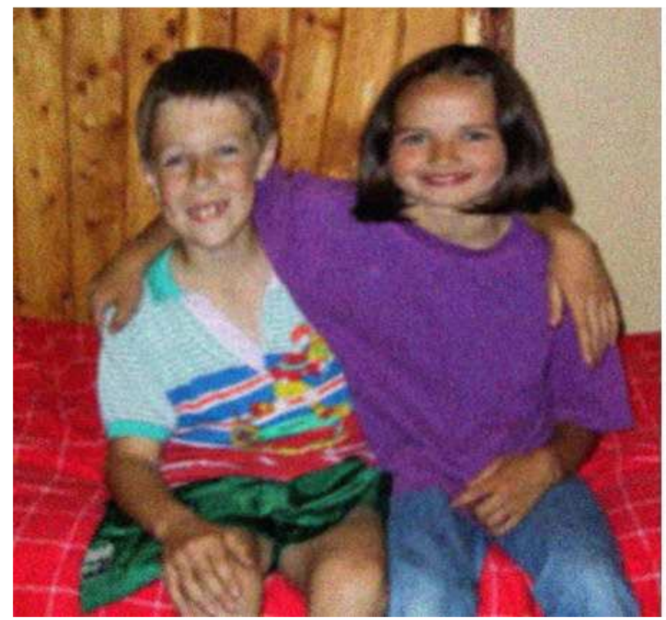
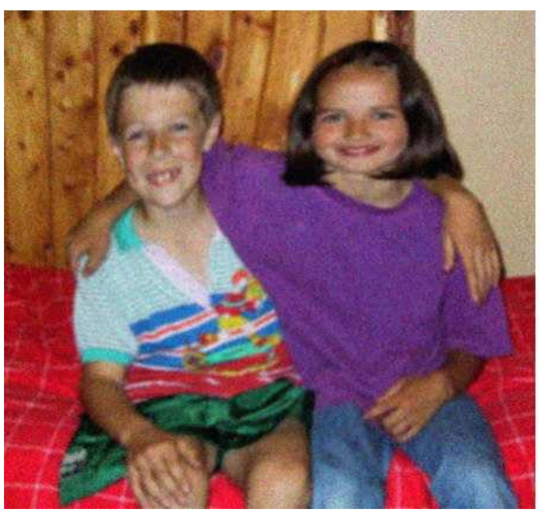
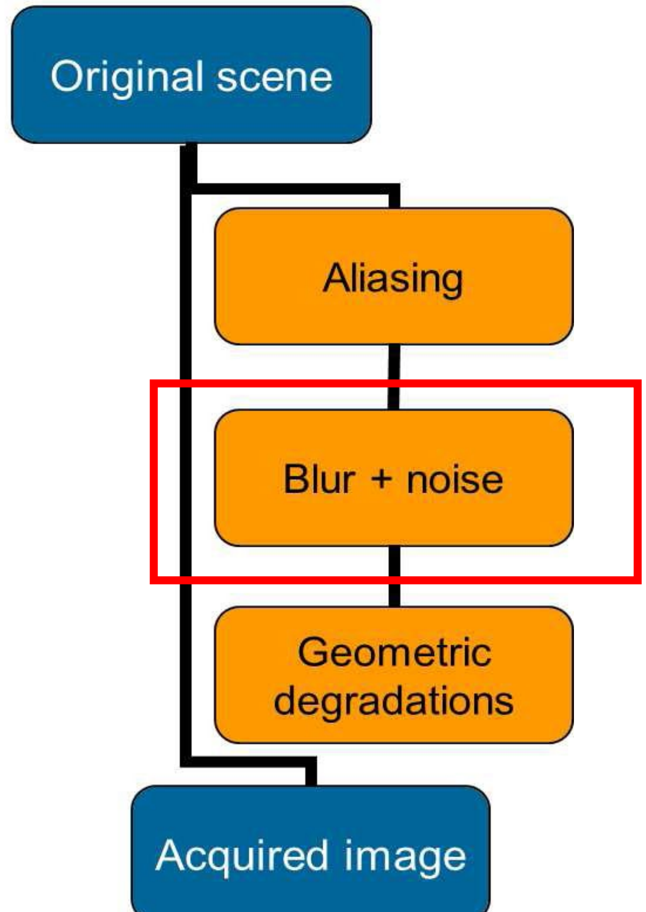
# Image restoration

# Acquired image is a degraded version of the original scene



# Image degradation model





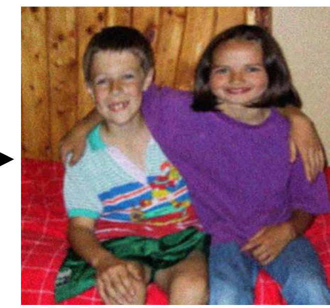
# Image acquisition model

original scene

degradations

$$\mathcal{D} = \mathcal{Q}\mathcal{S}\mathcal{D}_{\mathcal{I}}\mathcal{D}_{\mathcal{G}}$$

noise



acquired image

$$\mathcal{D}_{\mathcal{G}}(f)(x,y) = f(\tau(x,y))$$

$$\mathcal{D}_{\mathcal{I}}(f)(x,y) = \int_{\Lambda}\int_{T}\int\int h(x,y,a,b,\lambda,t)f(a,b)\,da\,db\,d\lambda\,dt$$

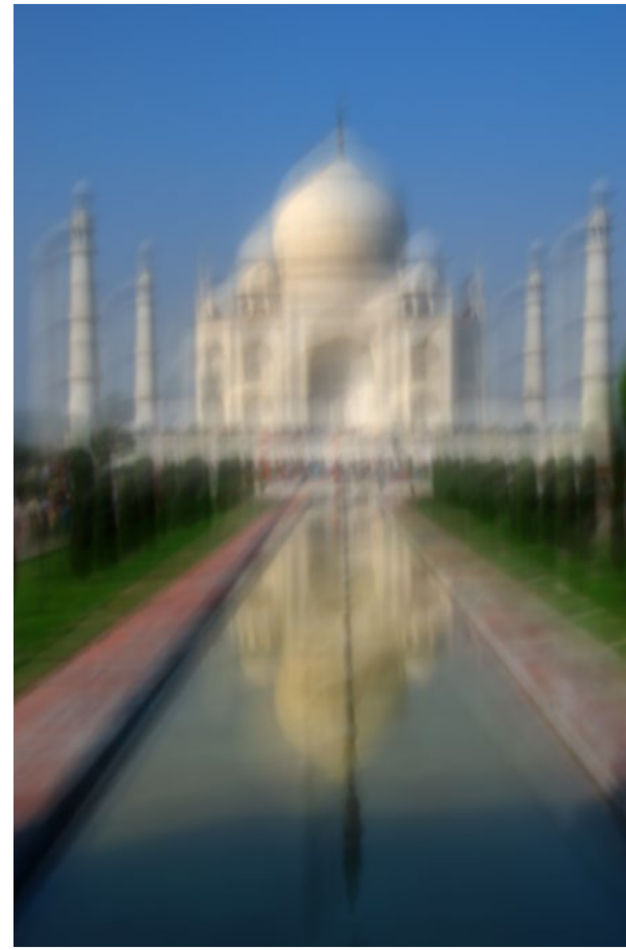
# Sources, models and appearance of individual degradations

## Blur

Image smoothing, low-pass filter
Suppression of high frequencies

$$\mathcal{D}_{\mathcal{I}}(f)(x,y) = \int_{\Lambda}\int_{T}\int\int h(x,y,a,b,\lambda,t)f(a,b)\,da\,db\,d\lambda\,dt$$

# Blur examples



# Typical blur sources

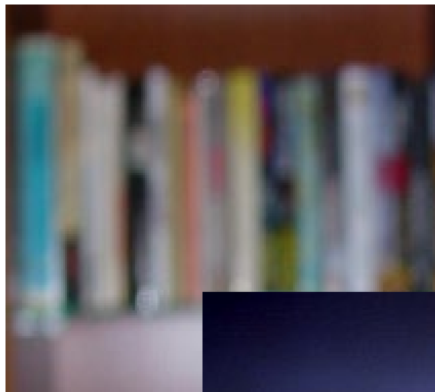
Camera shake/motion

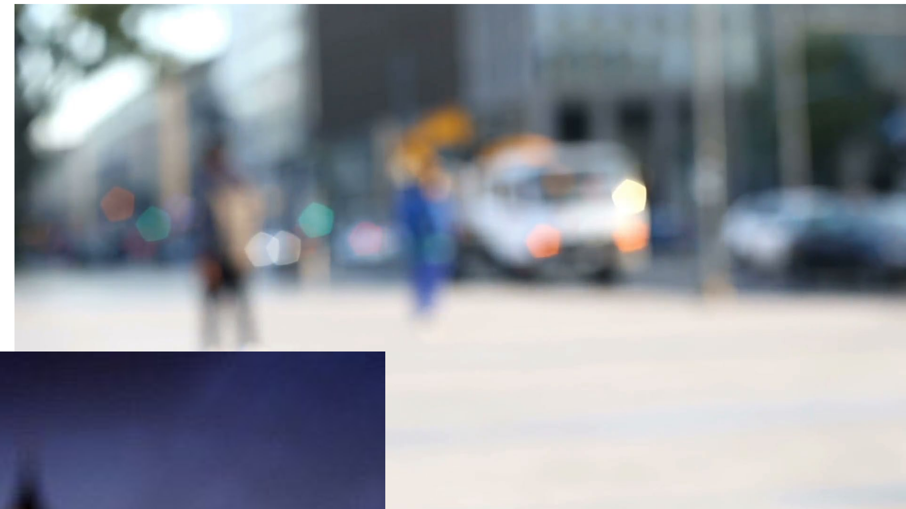
# Typical blur sources

Scene/object motion

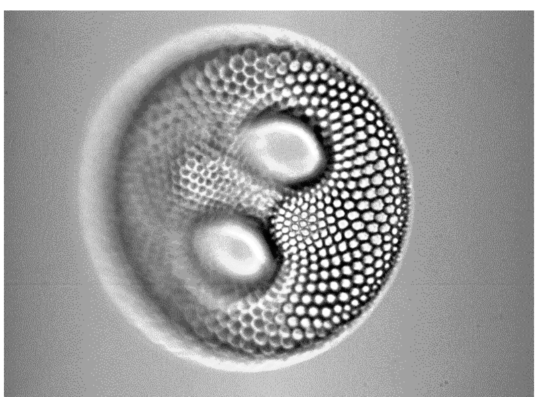
# Typical blur sources

Wrong focus



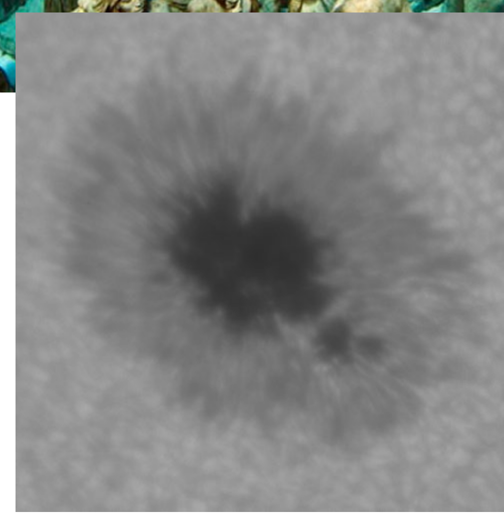


Karen Walrond

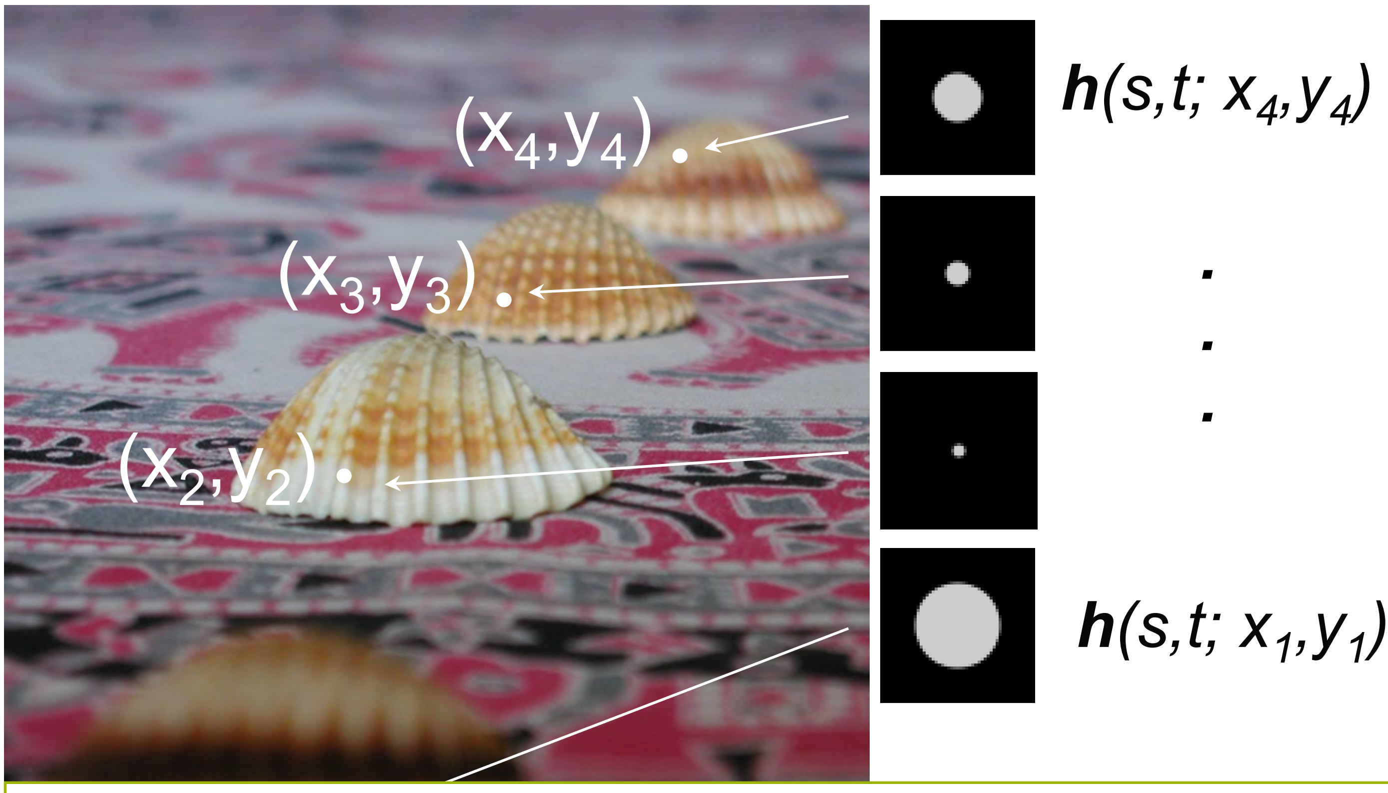


# Typical blur sources

Medium turbulence



# General space-variant blur model




$$\mathbf{z}(x,y) = \int_{\Omega} \mathbf{u}(x-s, y-t)\mathbf{h}(s,t; x-s, y-t)\, dsdt + \mathbf{n}(x,y)$$

# Simplified space-invariant blur model

- *Flat scene*

- *Constant*
- *motion*

$$z(x) = (h * u)(x) + n(x)$$

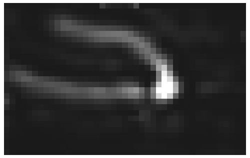
*h(x)* is the PSF of the camera

# Understanding PSF

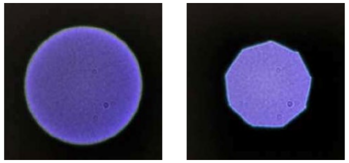
- PSF is the response to
- an ideal point source

$$z(x) = (h * u)(x) + n(x)$$

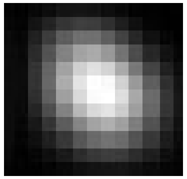
- *camera shake/motion*

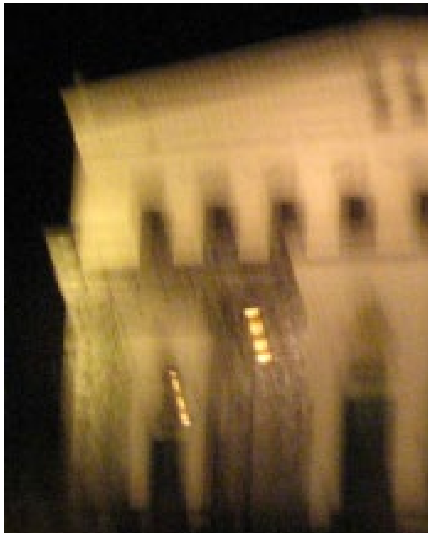


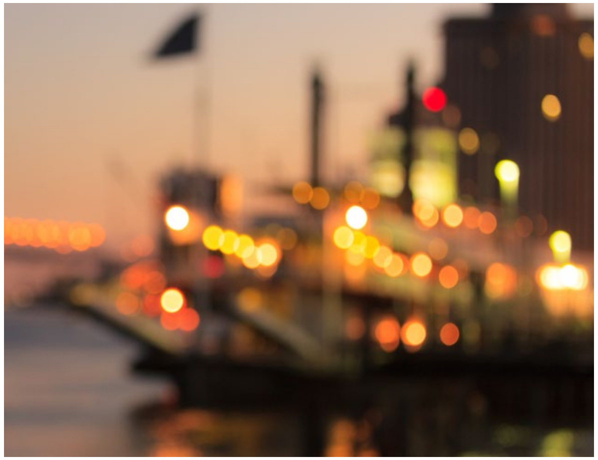
- *out-of-focus*



- *turbulence*

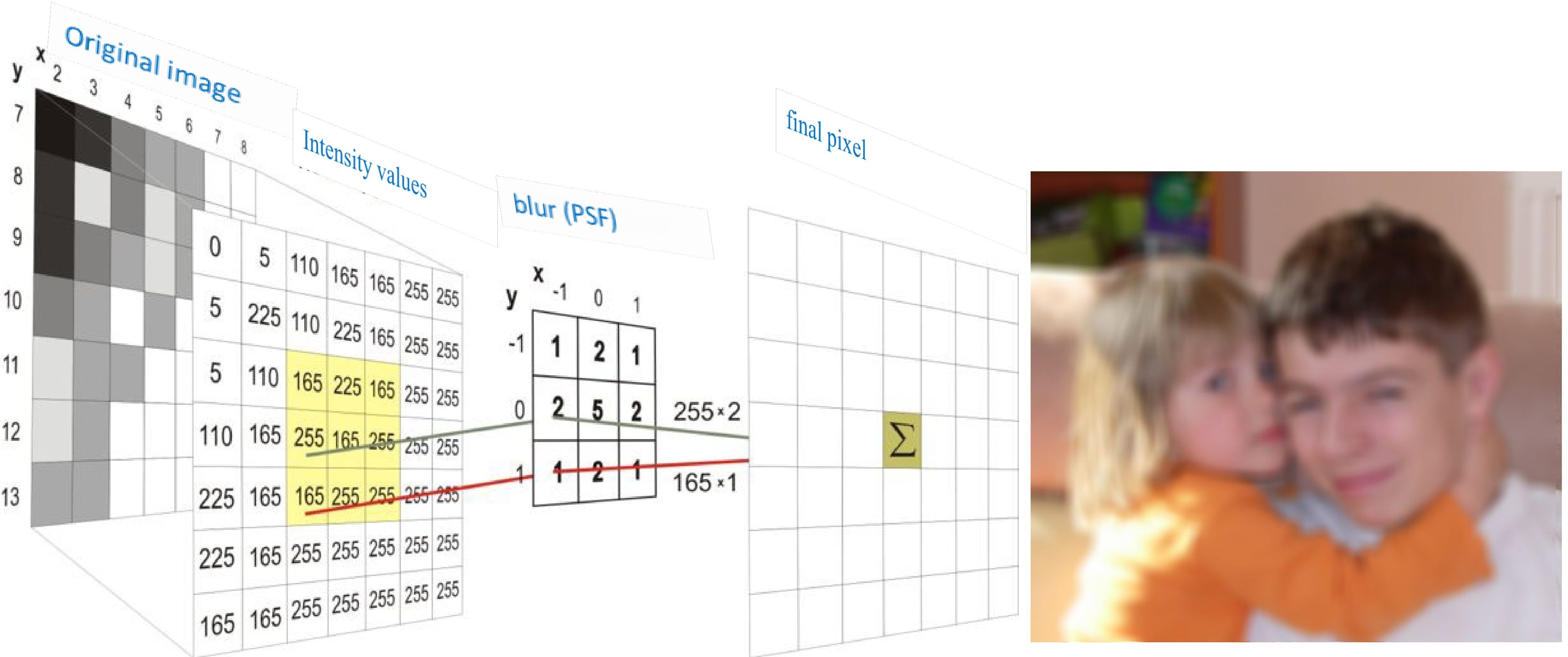






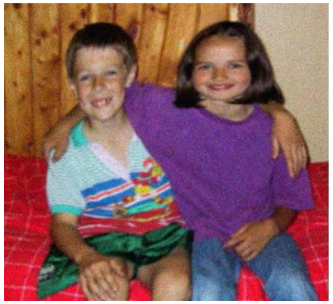
# Recalling convolution

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s, t) g(x - s, y - t) ds dt$$

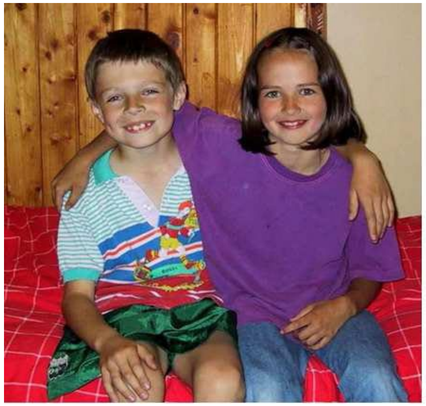


# Image restoration

"Inverting" the degradations $g = \mathcal{D}(f) + n$



?



Looking for f^ such that $\varrho(f, \hat{f})$ is minimized

# Noise-free case

$$\hat{f} = \mathcal{D}^{-1}(g) = \mathcal{D}_{\mathcal{G}}{}^{-1}\mathcal{D}_{\mathcal{I}}{}^{-1}\mathcal{S}^{-1}\mathcal{Q}^{-1}(g)$$

## Noisy case

$$g = \mathcal{D}(f) + n$$

Noise makes the problem ill-conditioned and difficult to handle

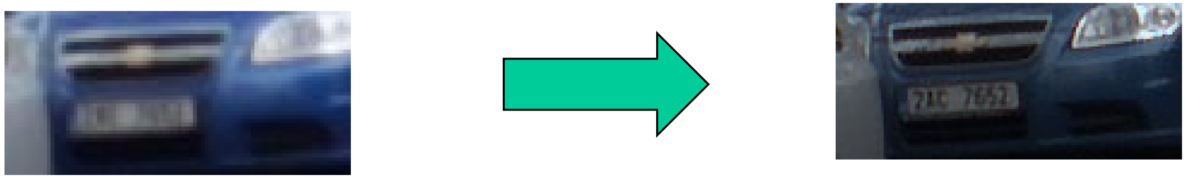
$$\hat{f} \neq \mathcal{D}^{-1}(g)$$

Denoising and/or regularization techniques are required.

# Historical remark

- Image restoration has been a very traditional area of image processing
- A. Rosenfeld: *Picture Processing by Computer*, Academic Press, 1969
- 2018 - 200 000 000 search results by Google
- - 3 000 000 results at Google Scholar

# Why is image restoration difficult?



- The problem is ill-conditioned and/or ill-posed

$$z(x) = (h * u)(x) + n(x)$$

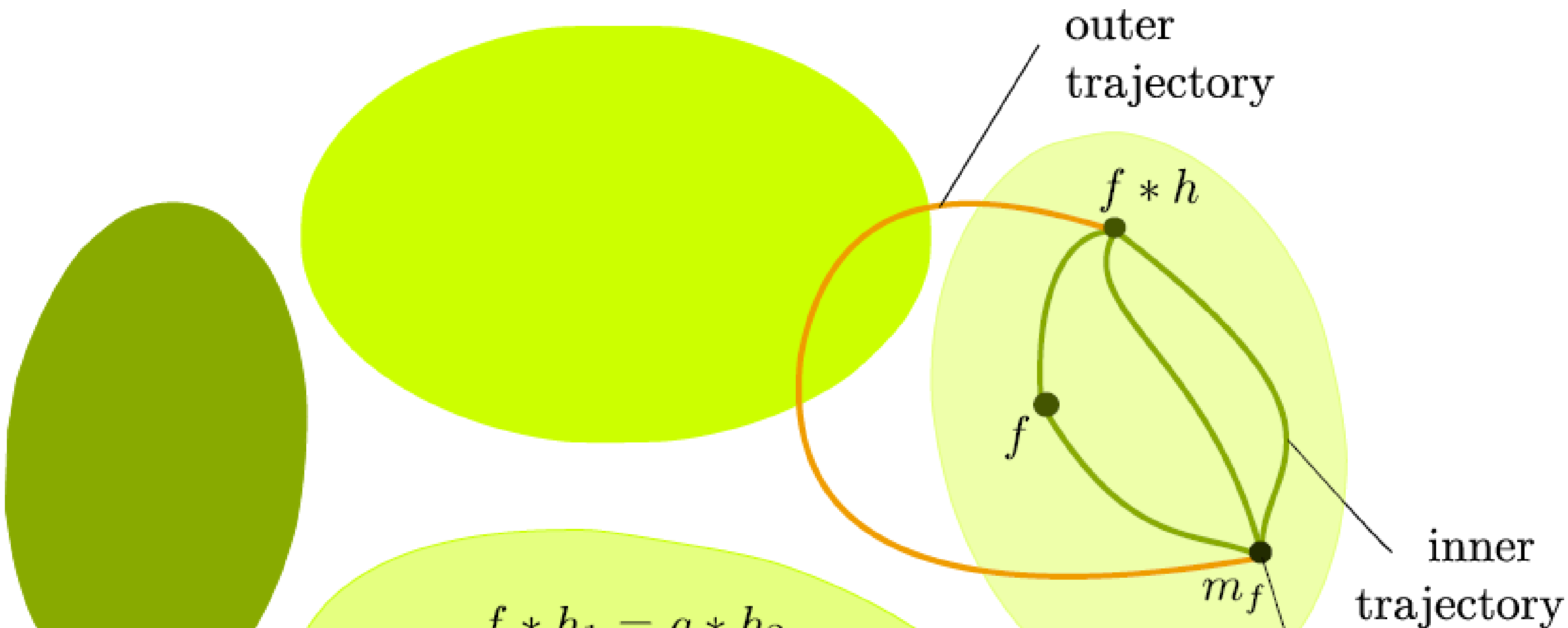
- Too many unknowns
- Even if the PSF is known, the noise makes the task ill-conditioned and requires a special care (regularization)

# Why is image restoration difficult?

- Even if we had a perfect deconvolution algorithm and no noise was present, there would be still a solution ambiguity

$$z(x) = ((h_1 * h_2 * \cdots * h_L) * u)(x)$$

# Understanding deconvolution




# What shall we do?

- We need to choose the correct trajectory in the image space and the point where to stop
- This can be supported by incorporating our preferences/priors/constraints into the restoration algorithms
- Both the original image $f$ and the PSF can be constrained

# Restoration categories

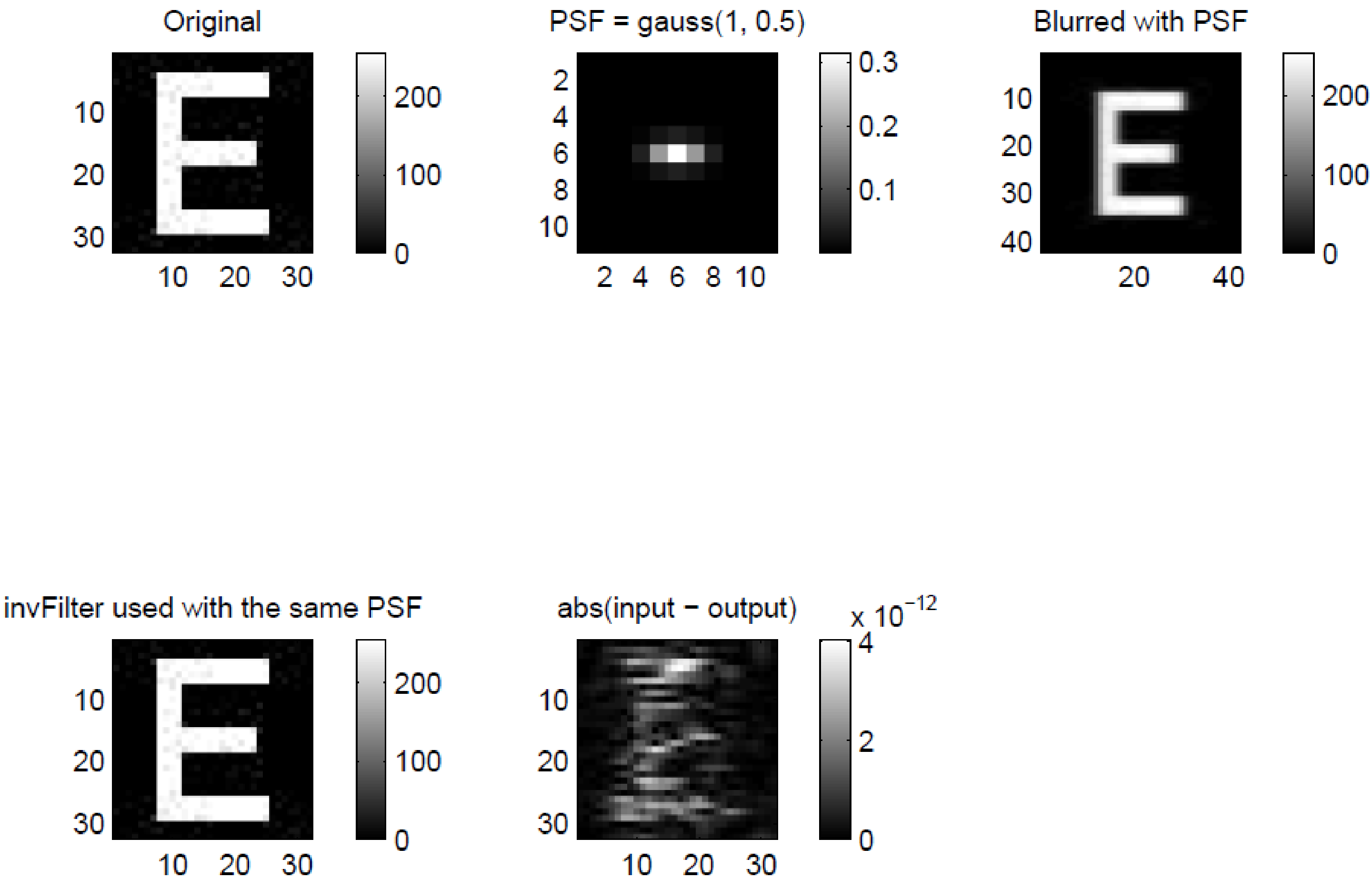
- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

# Intuitive solution to the inverse problem

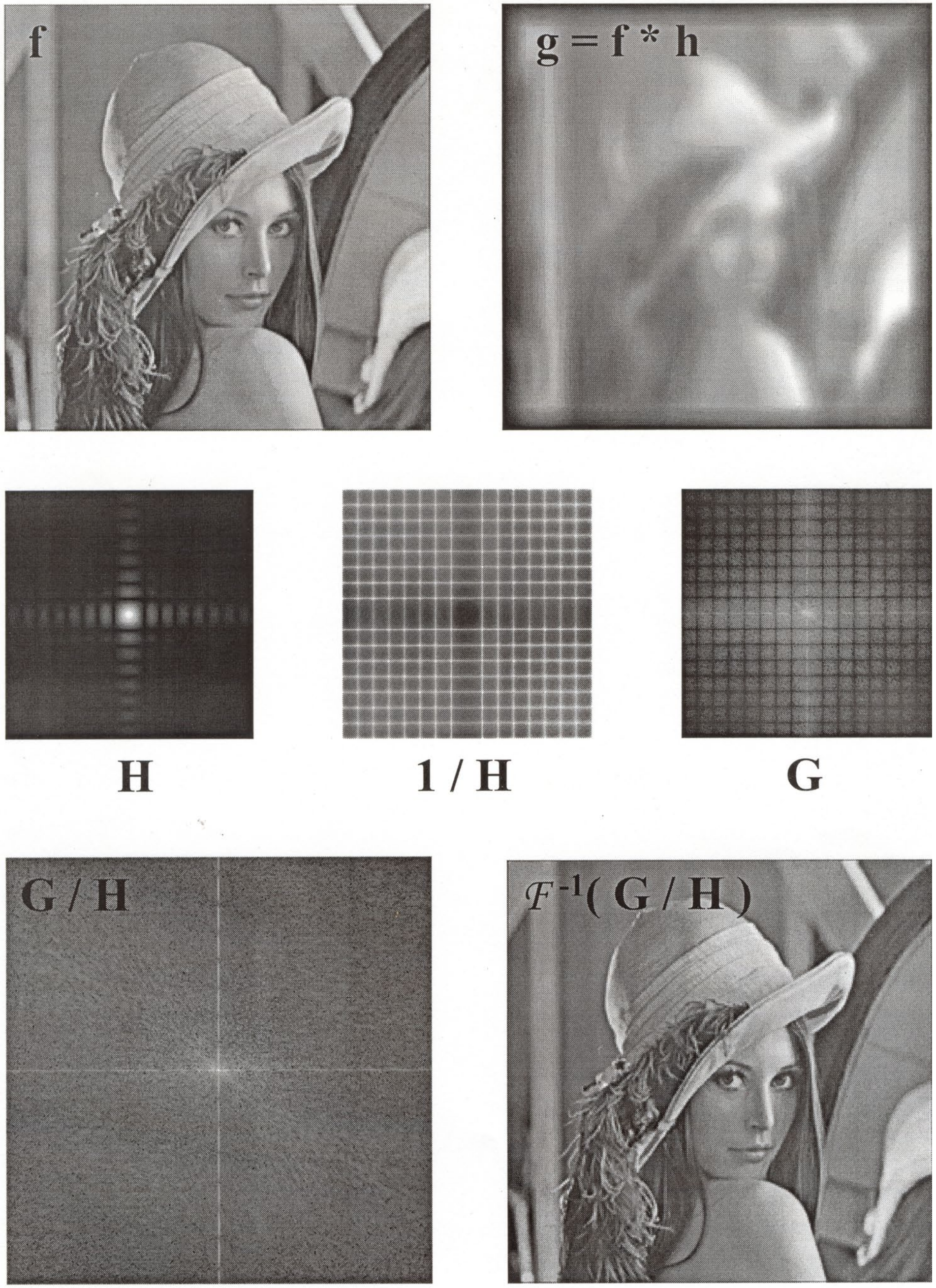
- No noise, PSF known – Fourier transform

$$G = F \cdot H$$

# Inverse filter in Fourier domain



# Inverse filter in Fourier domain



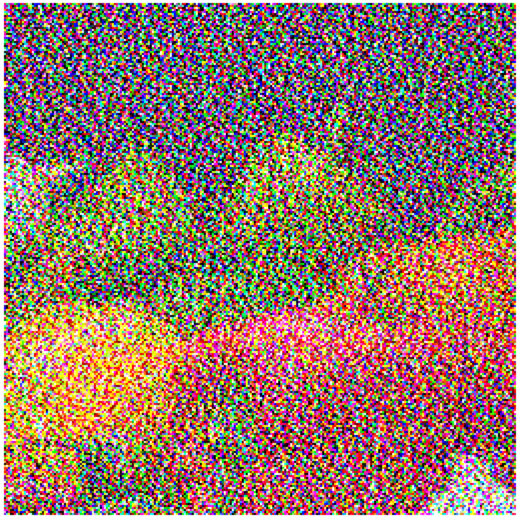

$$R(u, v) = \frac{1}{H(u, v)}$$

# Intuitive solution to the inverse problem

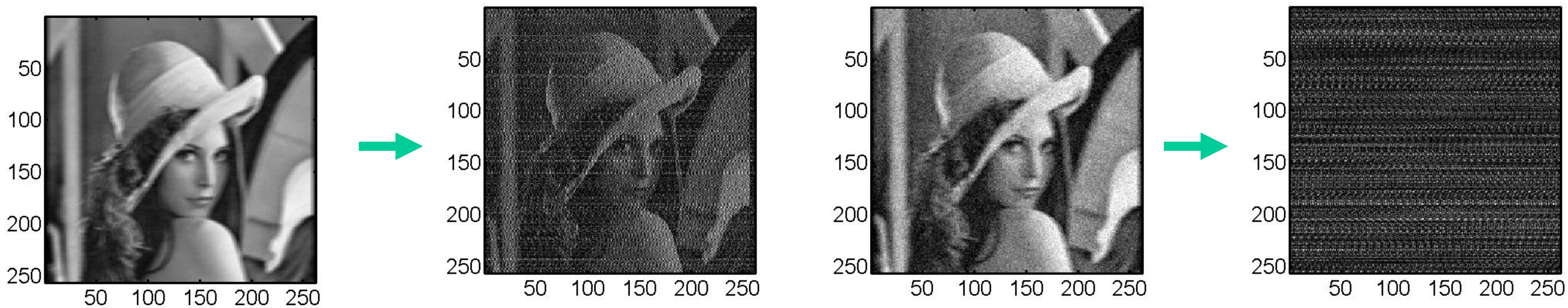
- ... does not work on images with noise

$$G = F \cdot H + N$$

$$F = \frac{G}{H} - \frac{N}{H}$$



- ... does not work on images with noise



# Wiener filter

$$E(\|f' - f\|^2) \to min$$

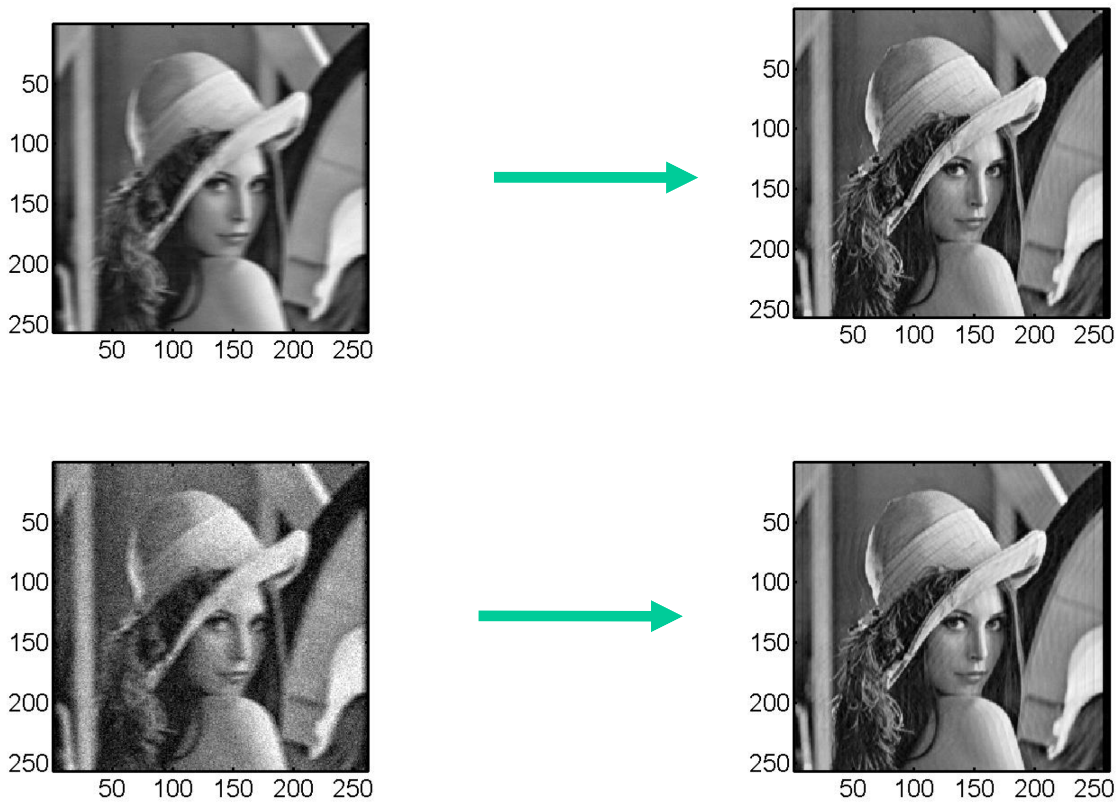
- Minimization on the set of linear filters F’ = RG

# Wiener filter

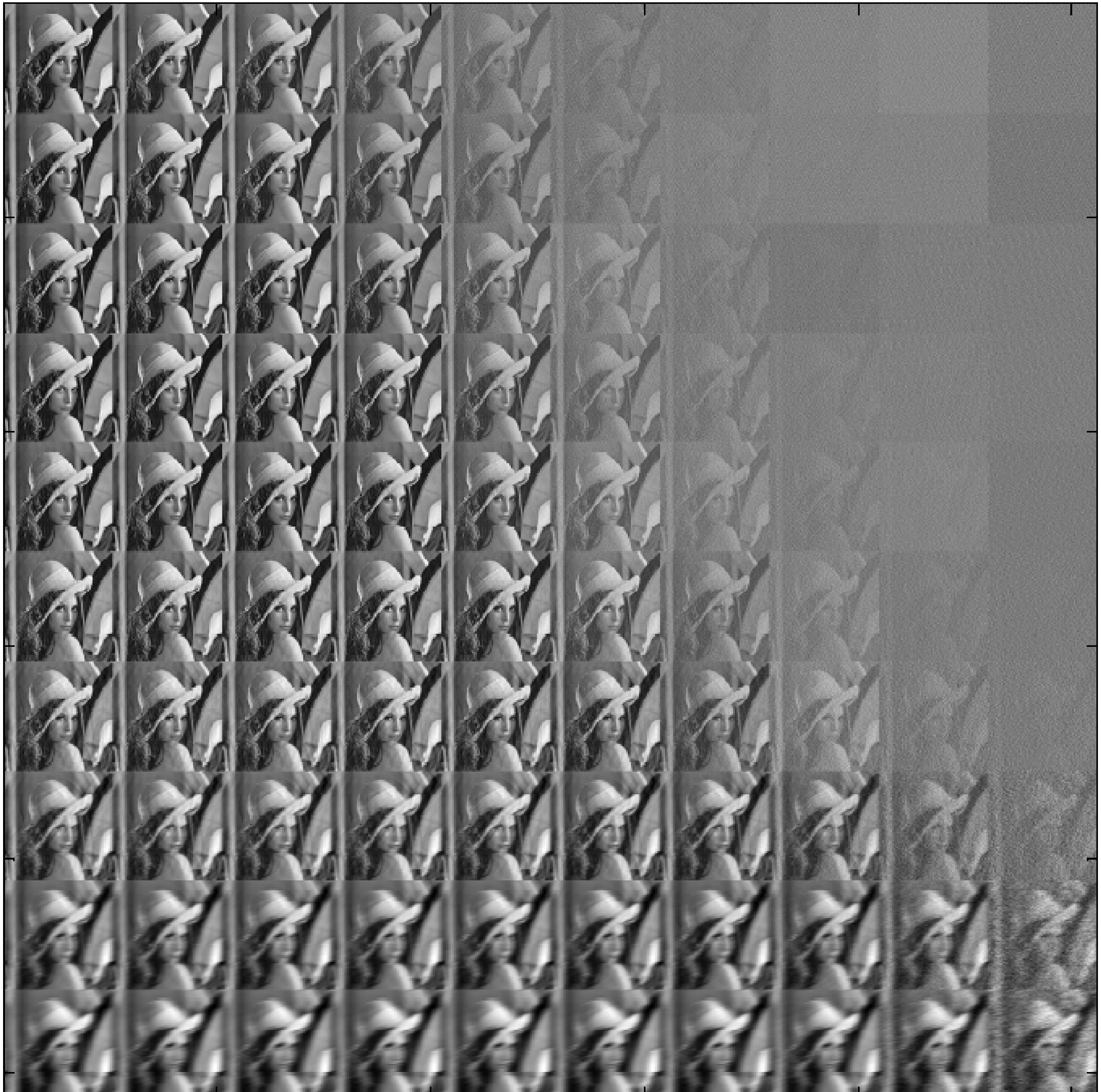
$$R(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)}$$

- $S_n(u,v)/S_f(u,v) \approx SNR^{-1}$

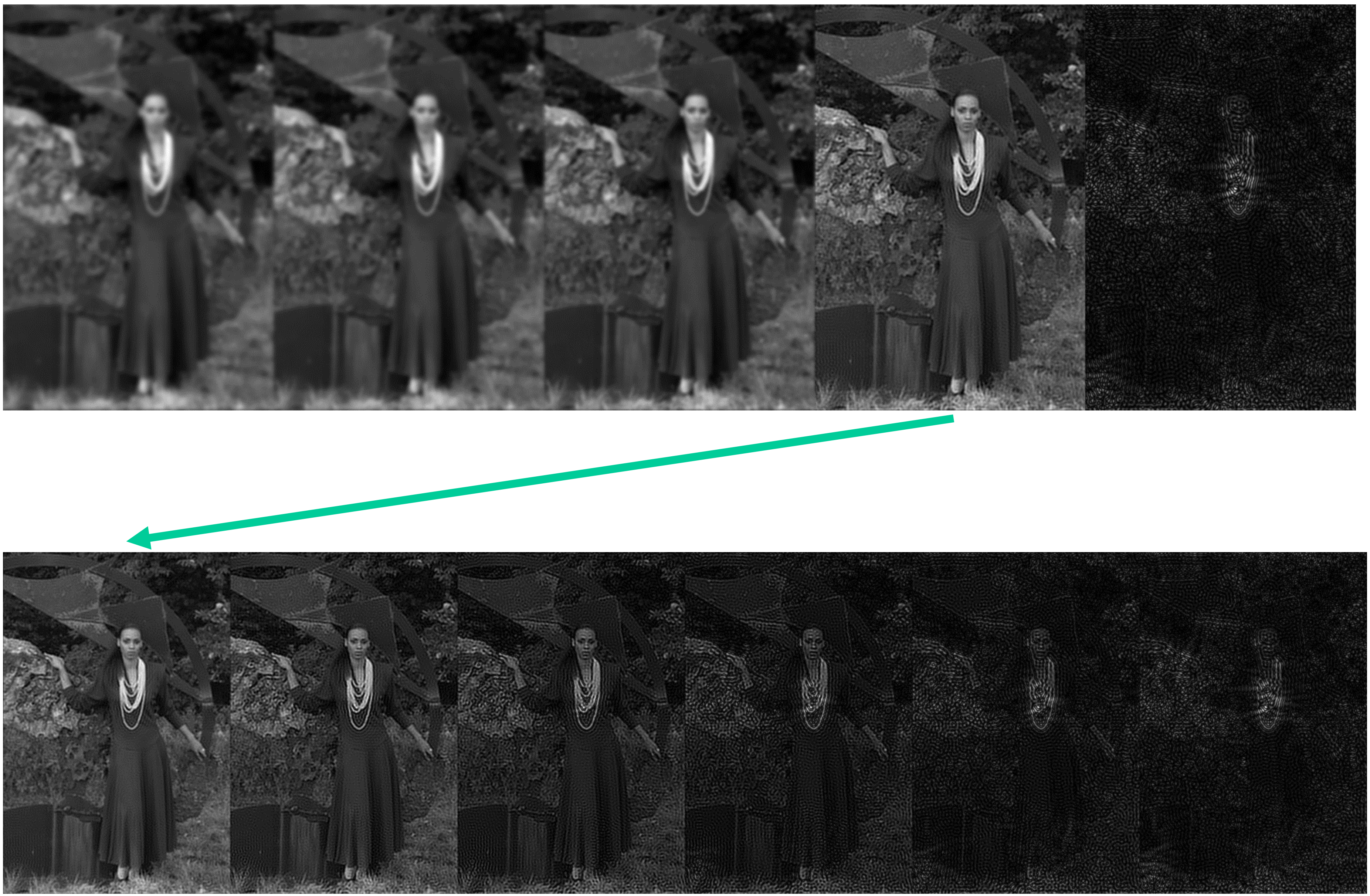
# Wiener filter



# The role of the SNR value



# The role of the PSF knowledge

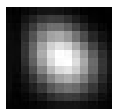


# Image restoration flowchart

Blurred image

# Image restoration flowchart

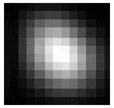
Blurred image



PSF estimation

# Image restoration flowchart

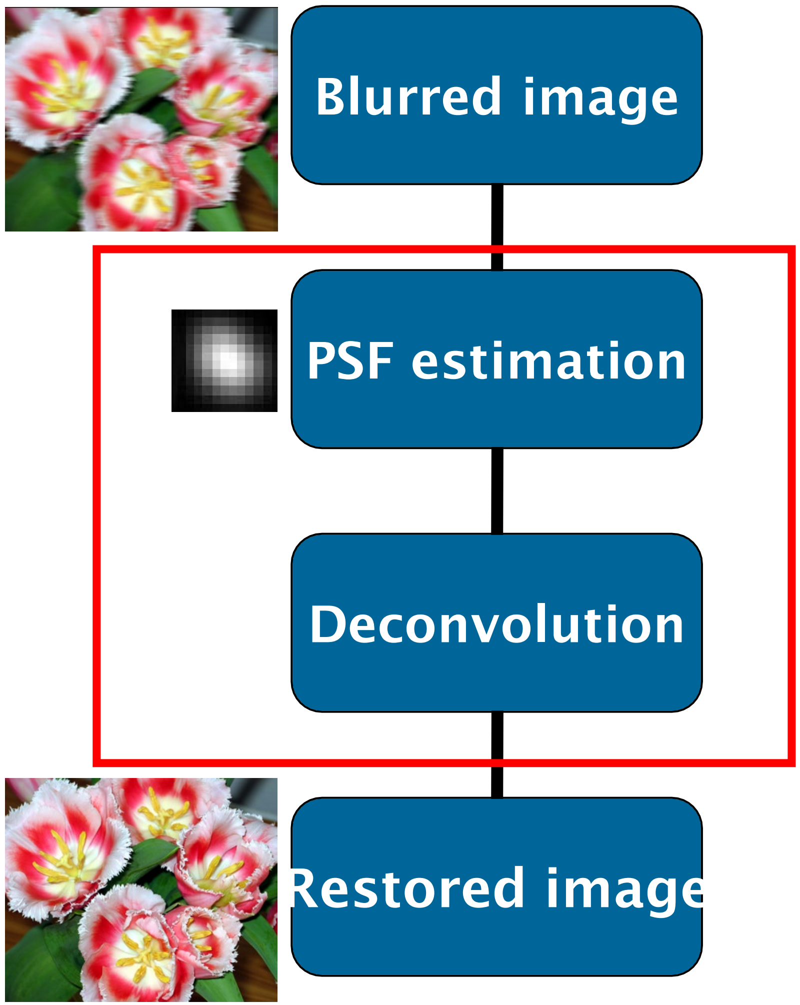
**Blurred image**



**PSF estimation**

**Deconvolution**

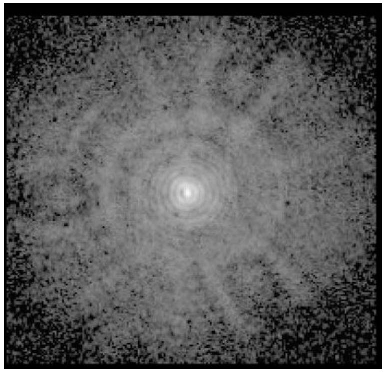
# Image restoration flowchart



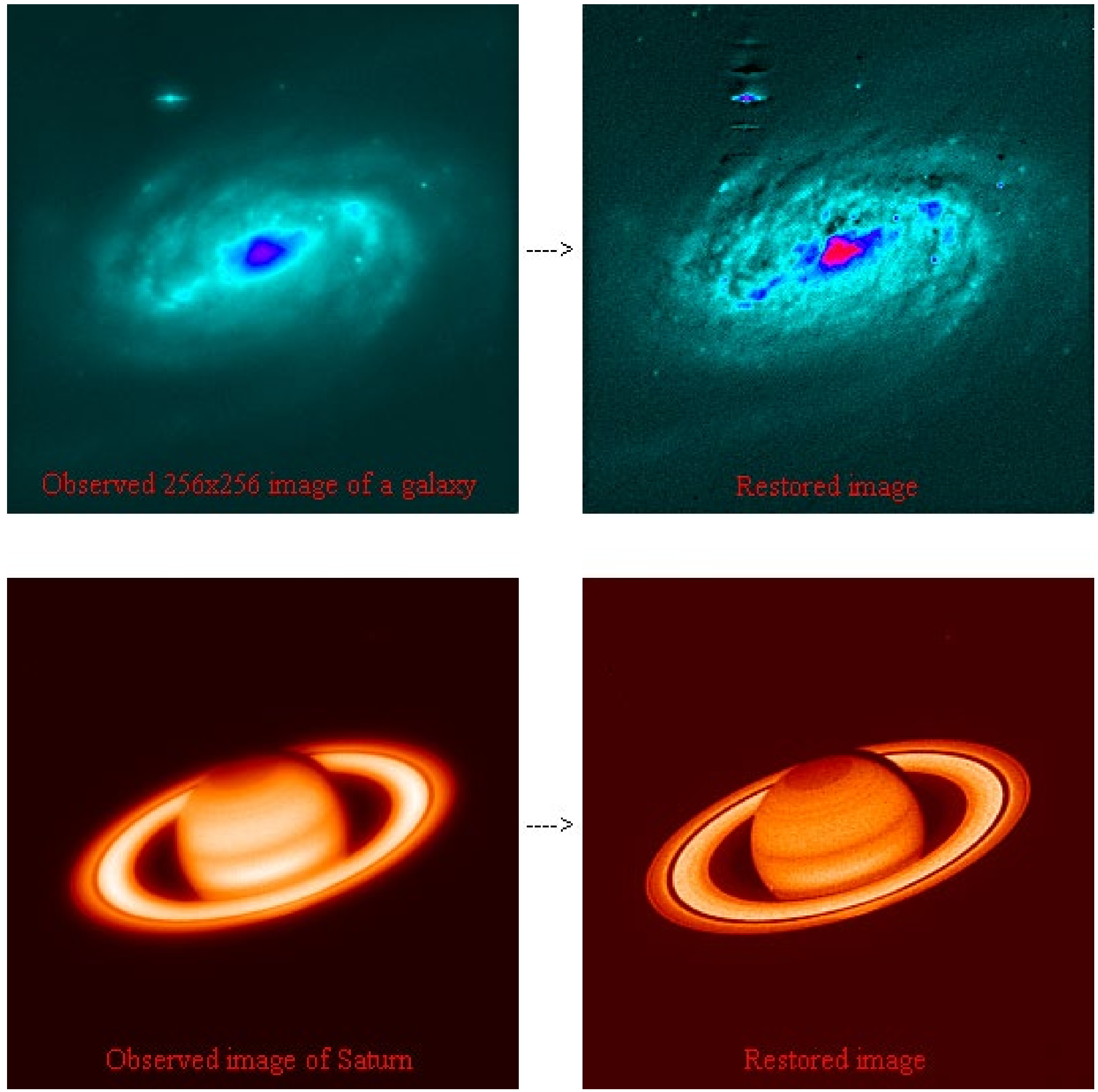

$$h * u = \int u(x-s, y-t)h(s,t)dsdt$$

# Images from the Hubble Space Telescope

S82E5937 1997:02:19 07:06:57

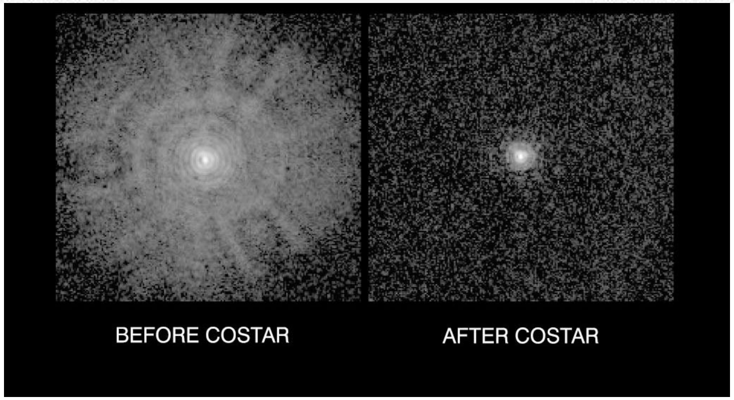


# Images from the Hubble Space Telescope

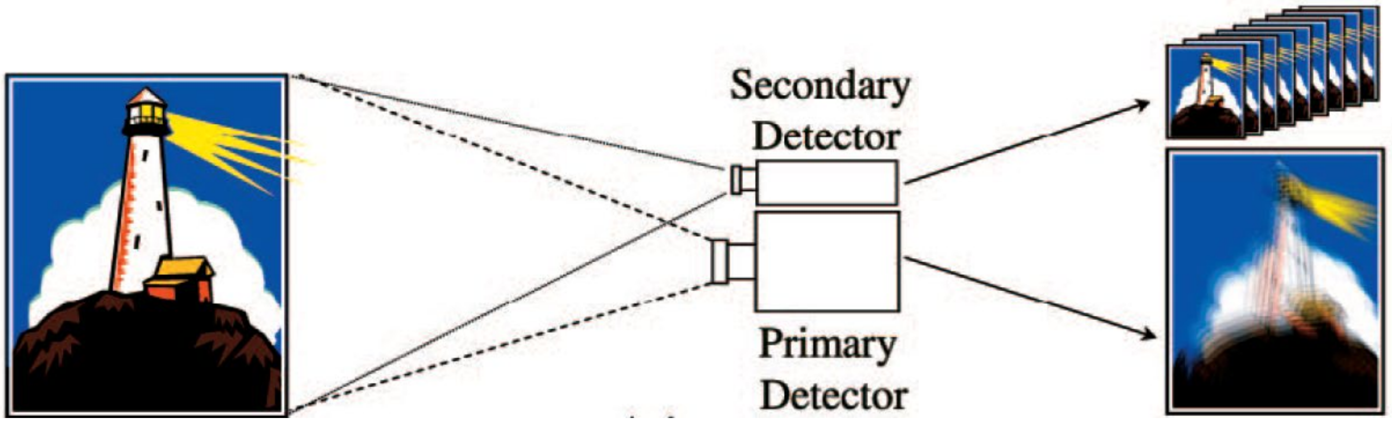


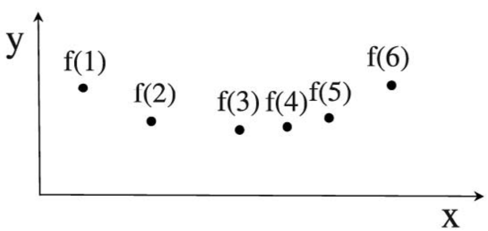
# Hubble Space Telescope  after COSTAR

Wide Field Planetary Camera 1 | Wide Field Planetary Camera 2

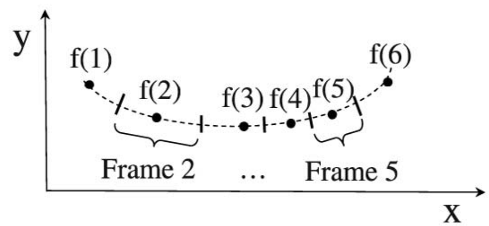


# High-speed camera

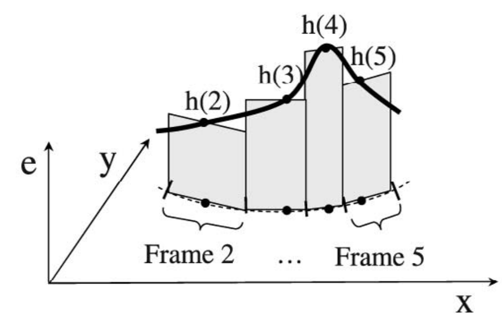



**Sampled trajectory (secondary**




**Interpolated trajectory**



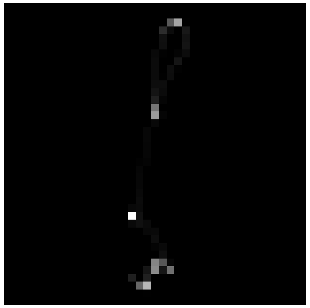

**Estimated PSF**

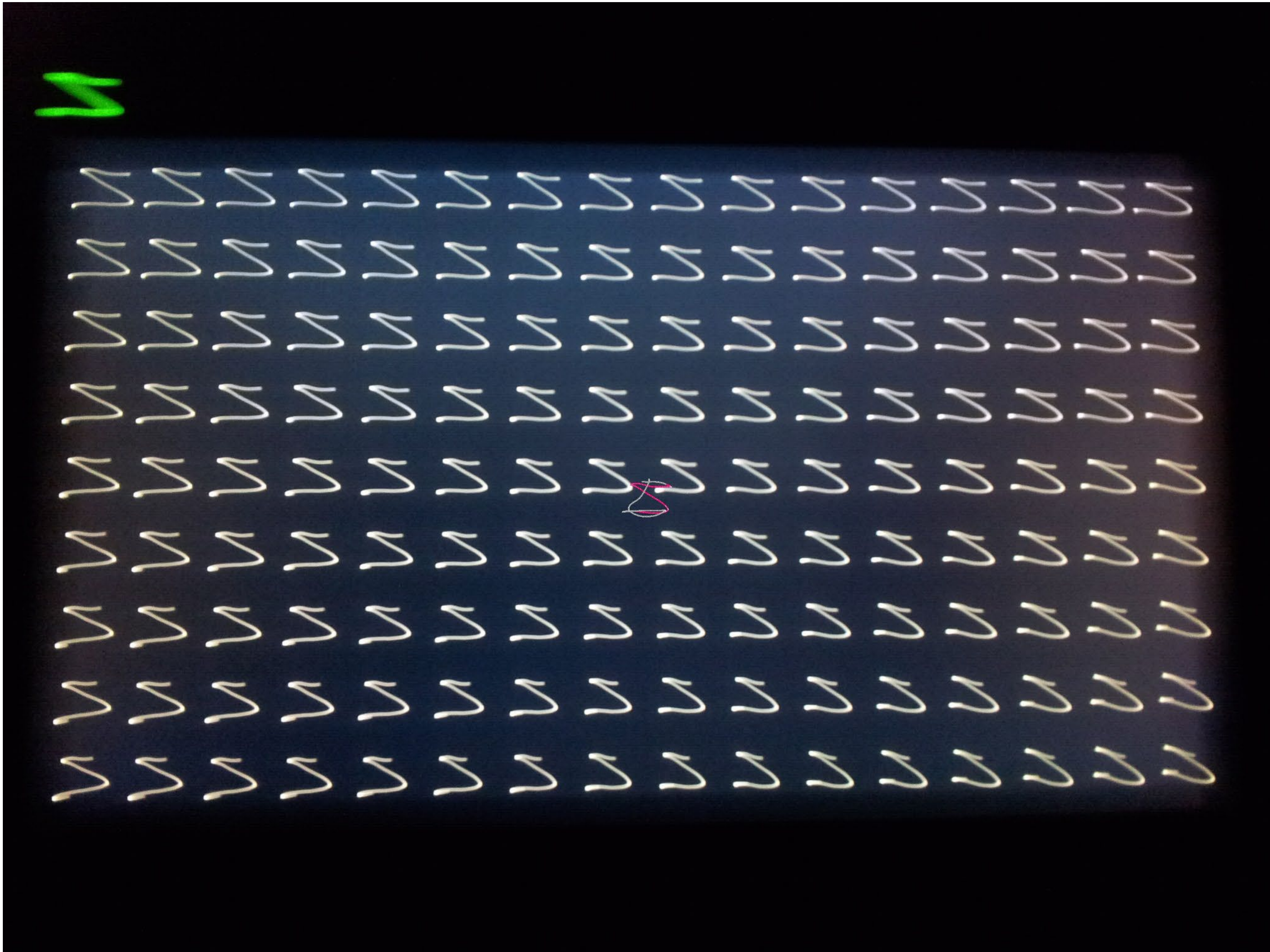
# PSF estimation in smartphones

- Camera shake blur
- Dominant motion - rotation

# PSF estimation in smartphones

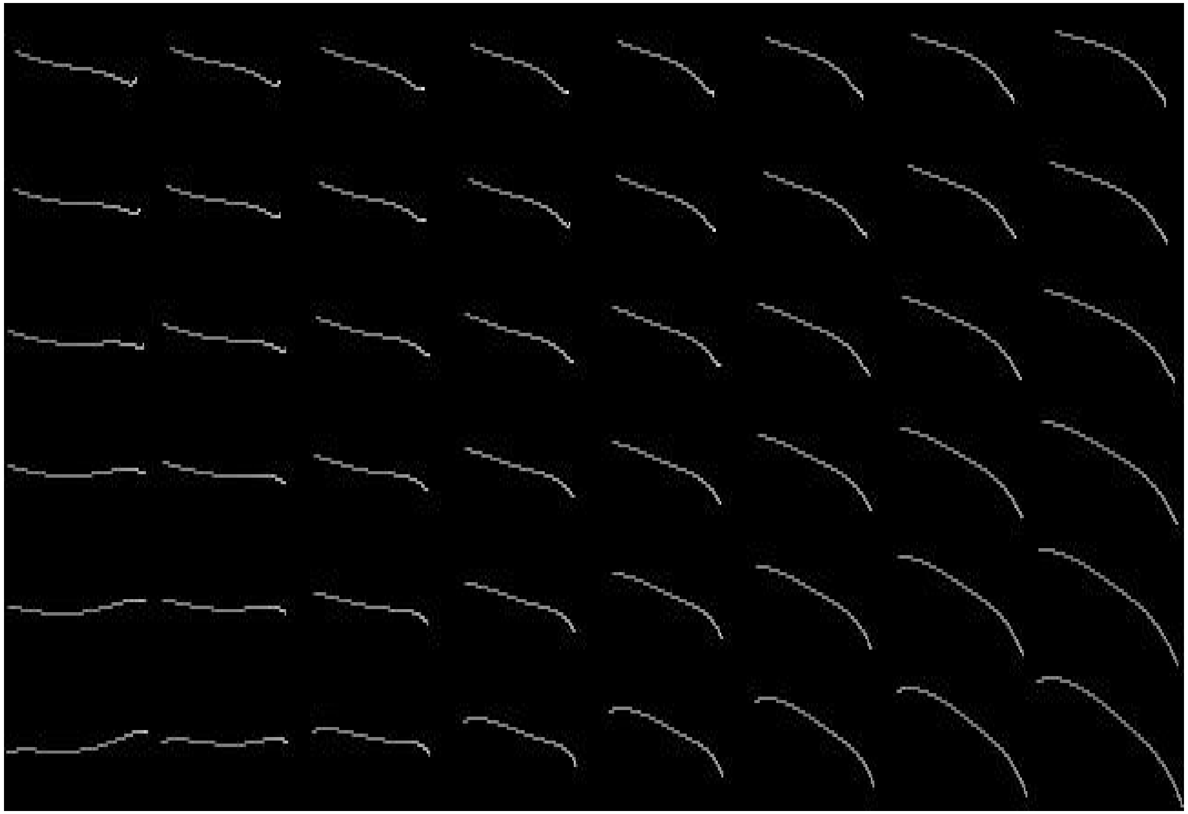
- Using accelerometers and/or gyroscopes
- Rotation and translation of the phone



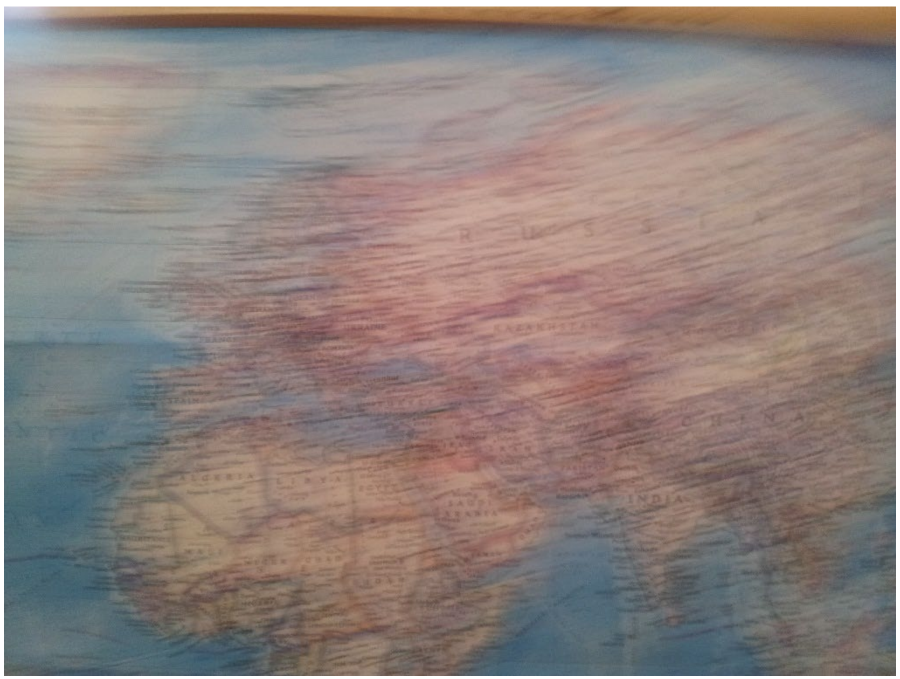

# Acquired blurred image

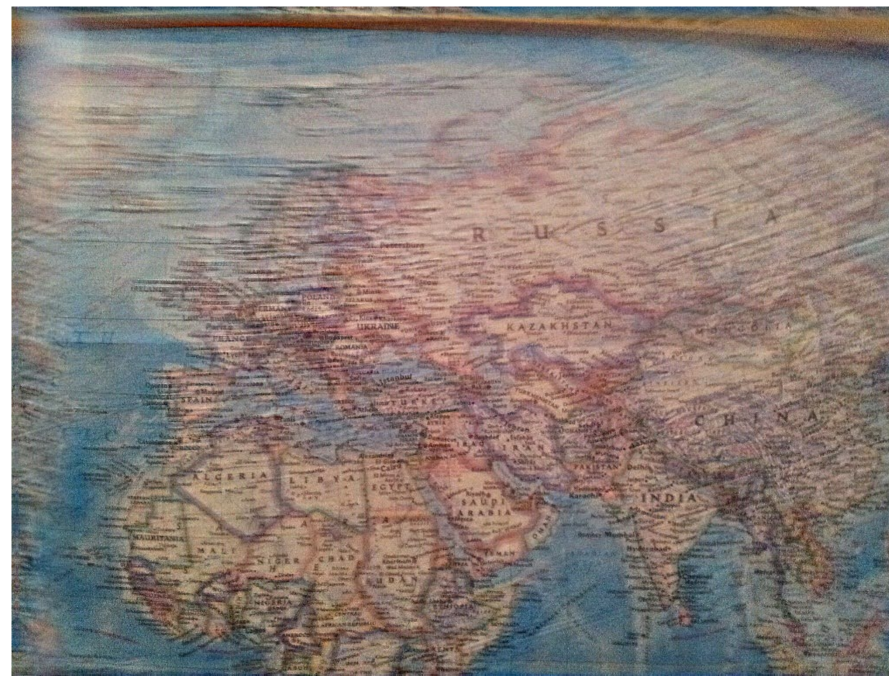

# PSF estimation

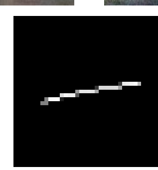



# Wiener filtering

# Wiener filtering in Samsung smartphone





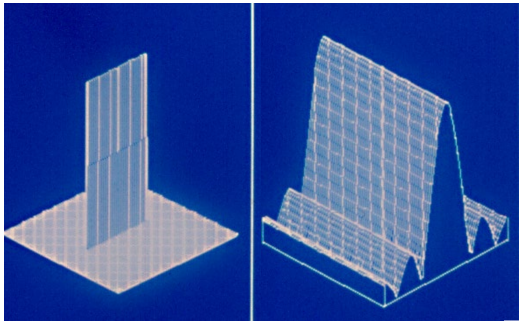



# Restoration categories

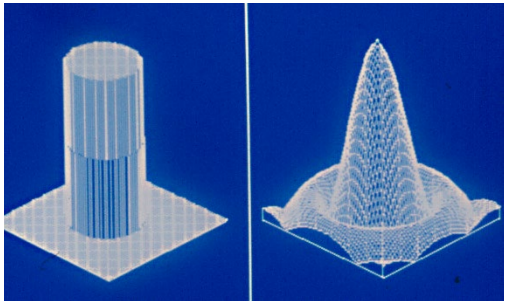
- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

# Common point-spread functions

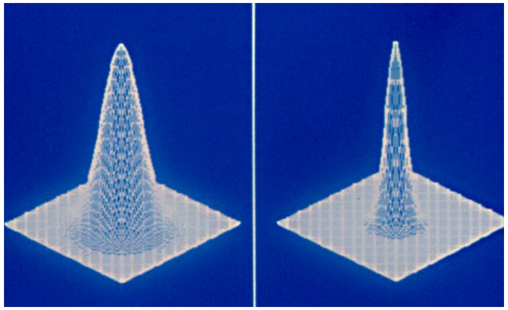
Motion blur: 1-D rectangular pulse, FT = sinc(u)

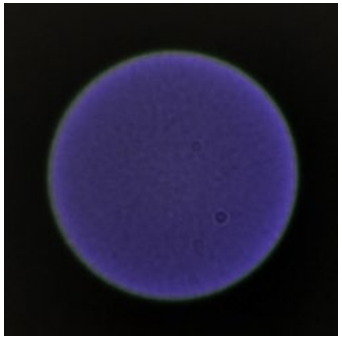


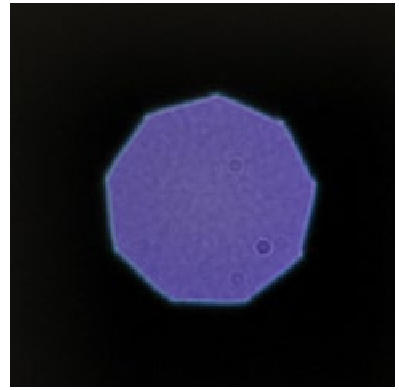
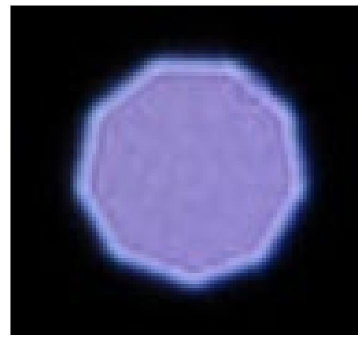
Out-of-focus blur: Cylinder, FT = B(r)/r

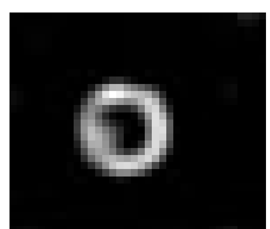


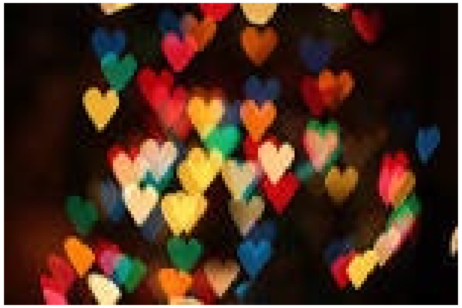
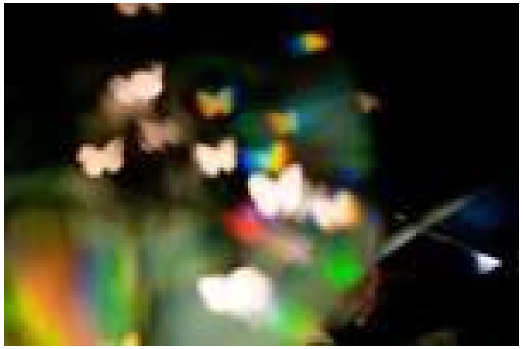
Atmospheric turbulence: Gaussian G(d), FT = Gaussian G(1/d)





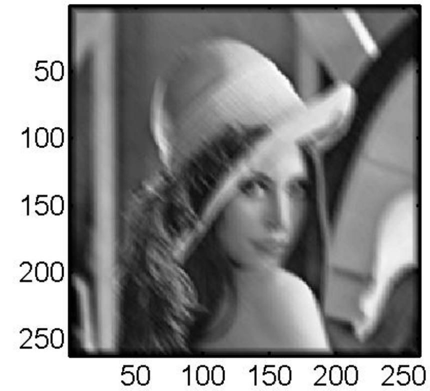



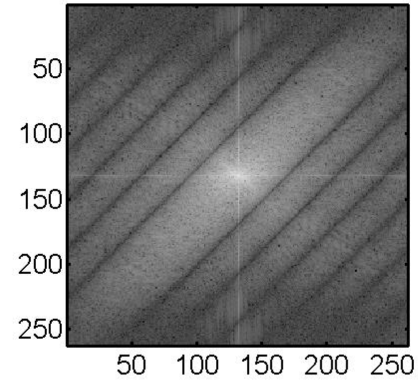


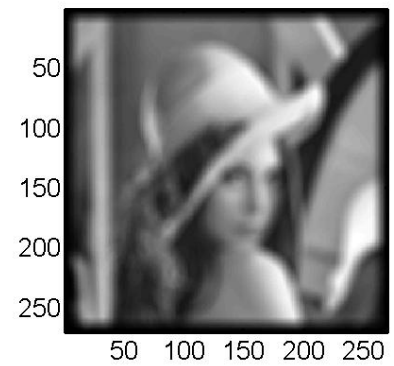
# Spectrum of a degraded image

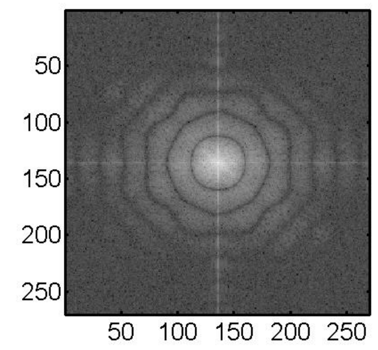
**Motion blur**



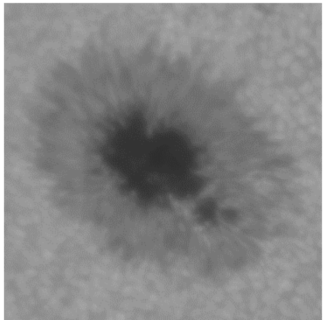



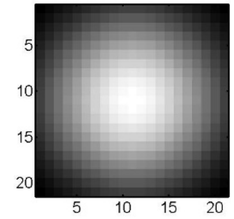

**Out-of-focus blur**




**Atmospheric turbulence blur**



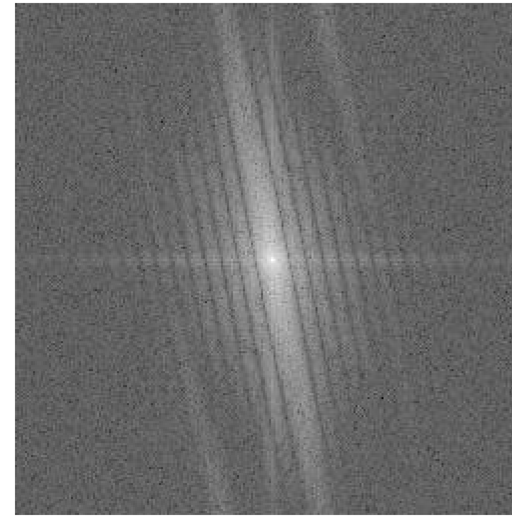


# Motion blur restoration by Wiener filter

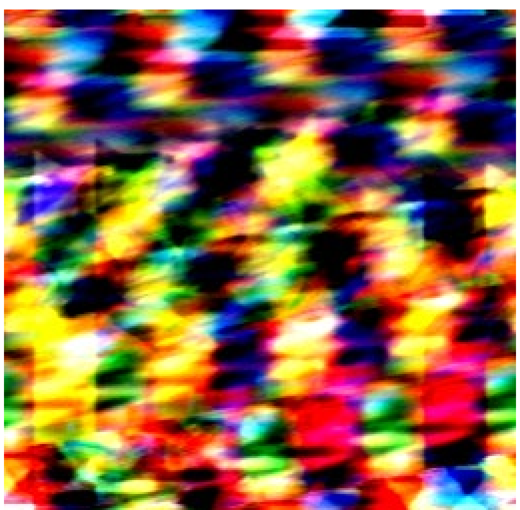
Original

Motion blurred

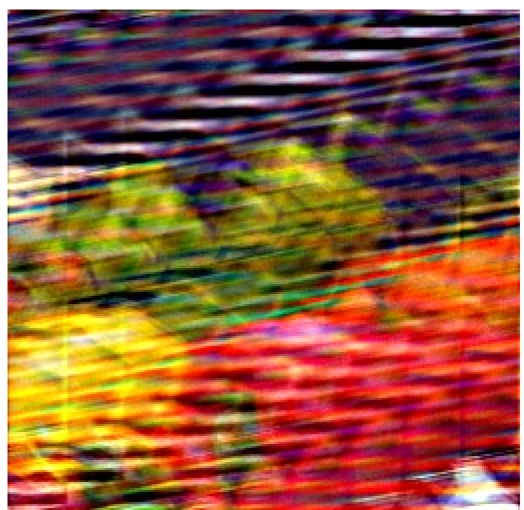
Spectrum



Wrong velocity



Wrong direction

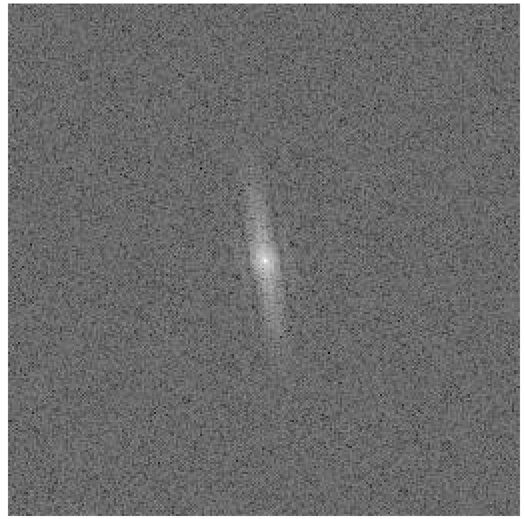


Correct PSF

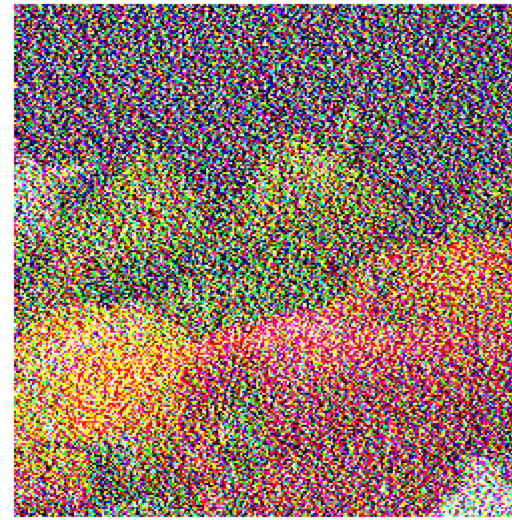
# Motion blur restoration by Wiener filter

Blur + noise

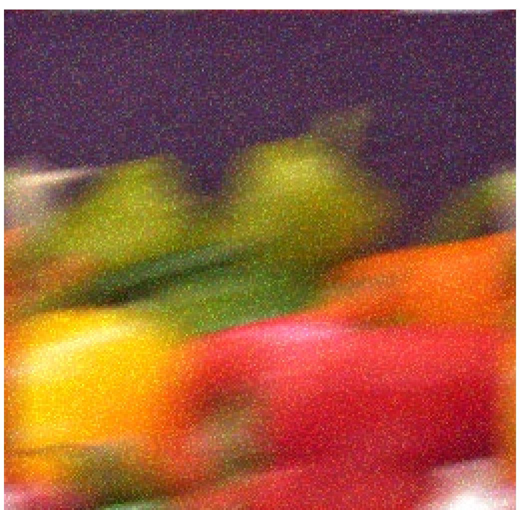
Spectrum



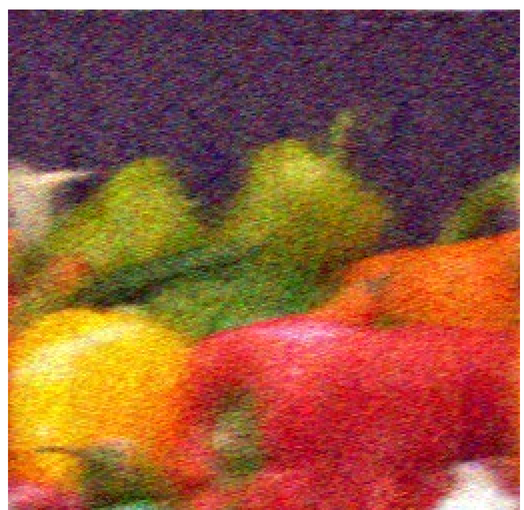
Inverse filtering



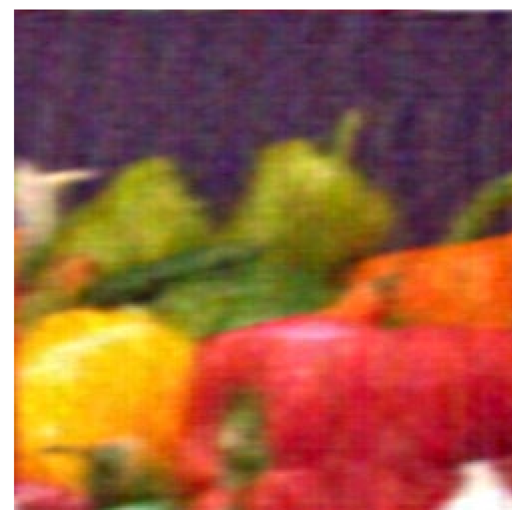
Constant SNR (good)



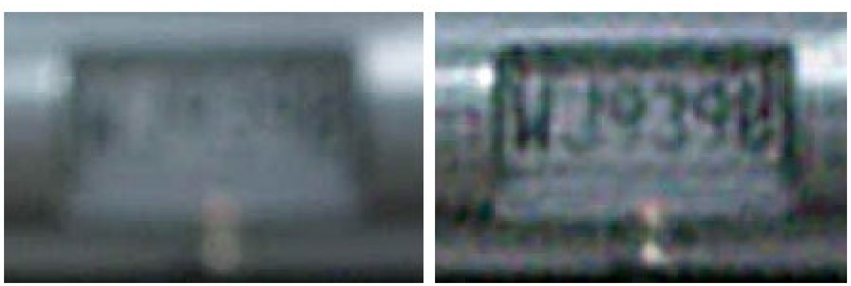
Constant SNR (wrong)



Actual SNR



# Licence plate recognition



# Restoration categories

- PSF is completely known
- PSF is constant and of a known parametric shape
- PSF is constant and unknown
- PSF is variable and unknown

# Blind deconvolution

$$z(x) = (h * u)(x) + n(x)$$

- almost impossible to resolve
- solution ambiguity

$$z(x) = ((h_1 * h_2 * \cdots * h_L) * u)(x) + n(x)$$

# Alternating Minimization

$$\min_{u,h} E(u,h) = \min_{u,h} \frac{1}{2}\|h * u - z\|^2 + \lambda Q(u) + \gamma R(h)$$

- Alternating Minimization

  1. *u-step:* $\tilde{u} = \arg\min_u E(u, \tilde{h})$

  2. *h-step:* $\tilde{h} = \arg\min_h E(\tilde{u}, h)$

  3. *repeat 1 and 2.*

# Example of blind deconvolution


Blurred image
$z(x)$

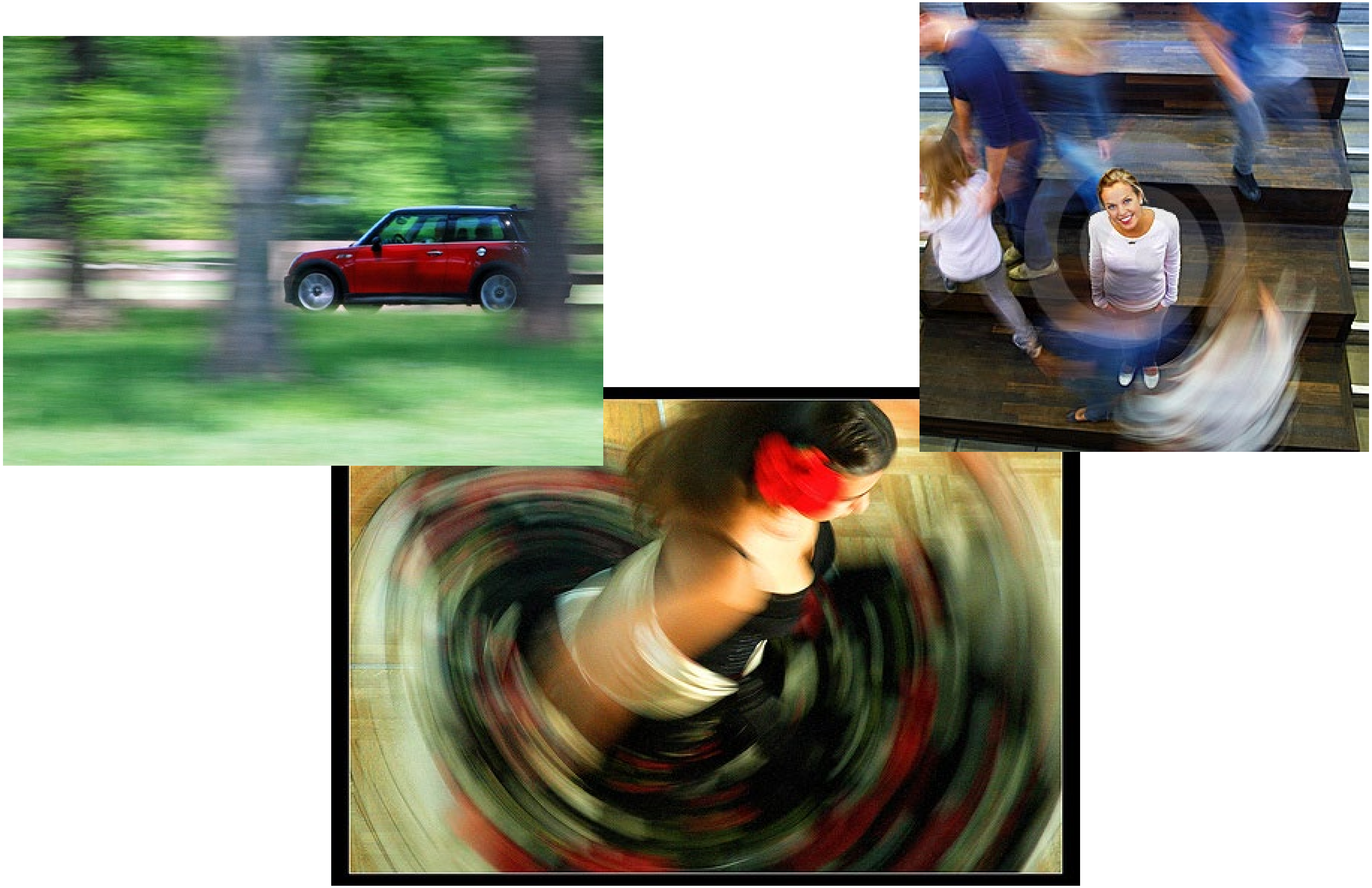

Reconstructed image
$\tilde{u}(x)$

# Image blind deconvolution

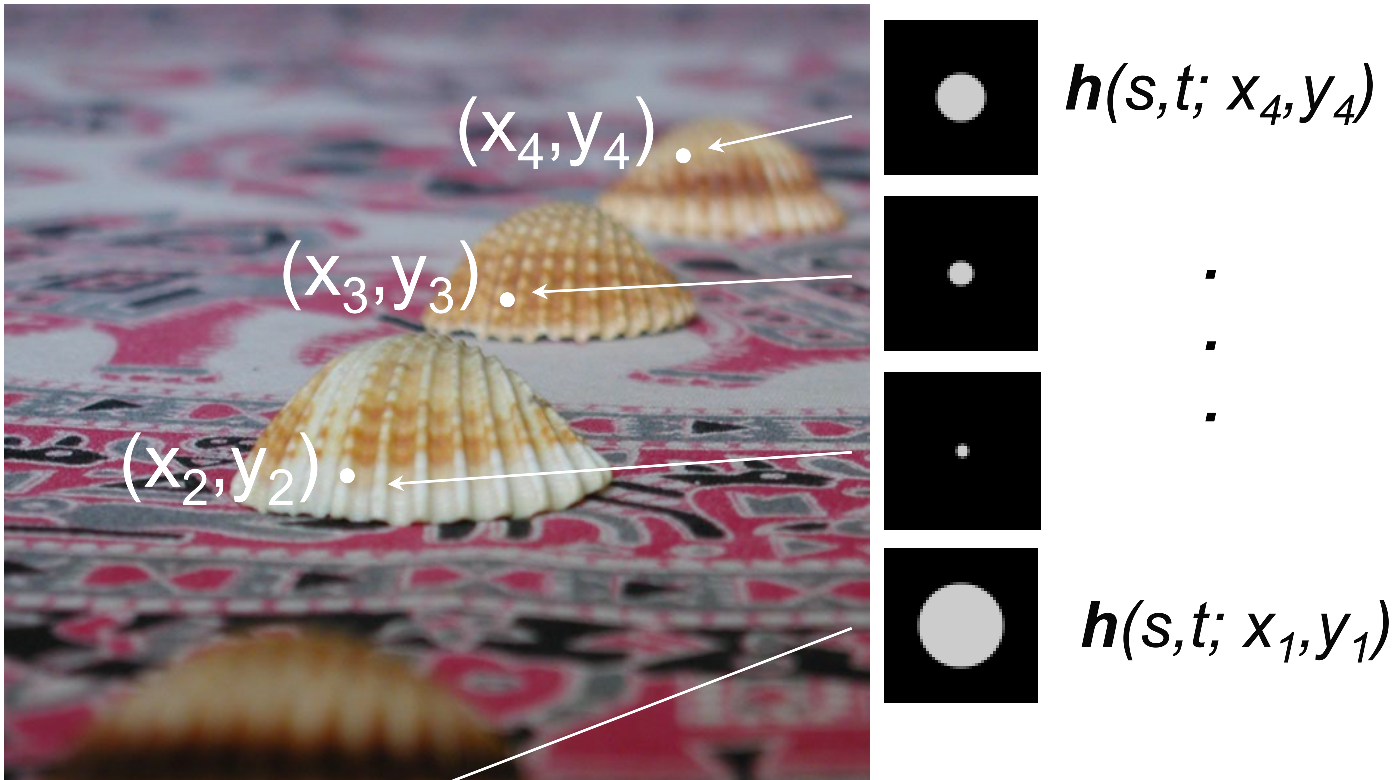
# Restoration categories

- **PSF is completely known**
- **PSF is constant and of a known parametric shape**
- **PSF is constant and unknown**
- **PSF is variable and unknown**

# Space-variant PSF



# Space-variant PSF

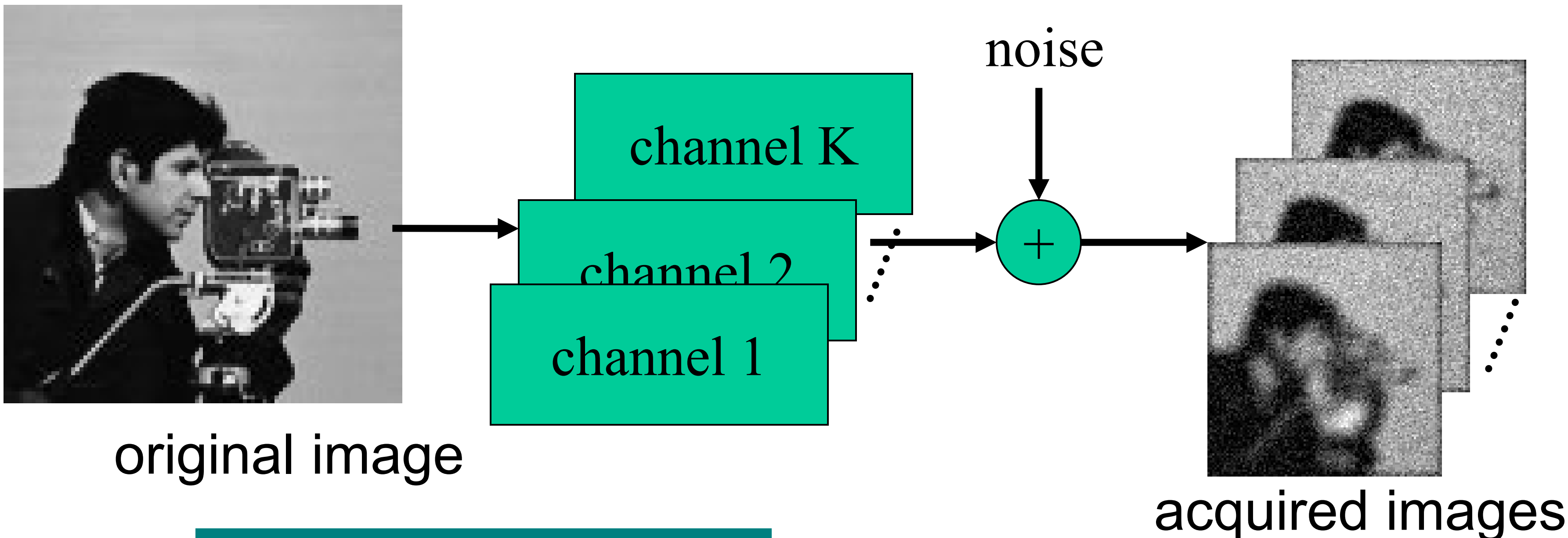



$$\mathbf{z}(x,y) = \int_{\Omega} \mathbf{u}(x-s,y-t)\mathbf{h}(s,t;x-s,y-t)\,dsdt + \mathbf{n}(x,y)$$

# Multichannel image restoration

- Assumptions:
- Several input images of the same scene are available
- They are blurred by convolution with different convolution kernels
- The original scene does not change during the acquisitions

# Multichannel acquisition model




$$[u * h_k](x, y) + n_k(x, y) = z_k(x, y)$$

# MC Blind Deconvolution

- System of integral equations

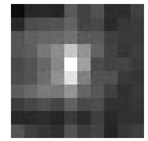
  (ill-posed, underdetermined)

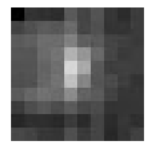
$$z_k(x) = (h_k * u)(x) + n_k(x)$$

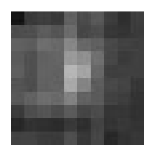
- Energy minimization problem (well-posed)

$$E(u, \{h_i\}) = \frac{1}{2} \sum_{i=1}^{K} \|h_i * u - z_i\|^2 + \lambda Q(u) + \gamma R(\{h_i\})$$

# Out-of-focus Camera





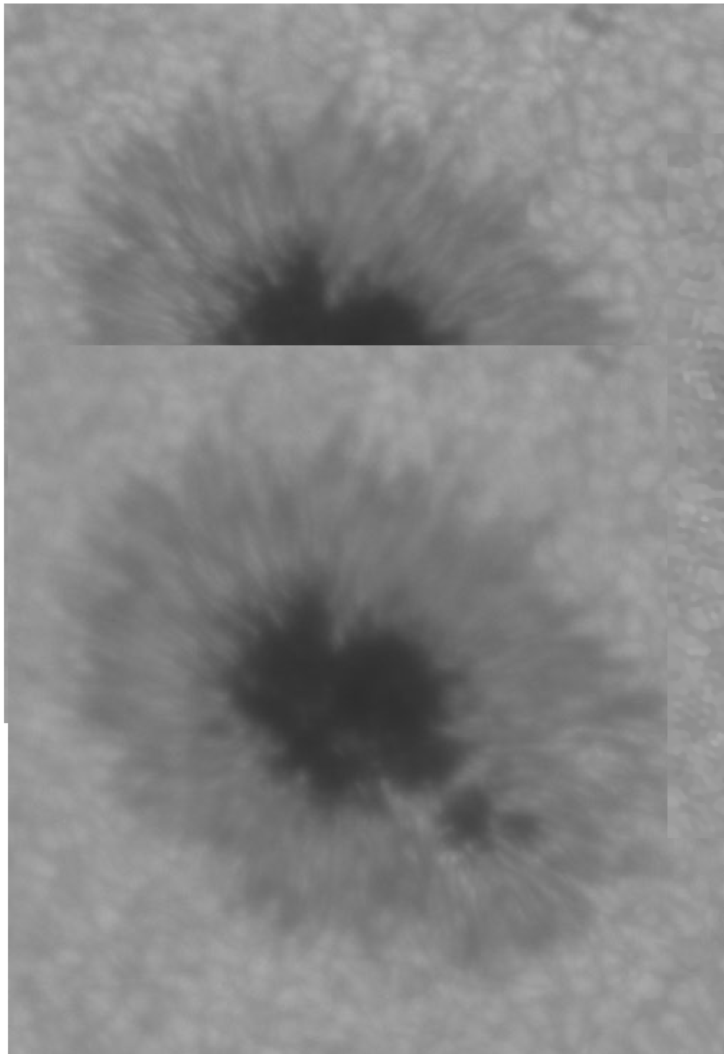


reconstructed

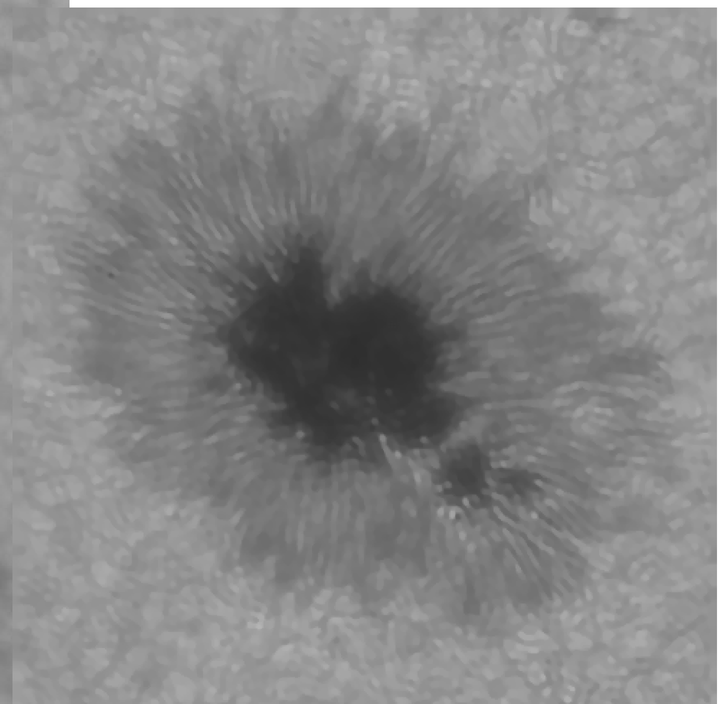

in focus

# Astronomical images

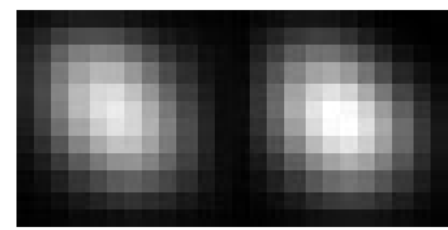
Degraded images



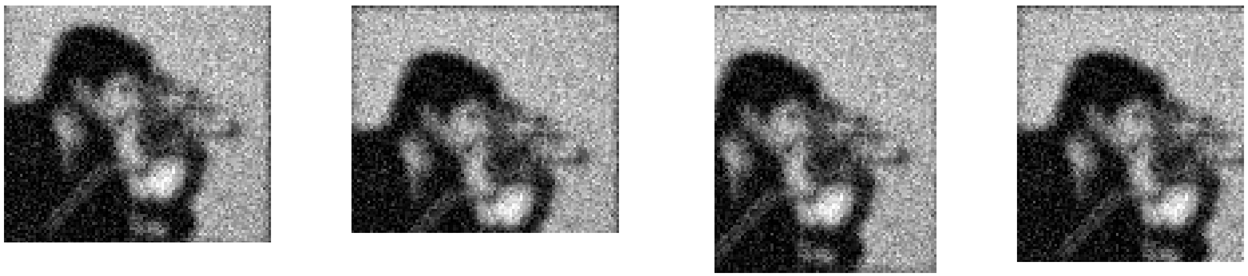
Reconstructed image



PSF estimation

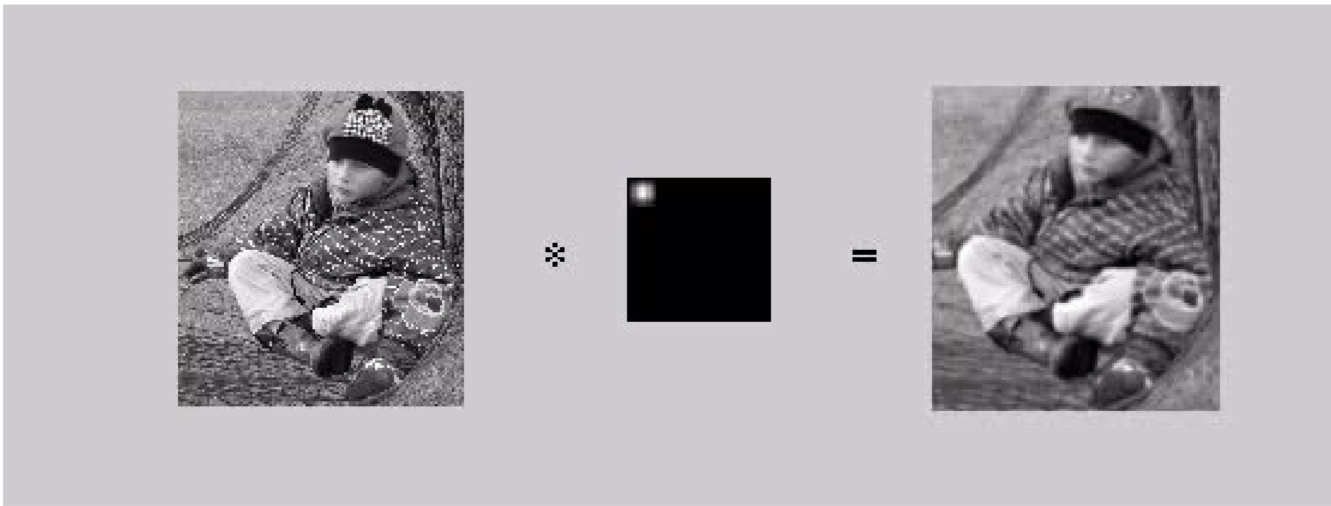


# Misregistration of the channels



... leads to artefacts if not handled properly

# Incorporating a between-image shift

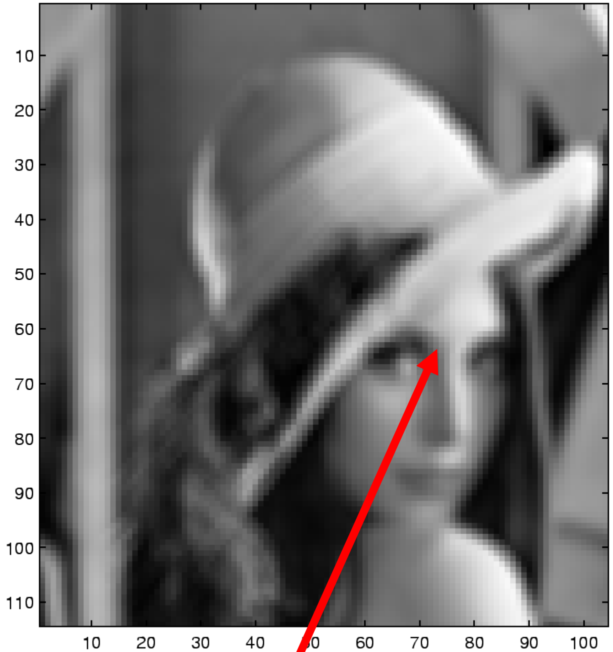


original image PSF degraded image

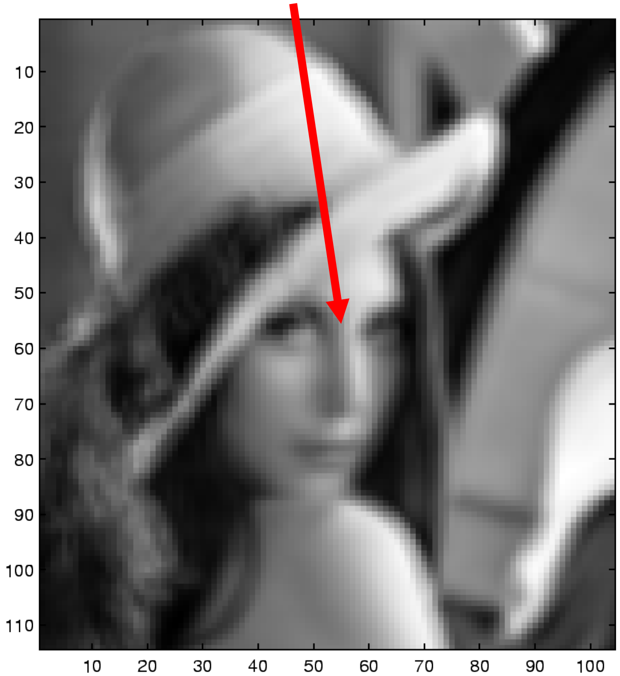
$$[u * h_k](\tau_k(x,y)) + n_k(x,y) = z_k(x,y)$$

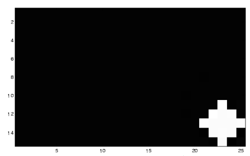
$$[u * g_k](x,y) + n_k(x,y) = z_k(x,y)$$

# Simulated blurring



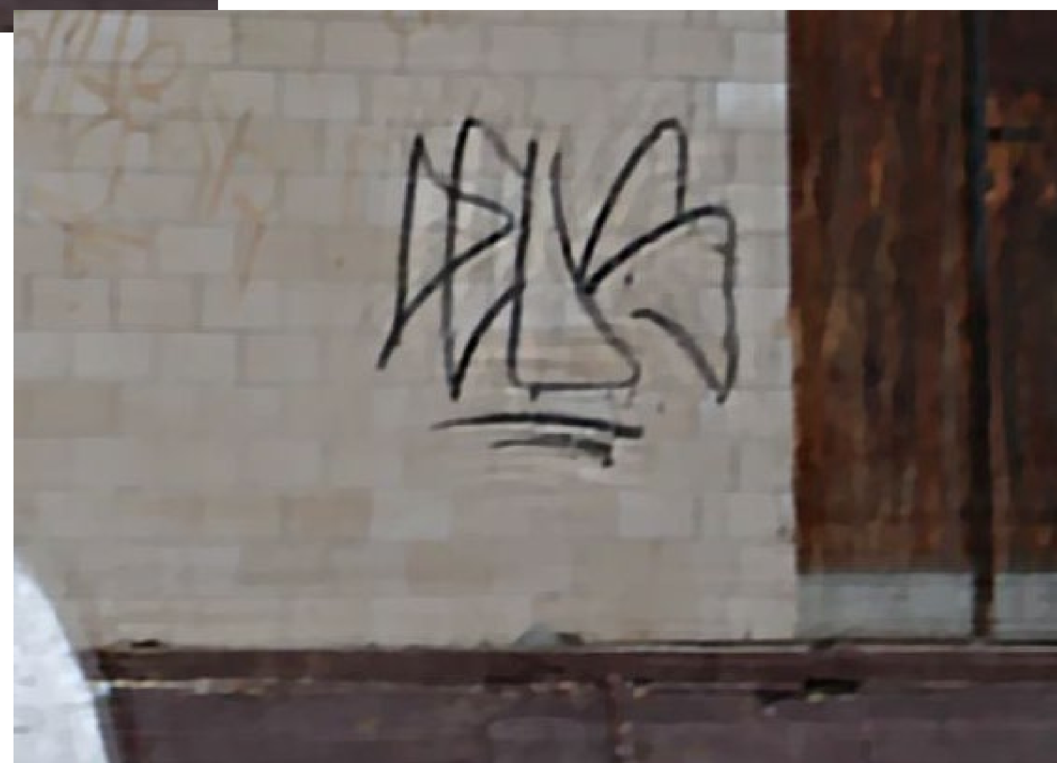
misalignment



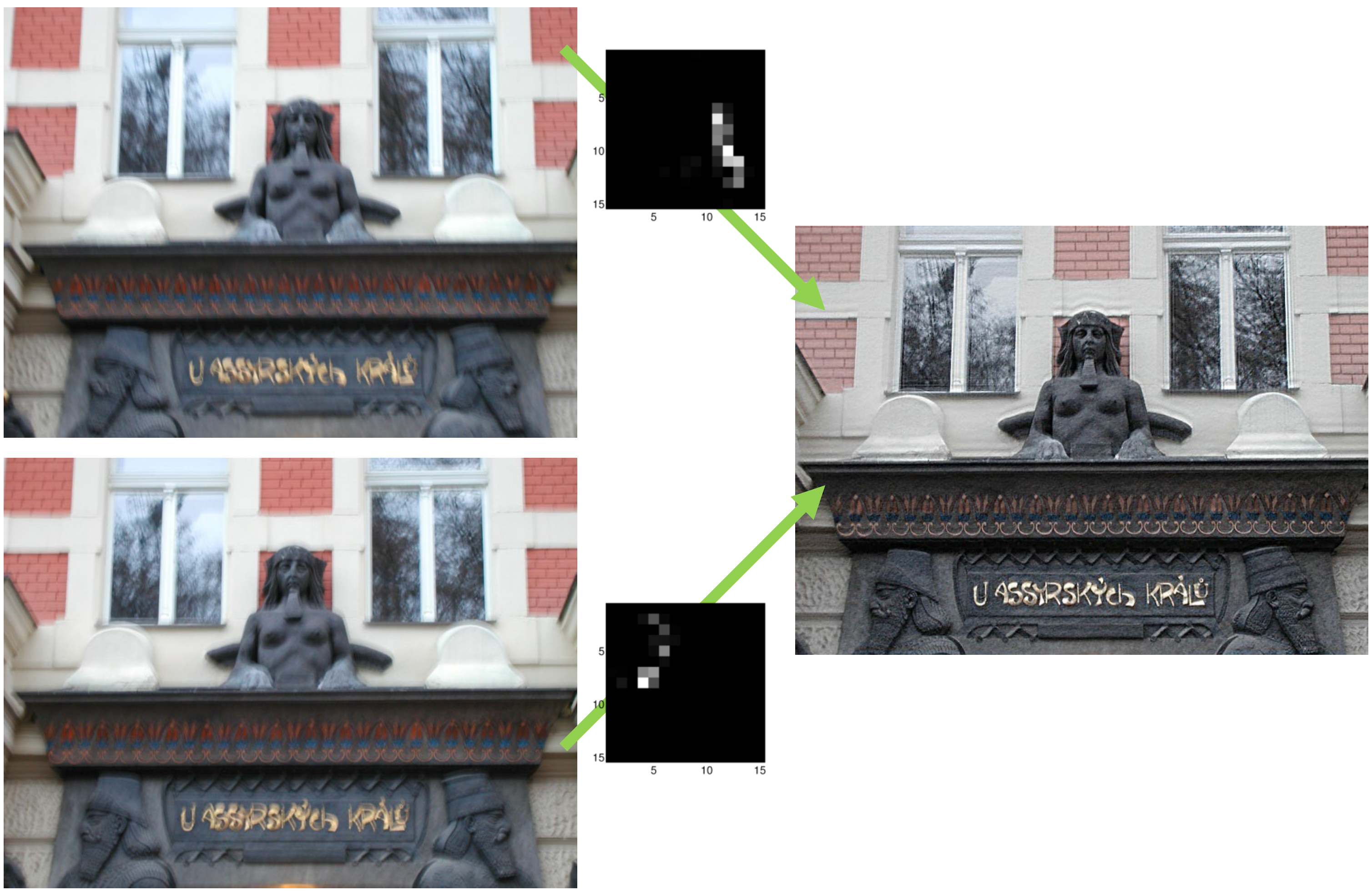


misalignment compensation

reconstructed image



# Long-time exposure I

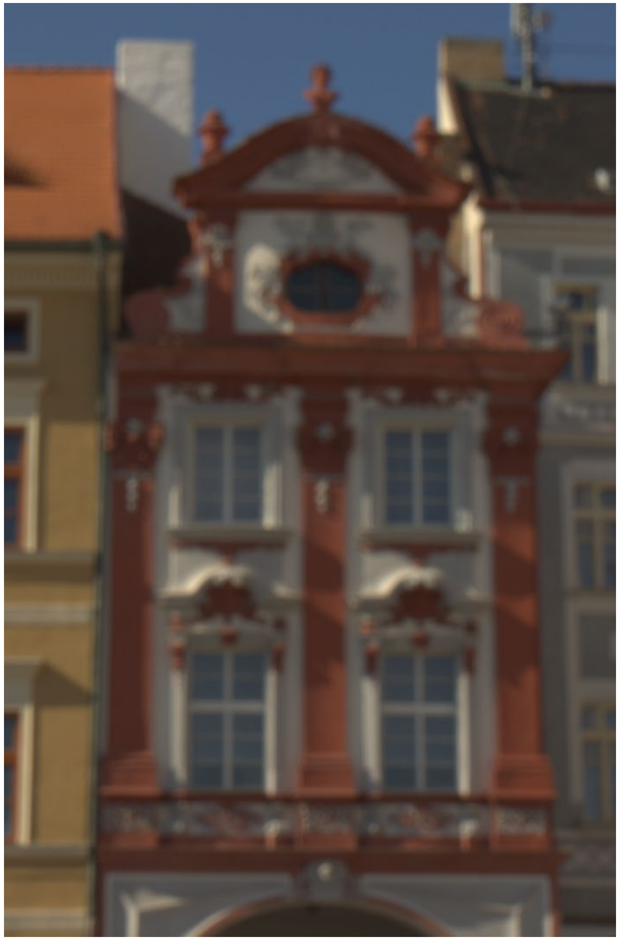


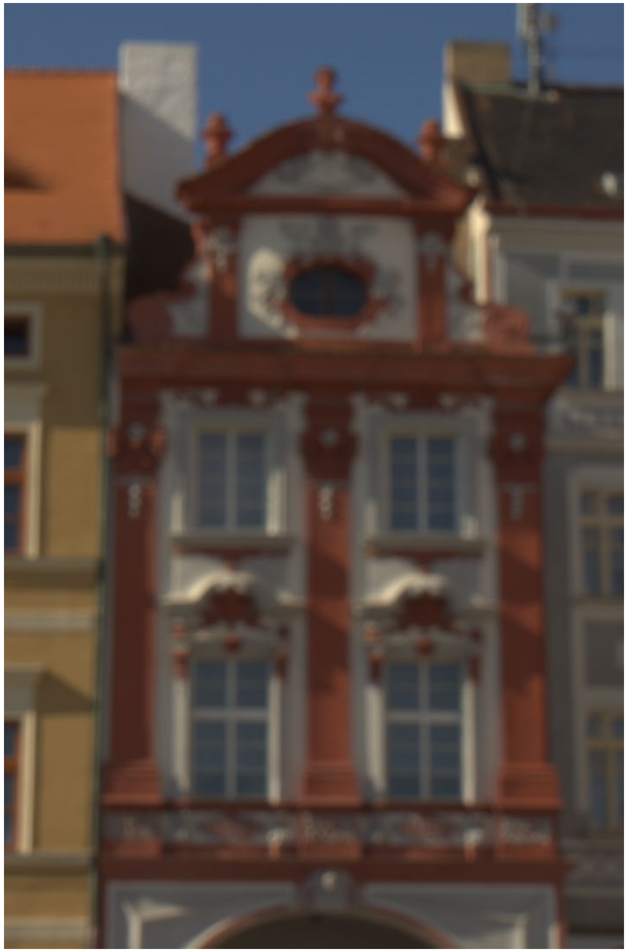
(a) Blurred input images, $1024 \times 768$ pixels

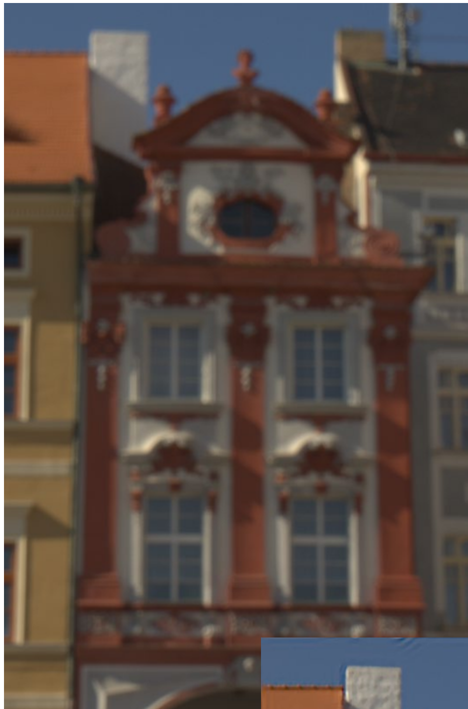
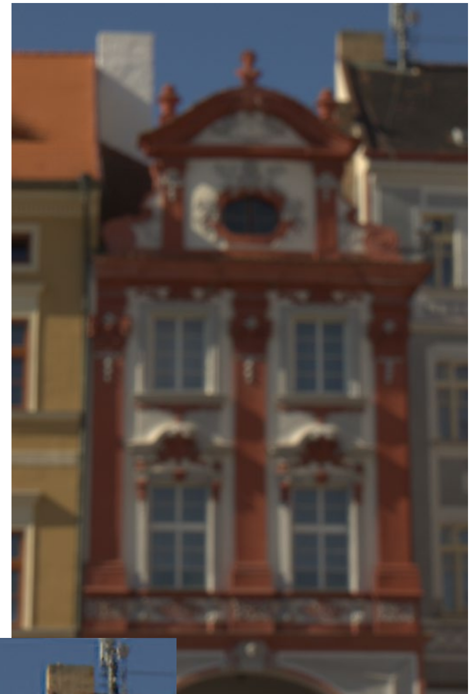
(b) Deconvolution from only first image

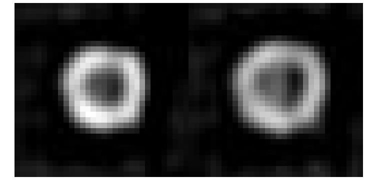
(c) Result of multi-image deconvolution

# Out of focus blur








# Super-resolution imaging

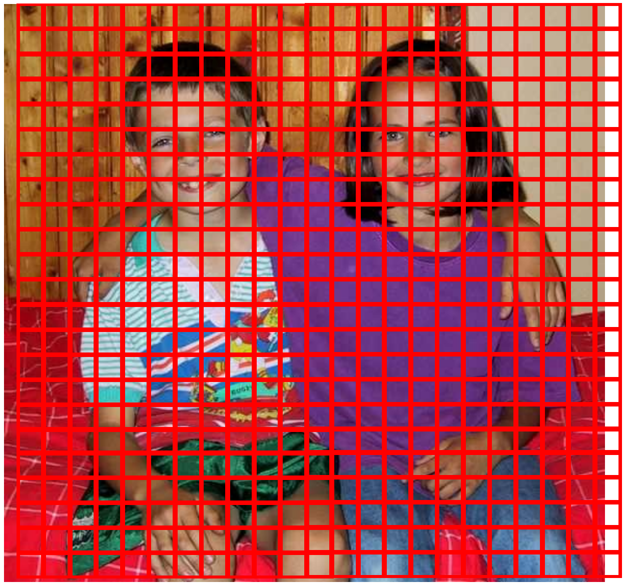
**Method:** Multiple acquisitions of the same scene with subpixel shift

# Aliasing

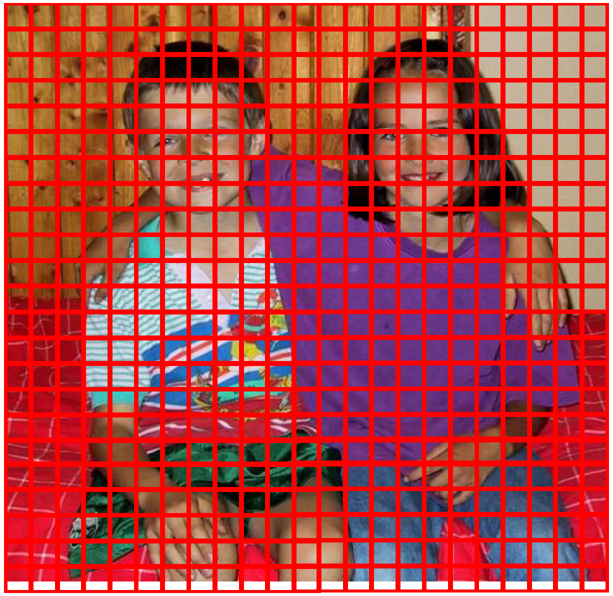
- The loss of the high frequencies (details) due to an insufficient resolution of the camera

# To suppress aliasing, apply multiple acquisitions with sub-pixel shifts

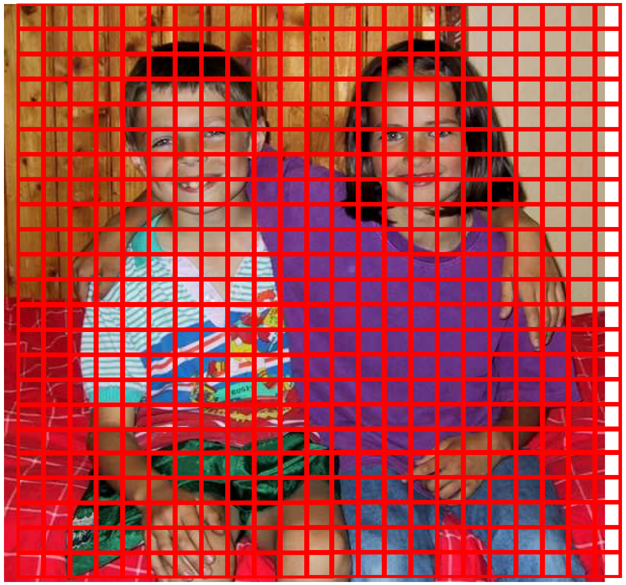
# Multiple acquisitions with shifts



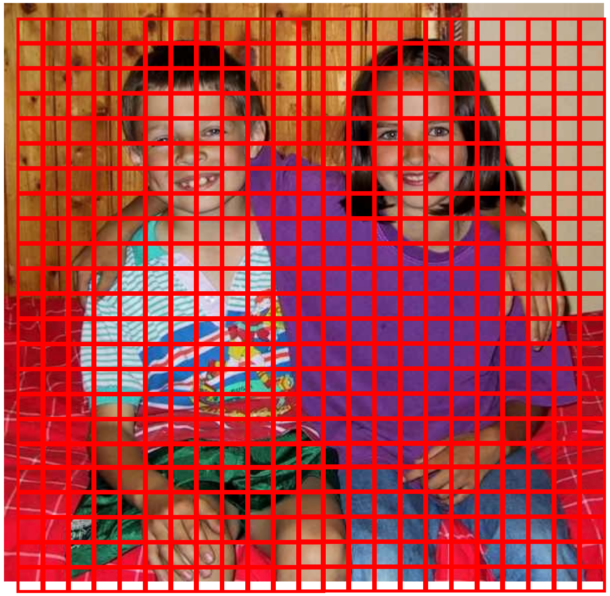
# Multiple acquisitions with shifts



# Multiple acquisitions with shifts



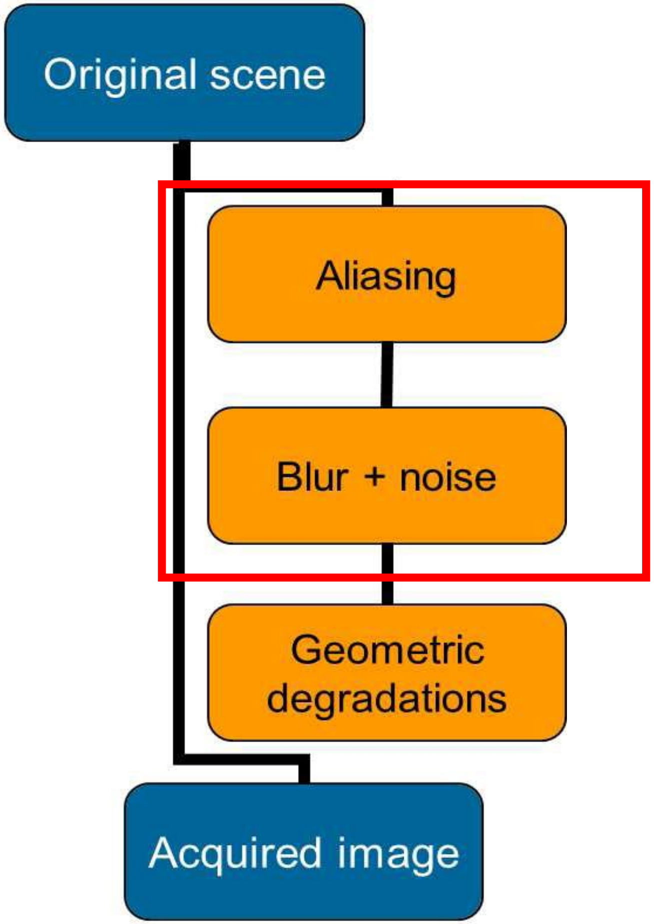
# Multiple acquisitions with shifts



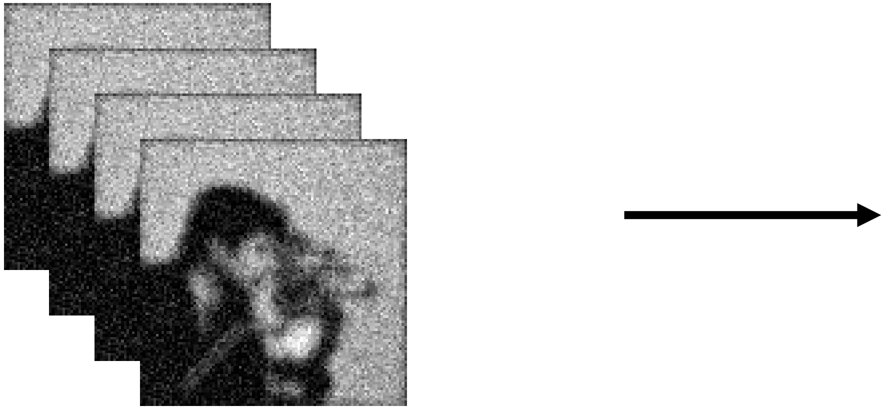

The image resolution has increased
This process is called super-resolution

# Realistic superresolution

SR must include also de-blurring



# Realistic superresolution



$$E(u, \{g_i\}) = \frac{1}{2} \sum_{i=1}^{K} ||D(g_i * u) - z_i||^2 + \lambda Q(u) + \gamma R(\{g_i\})$$