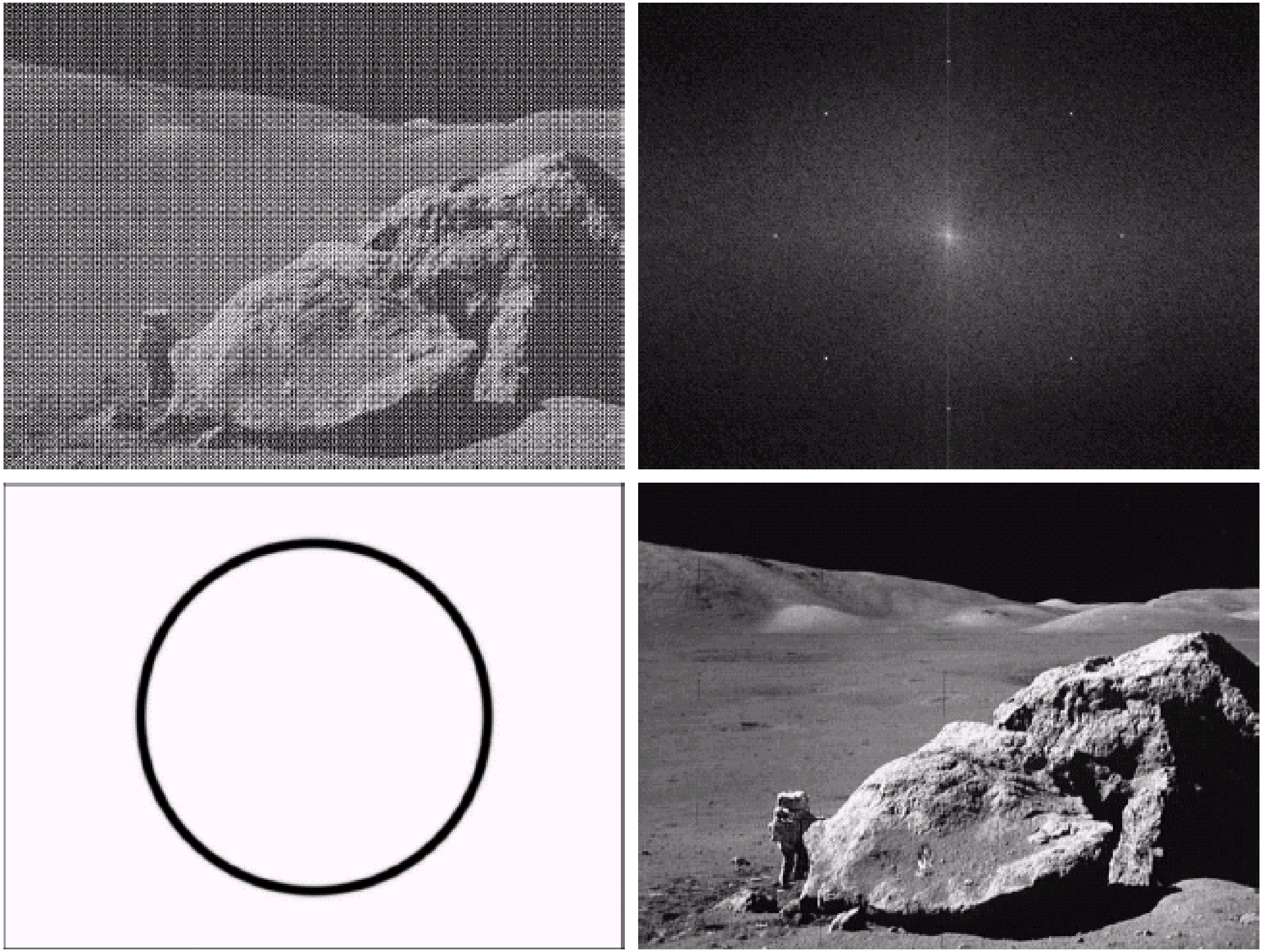
Gaussian low pass



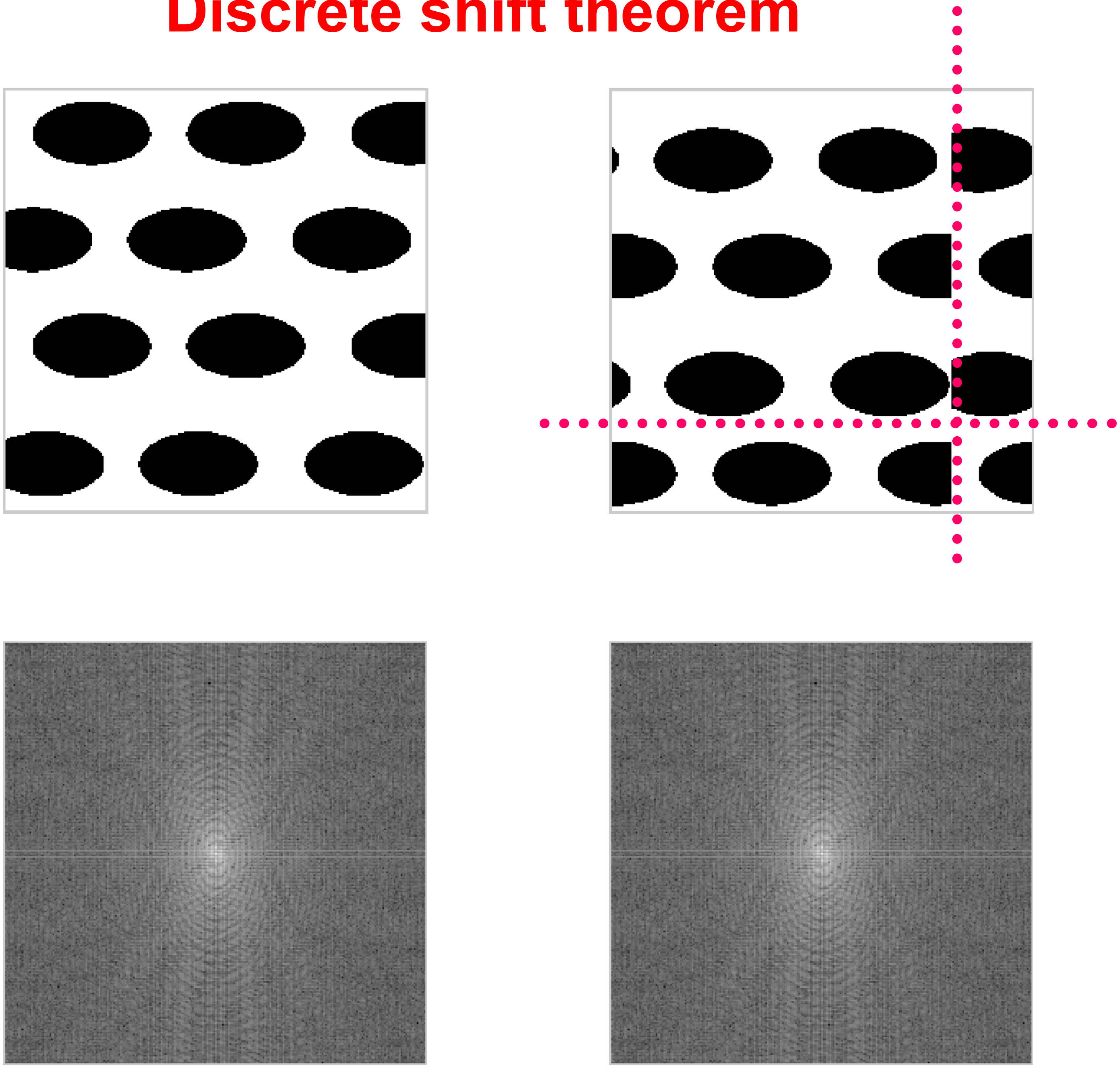
directional

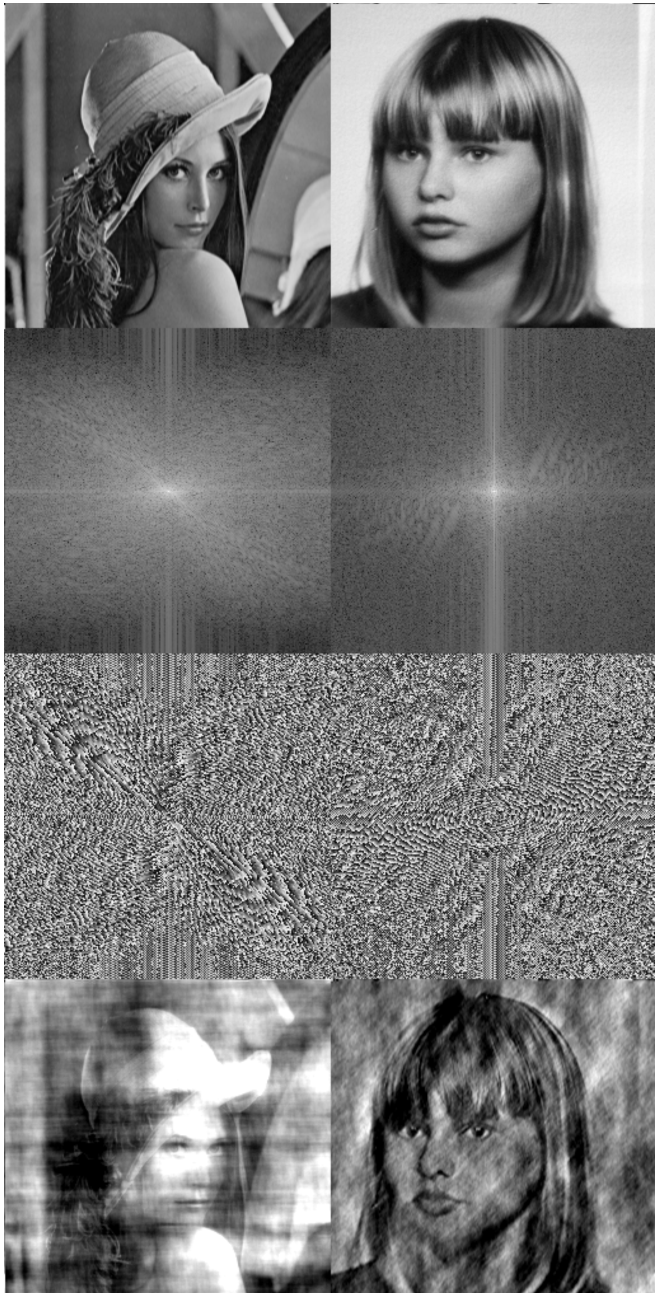






Discrete shift theorem





What is more important?

The phase!

Image whitening

$$w(x,y) = \mathcal{F}^{-1} \frac{F(u,v)}{|F(u,v)|}$$

original image

"whitened" image

Díky, pro dnešek končíme s FT!

Nějaké otázky ?