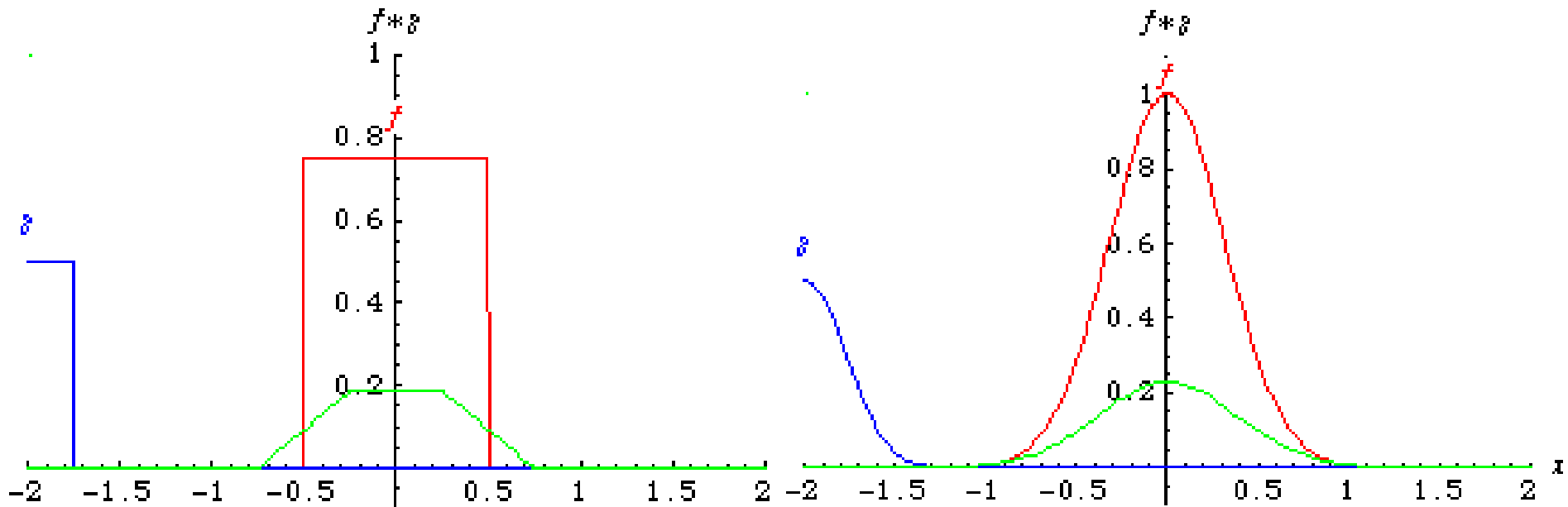


Konvoluce

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau$$

$$* : L_1 \times L_1 \rightarrow L_1$$



Vlastnosti

$$f * g = g * f$$

$$f * (g * h) = (f * g) * h$$

$$a(f * g) = (af) * g = f * (ag)$$

$$f * (g + h) = (f * g) + (f * h)$$

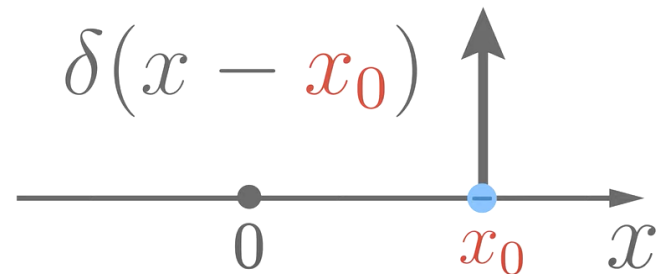
$$f * \delta = \delta * f = f$$

Dirac delta funkce

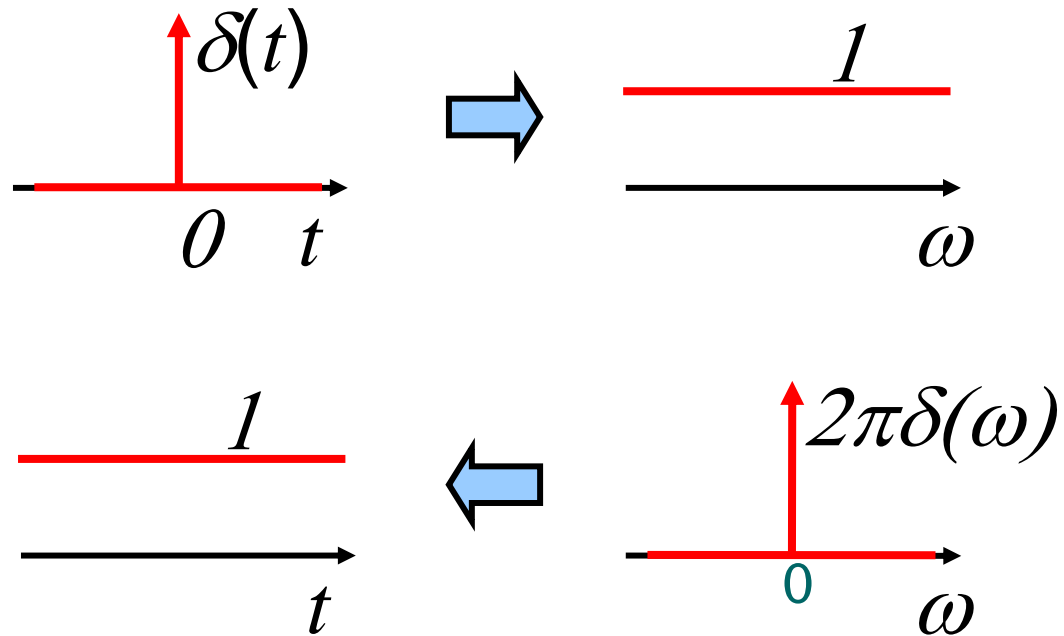
$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0, & x \neq 0 \end{cases} \quad \int \delta(x) dx = 1$$

$$\int_{-\infty}^{\infty} f(x)\delta(x - a) dx = f(a)$$

$$\delta(ax) = \frac{\delta(x)}{|a|}$$



Dirac delta funkce



Hranice



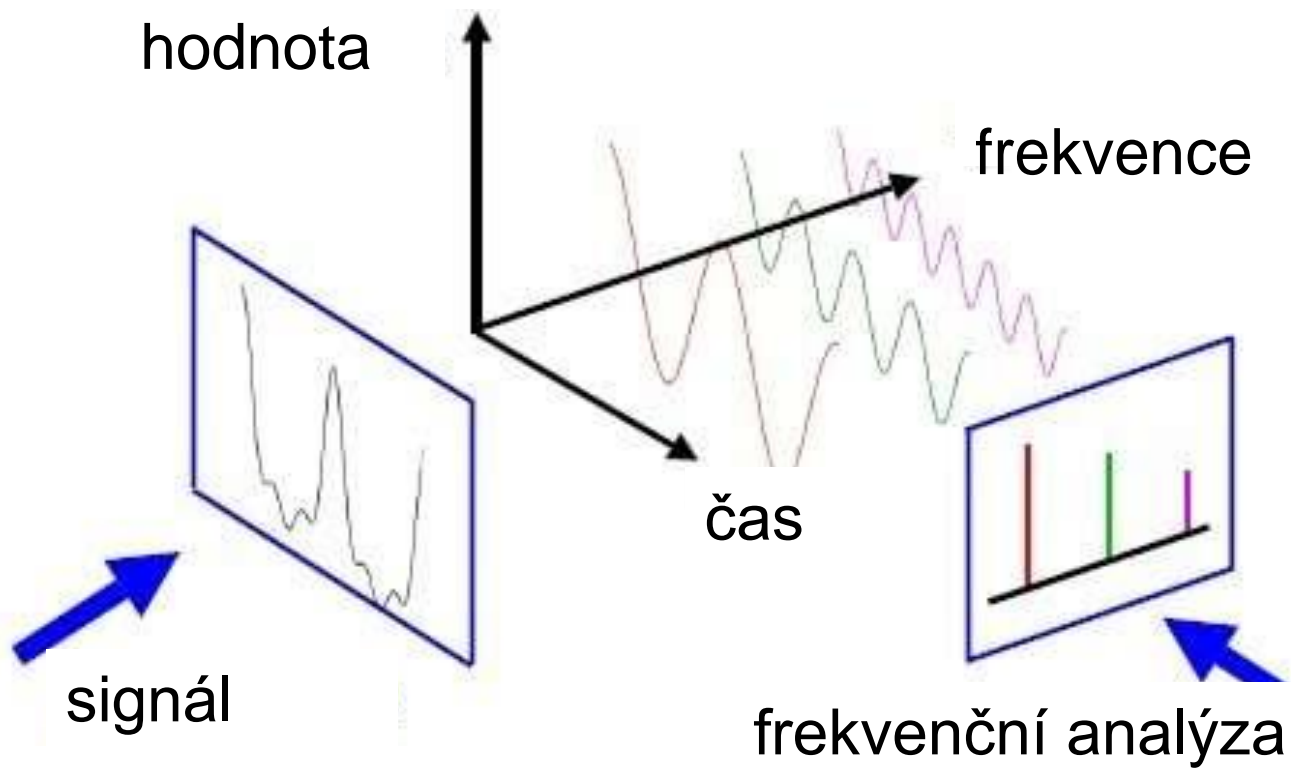
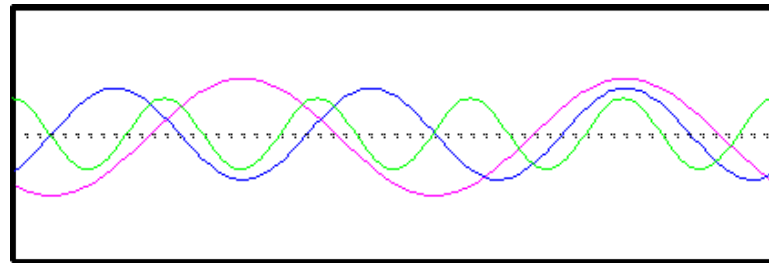
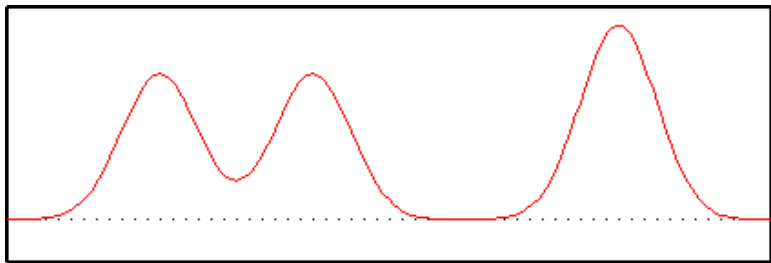
Convolution
mask



Ošetření hranice

- Zero padding
- Mirror extension
- Periodic extension

Vlnění



Fourier transform

Fourierovy řady

$\{\phi_k\}$ – UONS v \mathcal{H} , $f \in \mathcal{H}$

úplný ortonormální systém

$c_k = (f, \phi_k)$ – Fourierovy koeficienty

$$f = \sum_{i=1}^{\infty} c_k \phi_k$$



$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx),$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

Fourier transform

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx.$$

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x} dk$$

Existence FT – stačí $f \in L_1$

$$F = \mathcal{R}(F) + i\mathcal{I}(F) = |F| \cdot e^{i \cdot \text{ph}(F)}$$

Vlastnosti FT

- linearity

$$\mathcal{F}[af(x) + bg(x)] = a\mathcal{F}[f(x)] + b\mathcal{F}[g(x)] = aF(k) + bG(k).$$

- convolution

convolution theorem

$$\mathcal{F}[f]\mathcal{F}[g] = \mathcal{F}[f * g]$$

- shift

shift theorem

$$\mathcal{F}_x[f(x - x_0)](k) = e^{-2\pi i k x_0} F(k).$$

- rotation

rotational theorem

$$\mathcal{F}\{f(R_\theta \mathbf{r})\} = F(R_\theta \mathbf{k})$$

- scaling

similarity theorem

$$\mathcal{F}_x[f(ax)](k) = |a|^{-1} F\left(\frac{k}{a}\right).$$

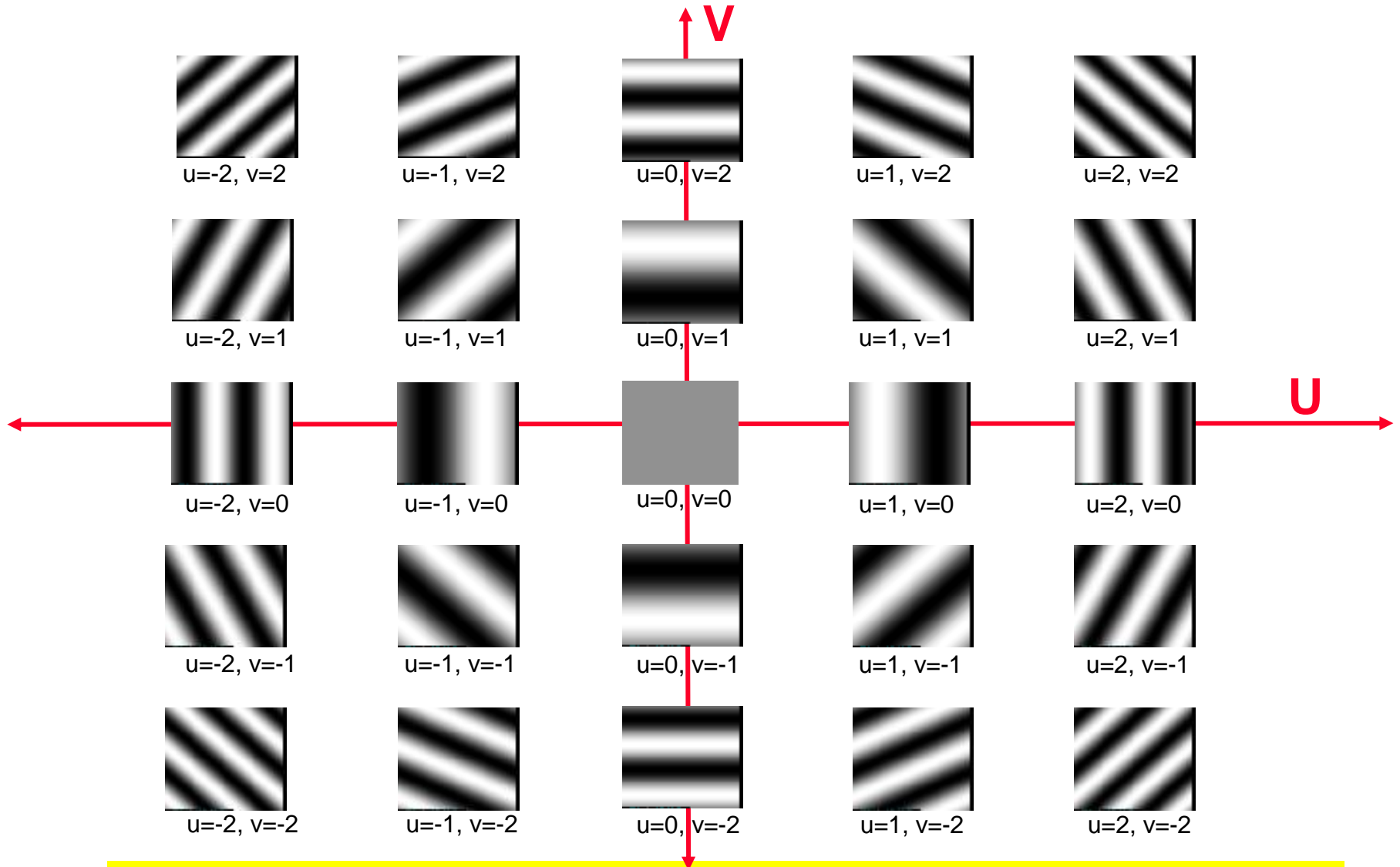
2D Fourier transform

$$\{\phi_{uv}(x, y)\} = \{e^{-2\pi i(ux+vy)}\}$$

$$F(u, v) = \int \int_{\mathbb{R}^2} f(x, y) e^{-2\pi i(ux+vy)} dx dy$$

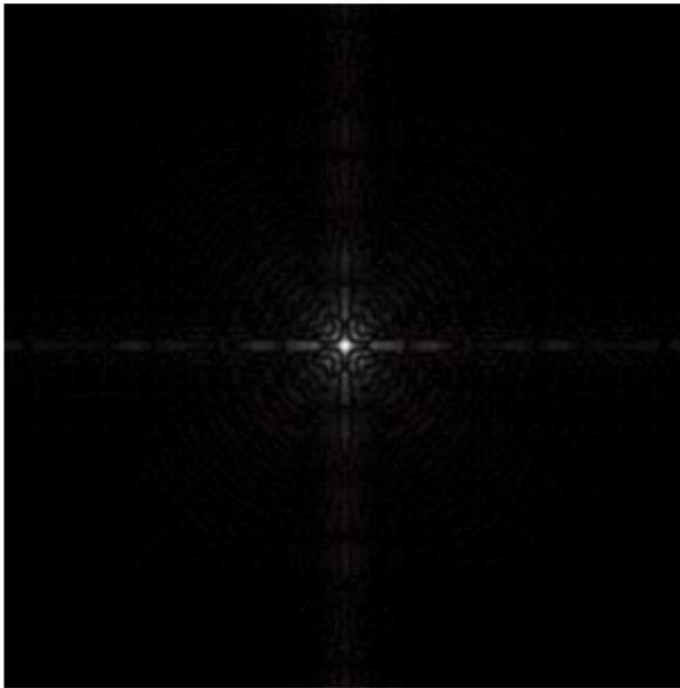
$$f(x, y) = \int \int_{\mathbb{R}^2} F(u, v) e^{-2\pi i(ux+vy)} du dv$$

The 2D Basis Functions $e^{2\pi i(ux+vy)}$

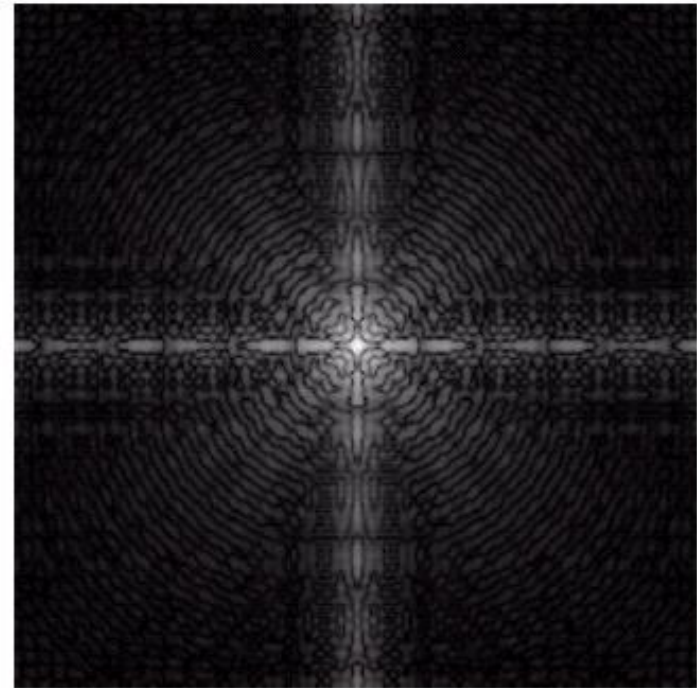


The wavelength is $1/\sqrt{u^2 + v^2}$. The direction is u/v .

Zobrazení spektra

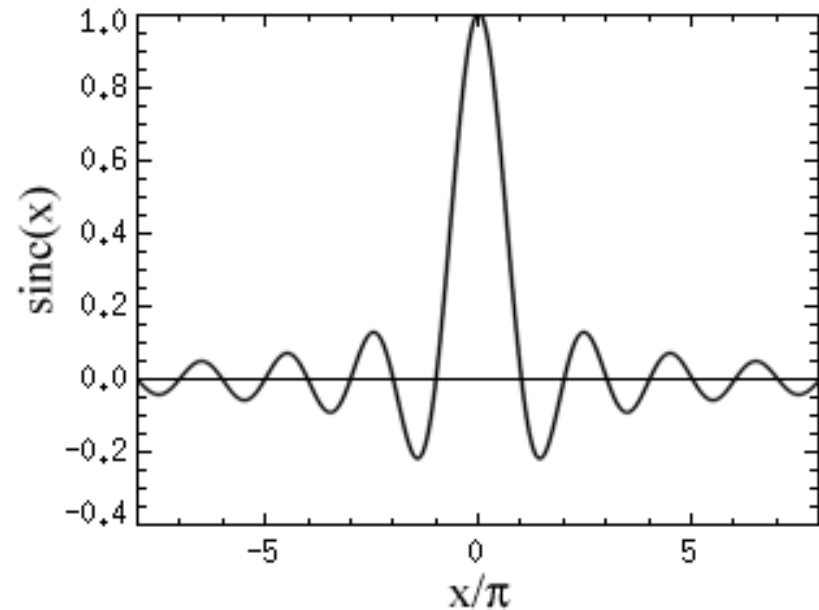
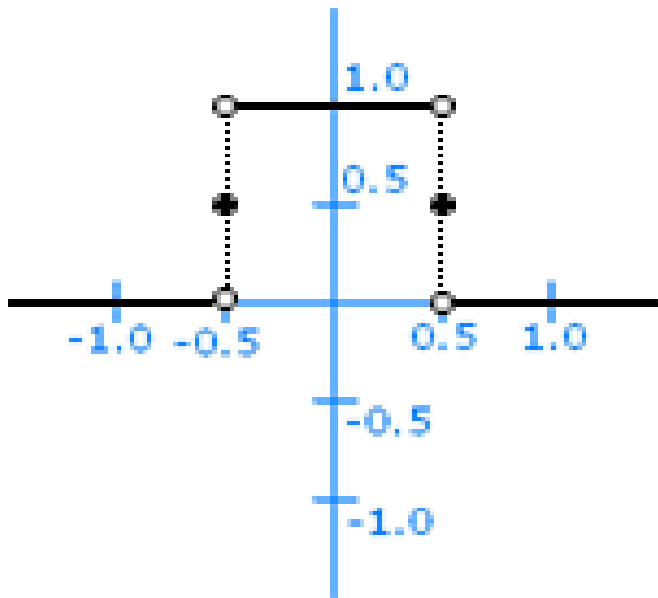


ampl



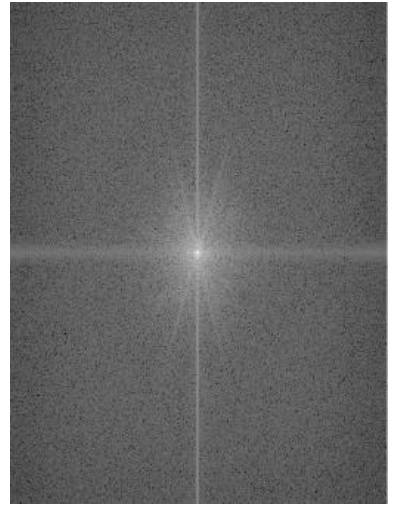
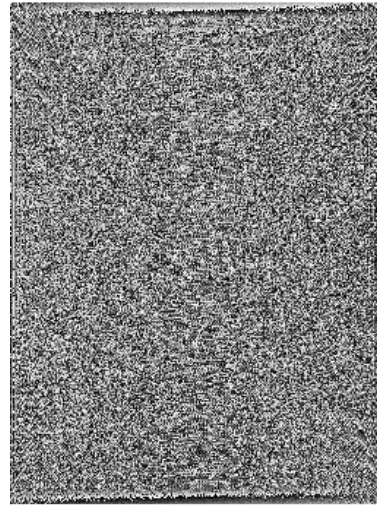
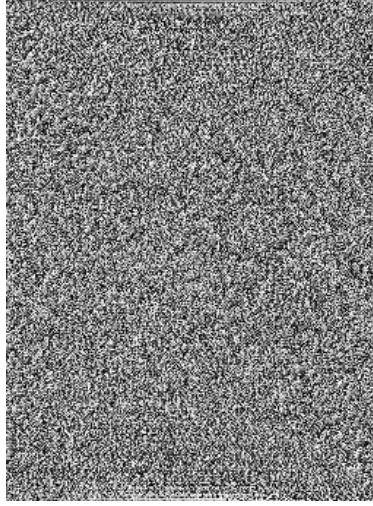
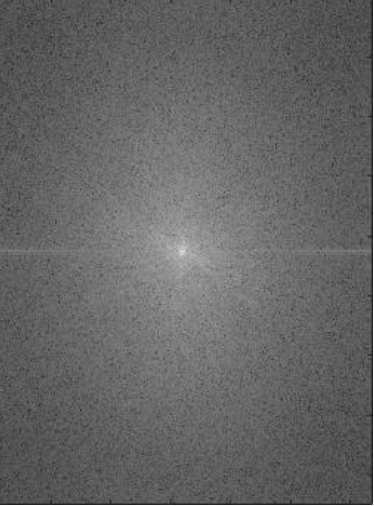
$\log(\text{ampl} + 1)$

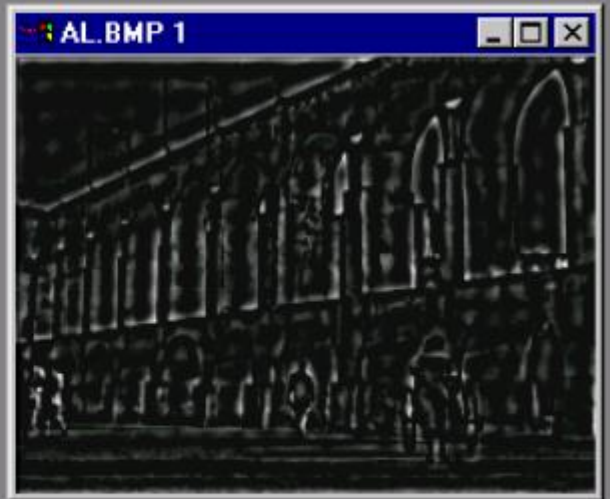
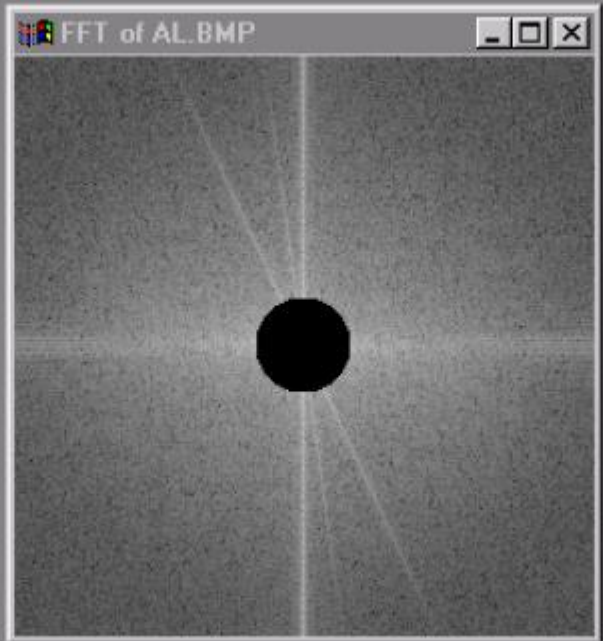
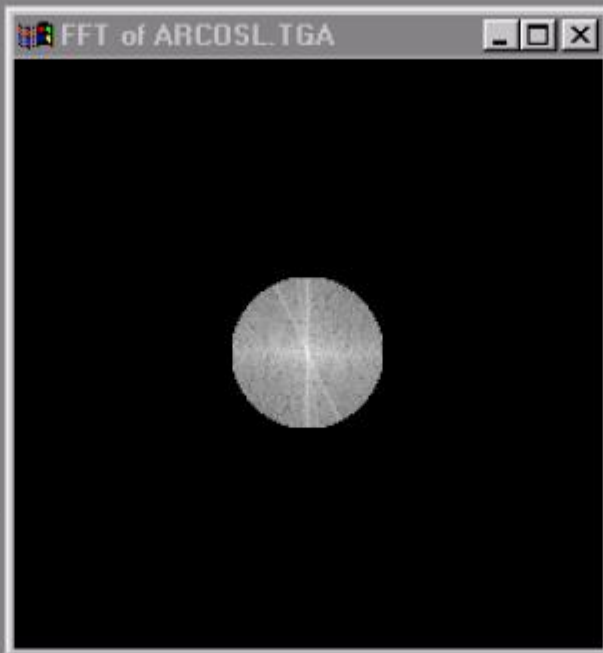
Obdélníkový puls



$$\Pi(t) = \begin{cases} 1 & \text{for } |t| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{F}\{\Pi(t)\}(x) = \frac{\sin(\pi x)}{\pi x} = \text{sinc}(x)$$

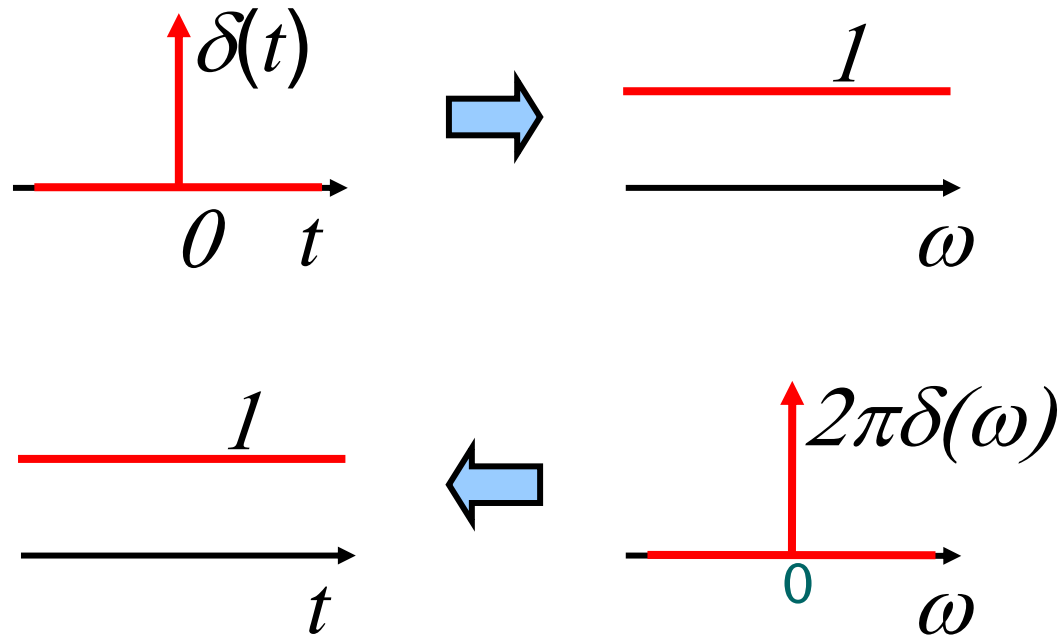




Otázky

- 1) Proofs & calculations
- 2) $FT(\delta) = ??$
- 3) Symmetry of the FT

Dirac delta funkce

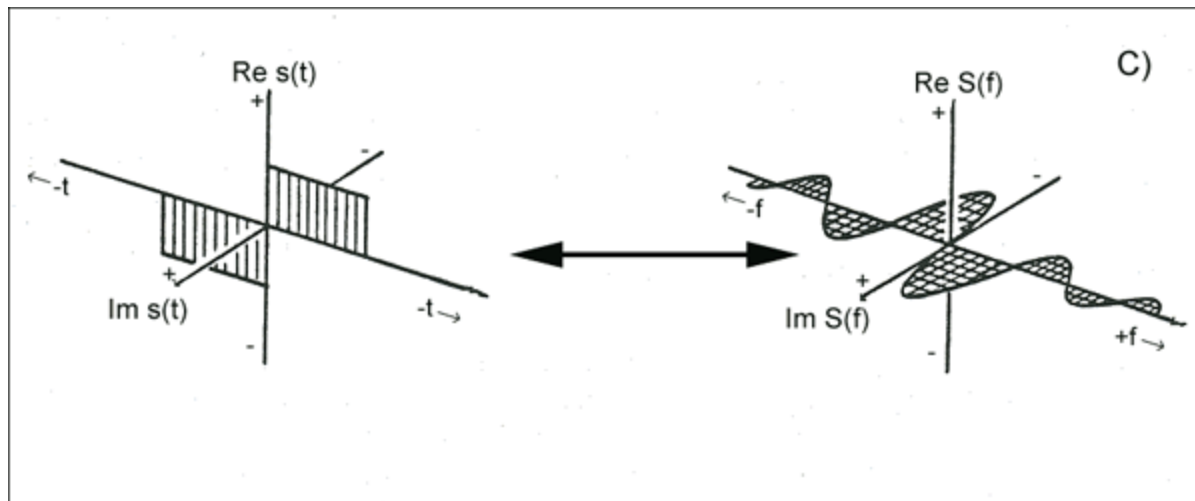
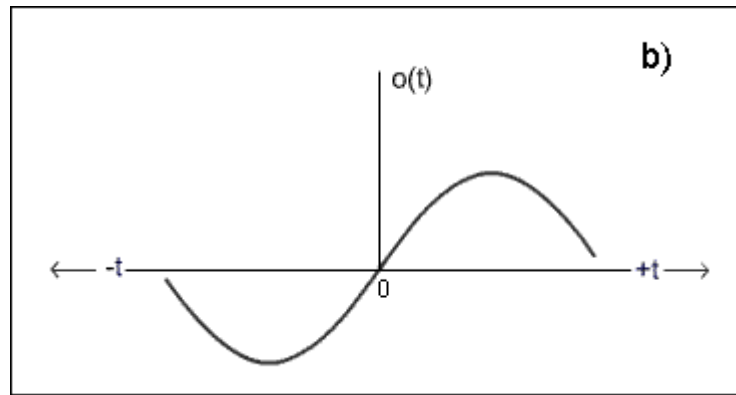
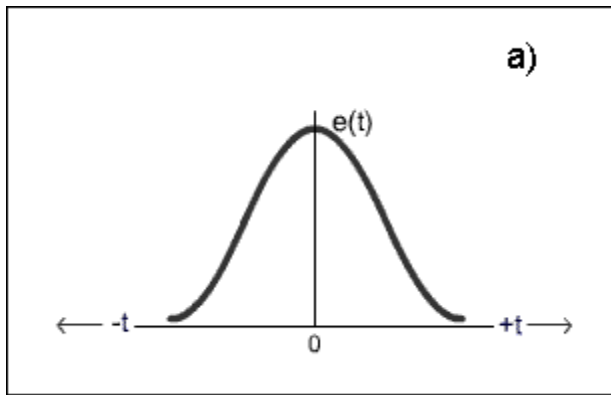


Example: What is the Fourier Transform of the Dirac Function?

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Proof : Consider any function $f(x)$

$$\begin{array}{ccc} f(x) * \delta(x) & = & f(x) \\ \downarrow & & \downarrow \\ F(u) \cdot F[\delta(x)] & = & F(u) \end{array}$$



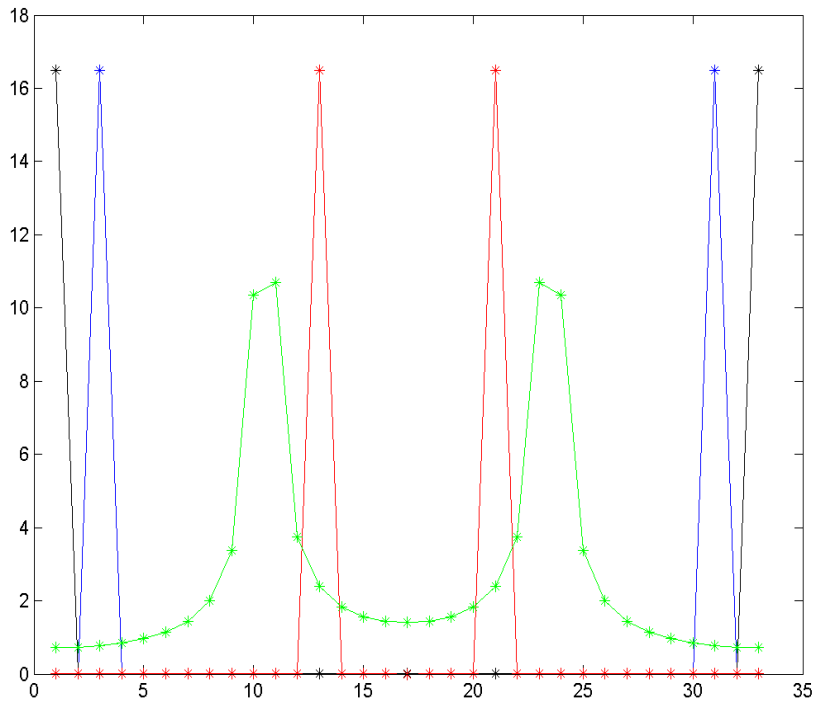
Diskrétní Fourier Transform

$$F_n = \sum_{k=0}^{N-1} f_k e^{-2\pi i n k / N}.$$

$$f_k = \frac{1}{N} \sum_{n=0}^{N-1} F_n e^{2\pi i n k / N}.$$

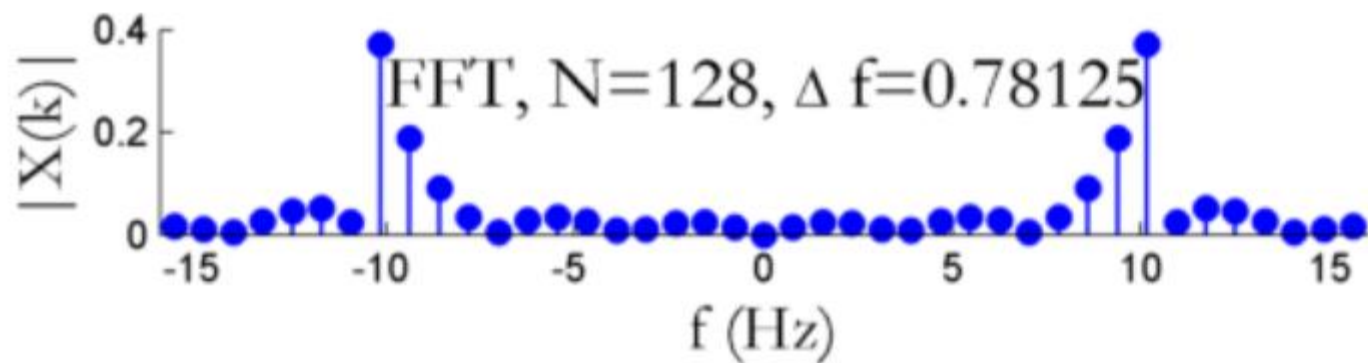
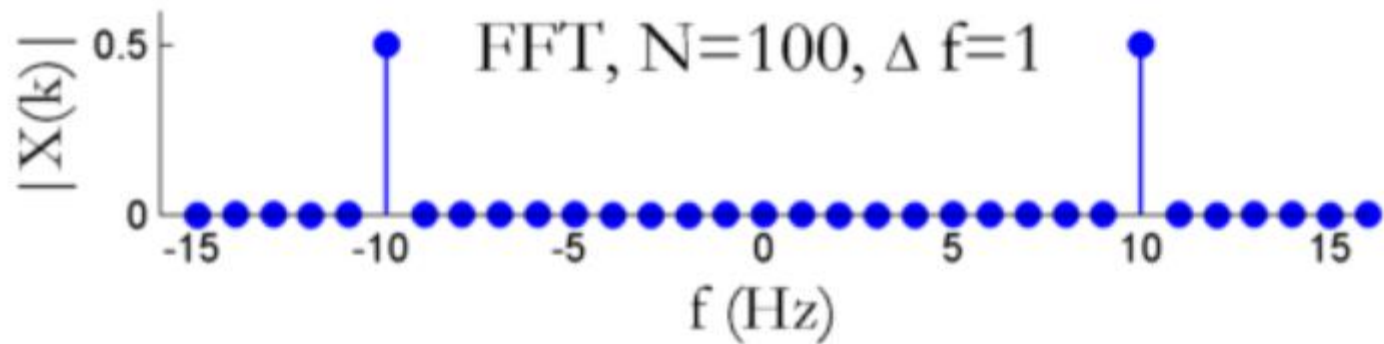
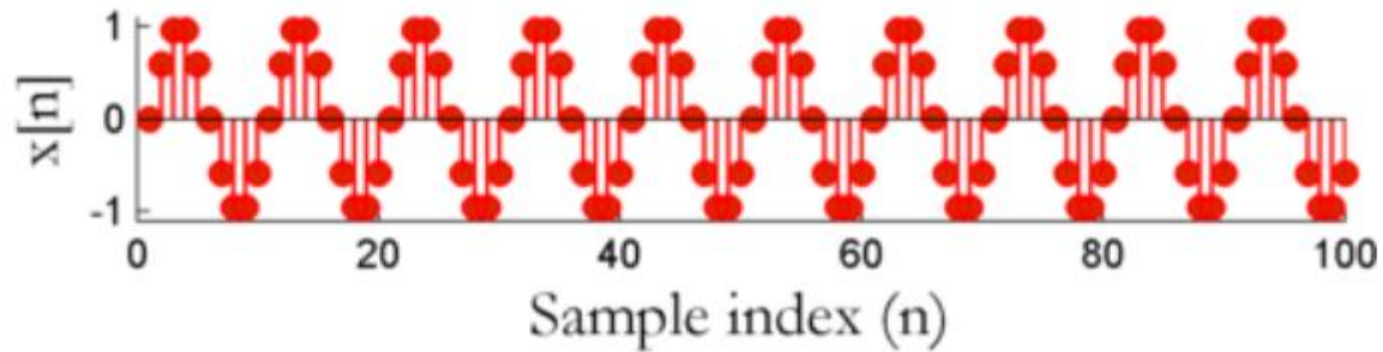
$$F_n \equiv \sum_{k=0}^{N-1} f_k e^{-2\pi i n k / N}.$$

$$f_k \equiv \frac{1}{N} \sum_{n=0}^{N-1} F_n e^{2\pi i n k / N}.$$

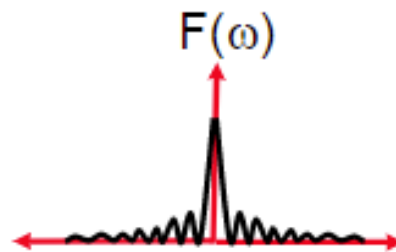
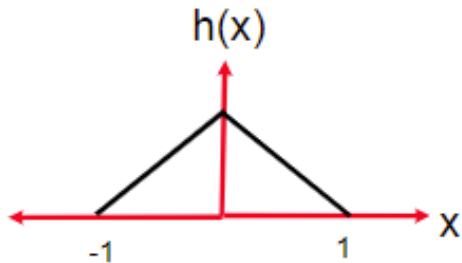


4 / N
 6.5 / N
 17 / N
 19 / N

Time domain



Example: What is the Fourier Transform of:



$$h(x) = \text{rect}(x/2) * \text{rect}(x/2)$$

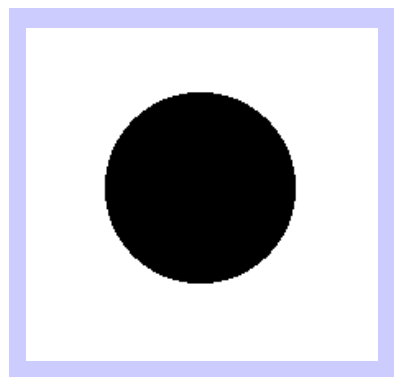
$$H(\omega) = \text{sinc}(\omega/2) \cdot \text{sinc}(\omega/2)$$

Convolution theorem

$$\mathcal{F}\{(f * g)(t)\} = F(\omega) \cdot G(\omega)$$

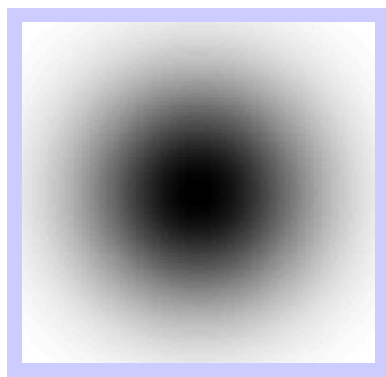
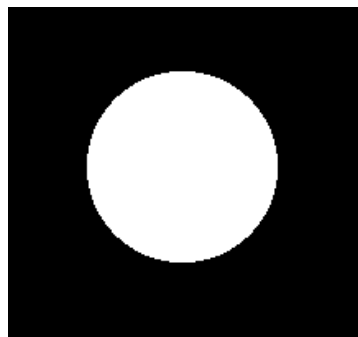
$$\mathcal{F}^{-1}\{F(\omega) \cdot G(\omega)\} = (f * g)(t)$$

Filtering in the Fourier domain



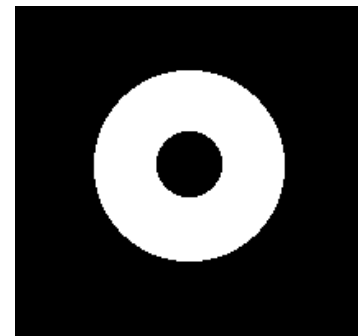
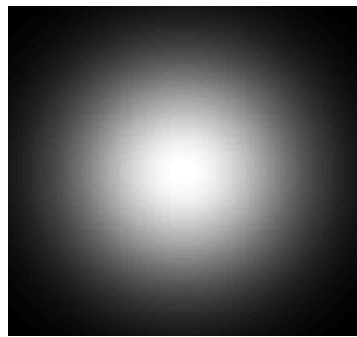
high pass

low pass



Gaussian high pass

Gaussian low pass



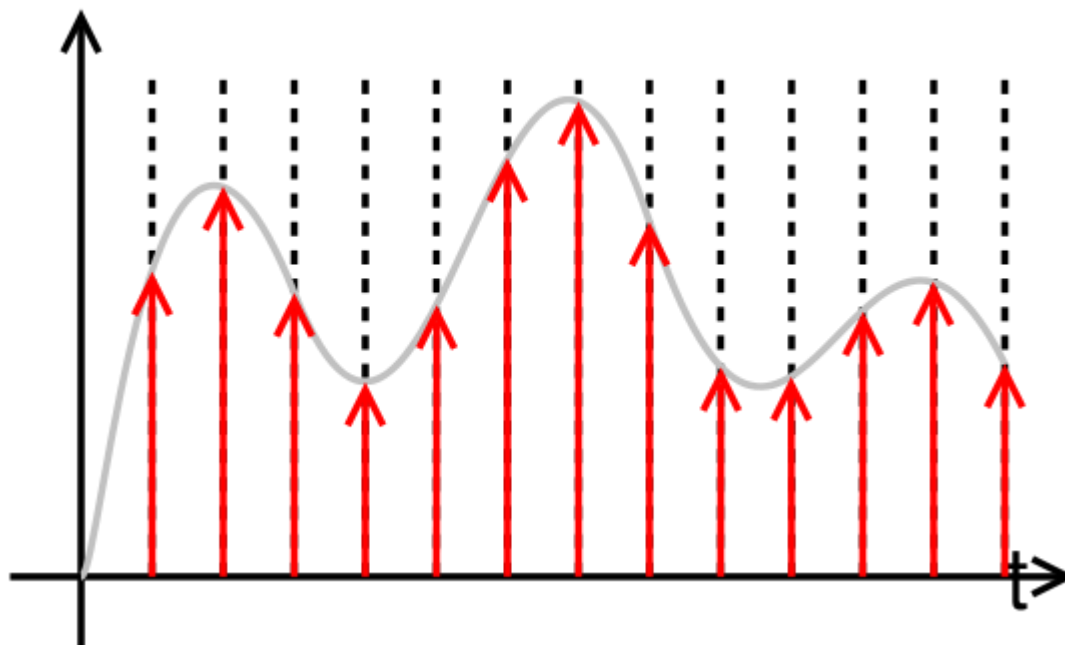
band pass

directional

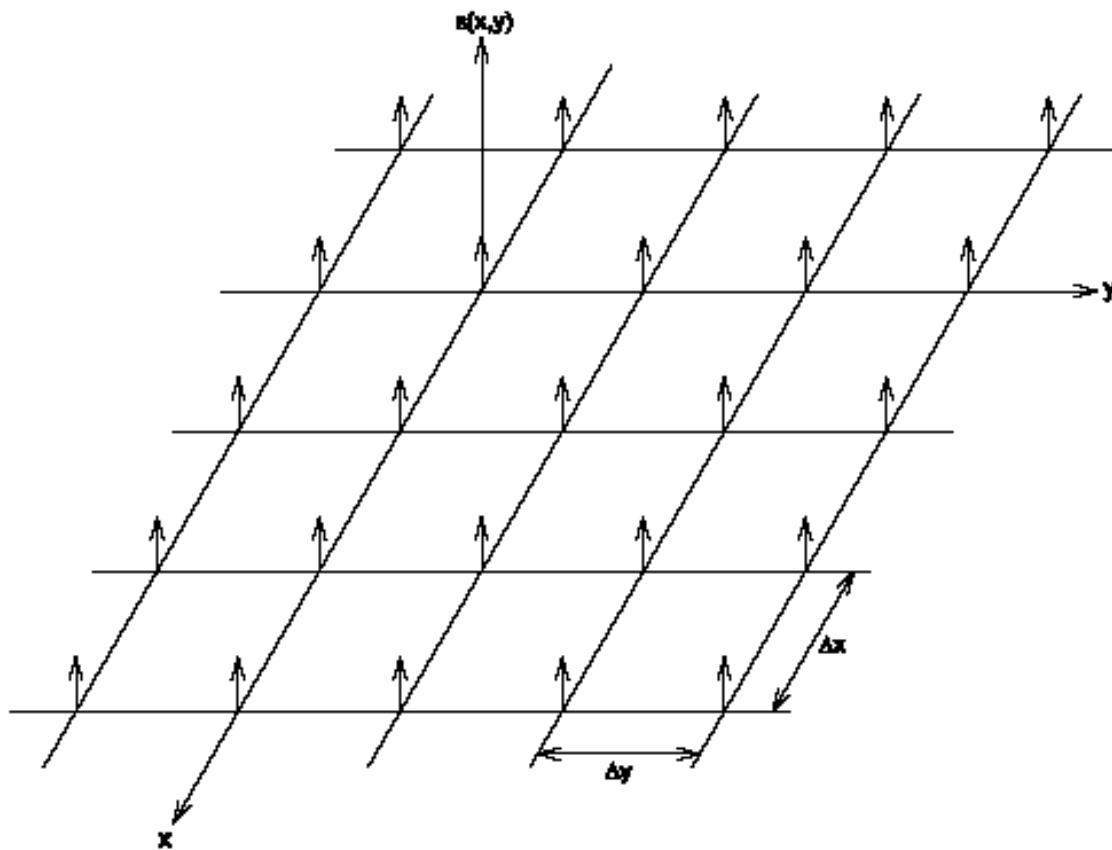


Digitalizace spojitého obrazu

- Vzorkování (sampling)



$$s(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \delta(x - i\Delta x, y - j\Delta y)$$



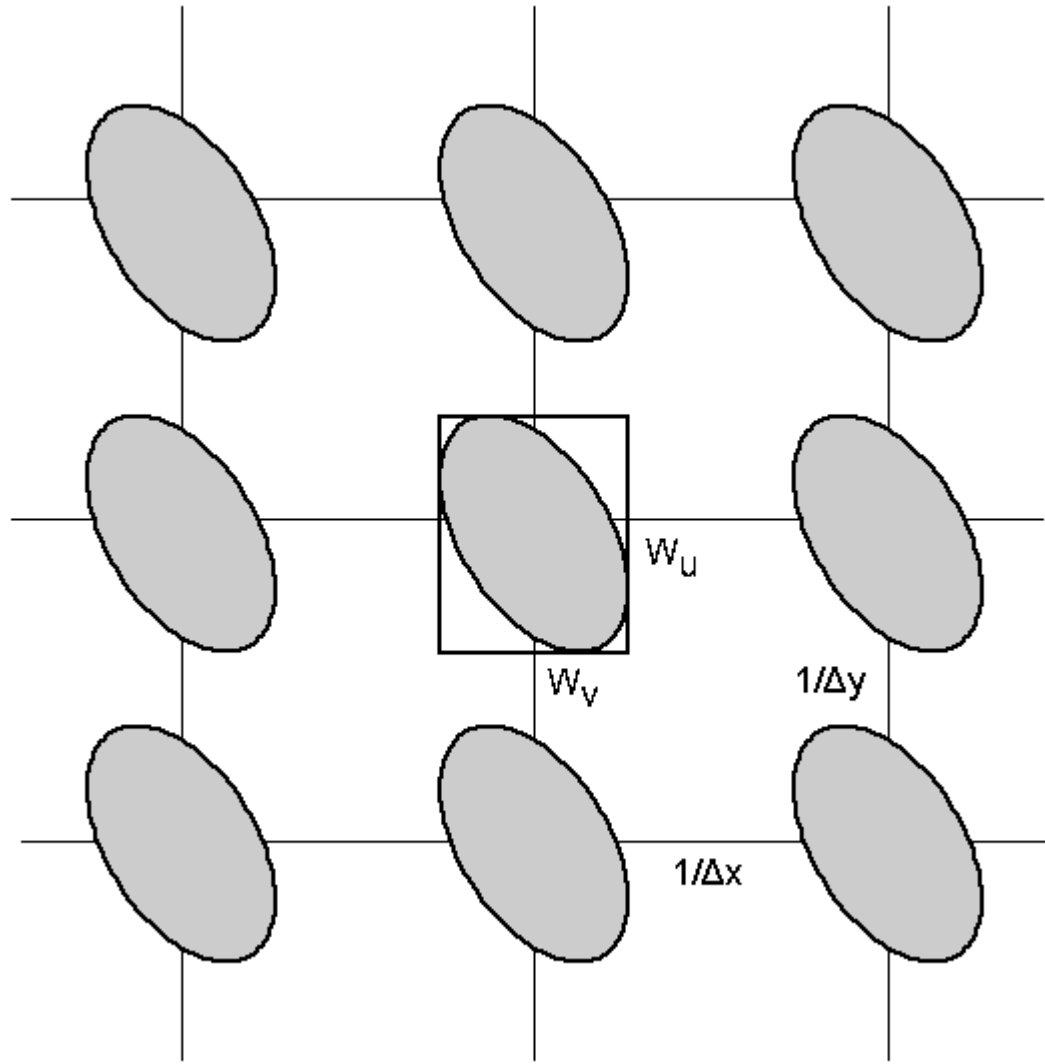
Matematický model vzorkování

Frekvenční oblast

$$D(u, v) = F(u, v) * S(u, v)$$

$$S(u, v) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \delta \left(u - i \frac{1}{\Delta x}, v - j \frac{1}{\Delta y} \right)$$

$D(u,v)$

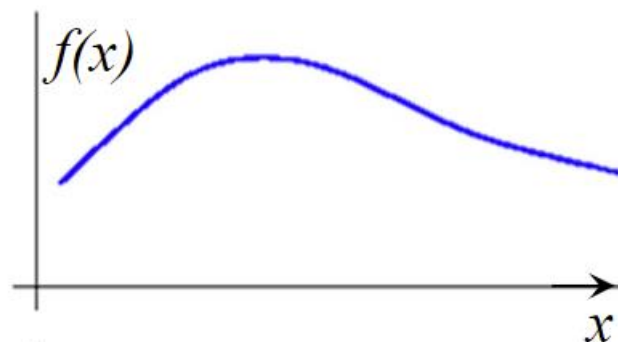
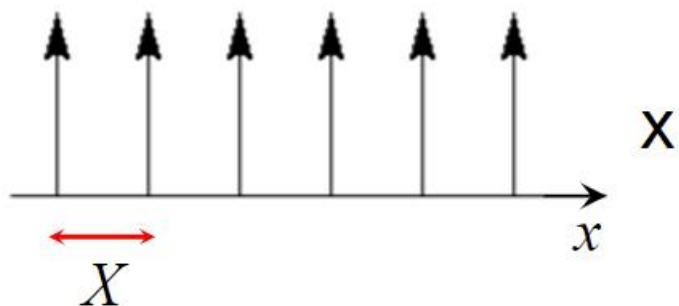


Matematický model vzorkování

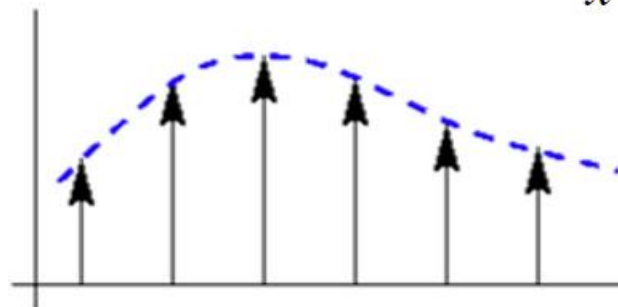
Obrazová oblast

$$f(x, y) \cdot s(x, y) = d(x, y)$$

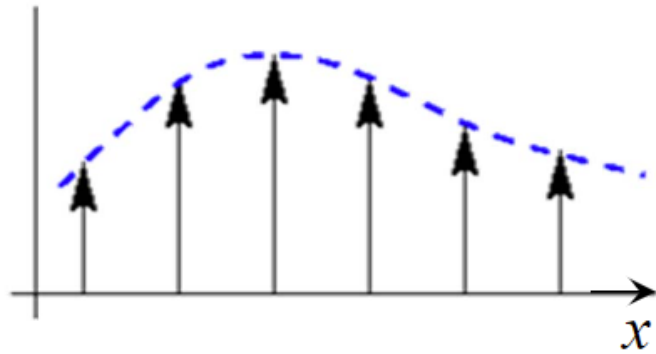
$$s(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \delta(x - i\Delta x, y - j\Delta y)$$



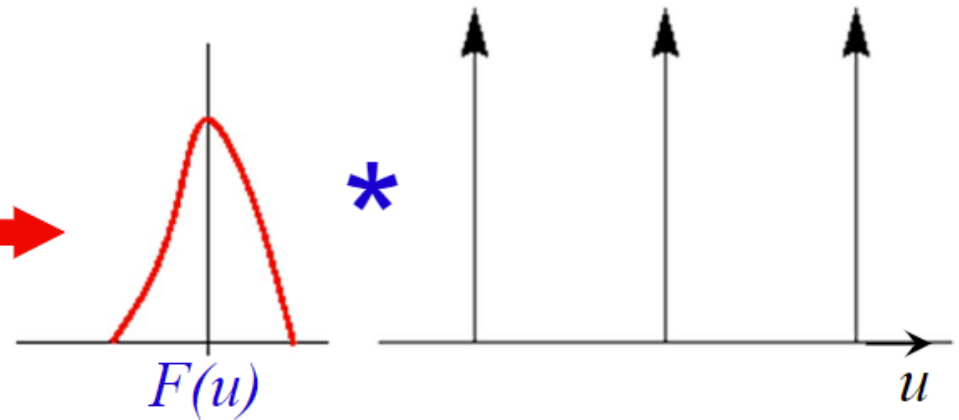
$$f_s(x) = \sum_{n=-\infty}^{\infty} \delta(x - nX) f(x)$$



spatial domain

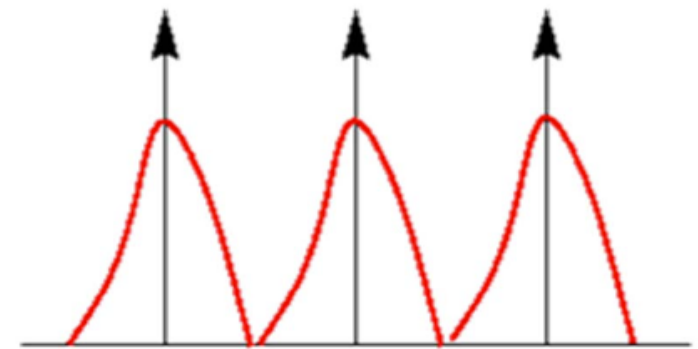


frequency domain



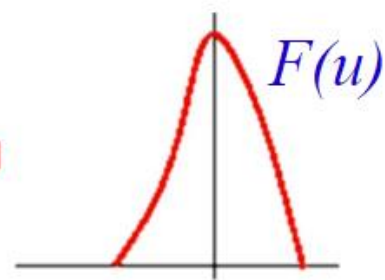
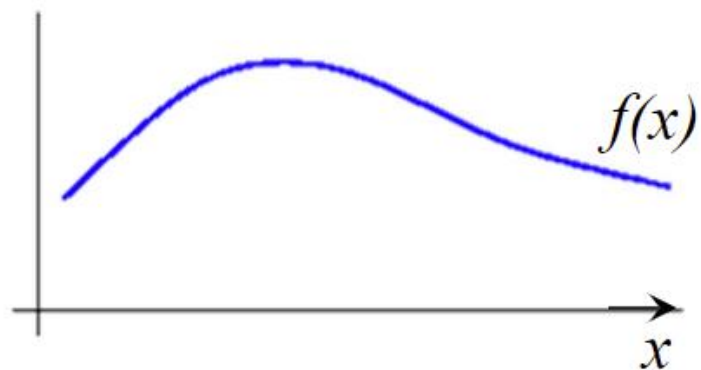
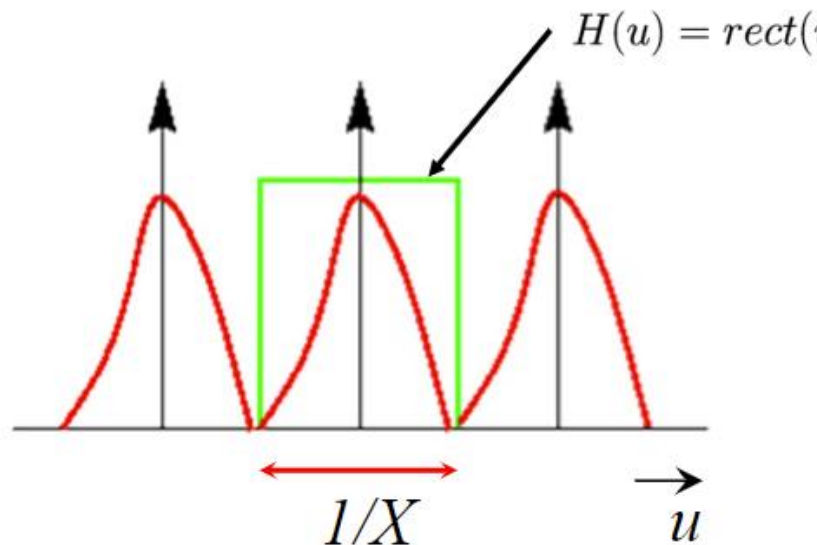
$$\begin{aligned} f_s(x) &= \sum_{n=-\infty}^{\infty} \delta(x - nX) f(x) \\ &= \sum_{n=-\infty}^{\infty} f(nX) \delta(x - nX) \end{aligned}$$

$$F_s(u) = \frac{1}{X} \sum_{n=-\infty}^{\infty} \delta(u - n/X) * F(u) = \frac{1}{X} \sum_{n=-\infty}^{\infty} F(u - n/X)$$



replicated copies of $F(u)$

Apply a box filter



$$F(u) = F_s(u)H(u)$$

$$f(x) = \sum_{n=-\infty}^{\infty} f(nX)\delta(x - nX) * \text{sinc}\frac{\pi x}{X}$$

$$= \sum_{n=-\infty}^{\infty} f(nX)\text{sinc}\frac{\pi}{X}(x - nX)$$

Nyquistova podmínka

Vzorkování bez ztráty informace

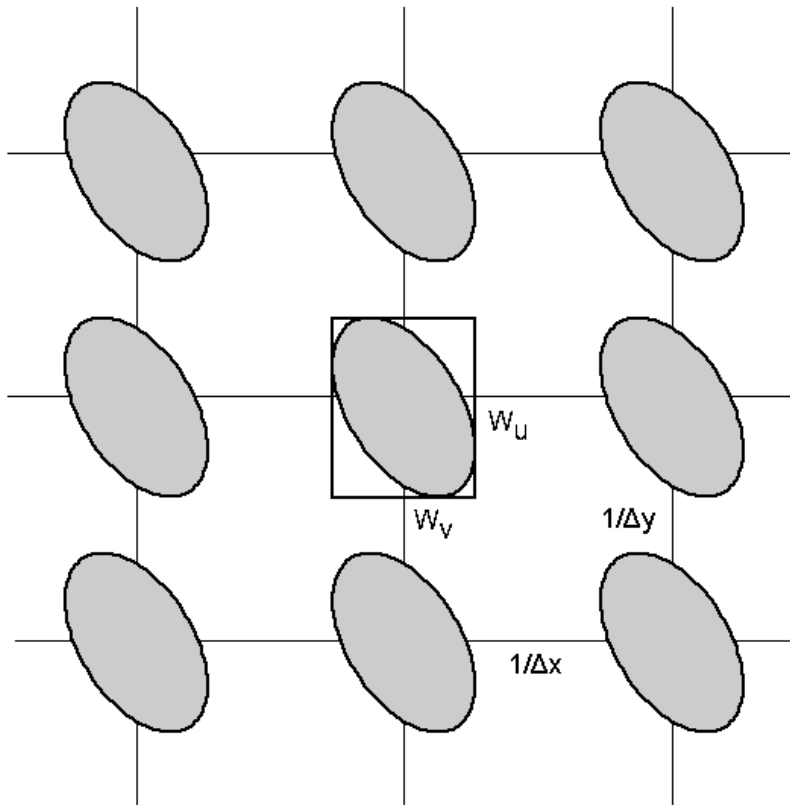
$$\Delta x \leq \frac{1}{2W_u}$$

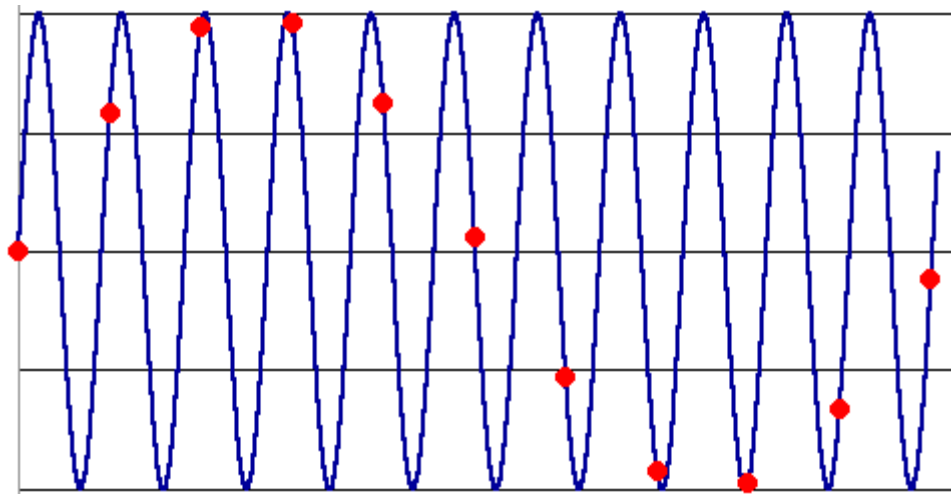
$$\Delta y \leq \frac{1}{2W_v}$$

Zpětná rekonstrukce obrazu

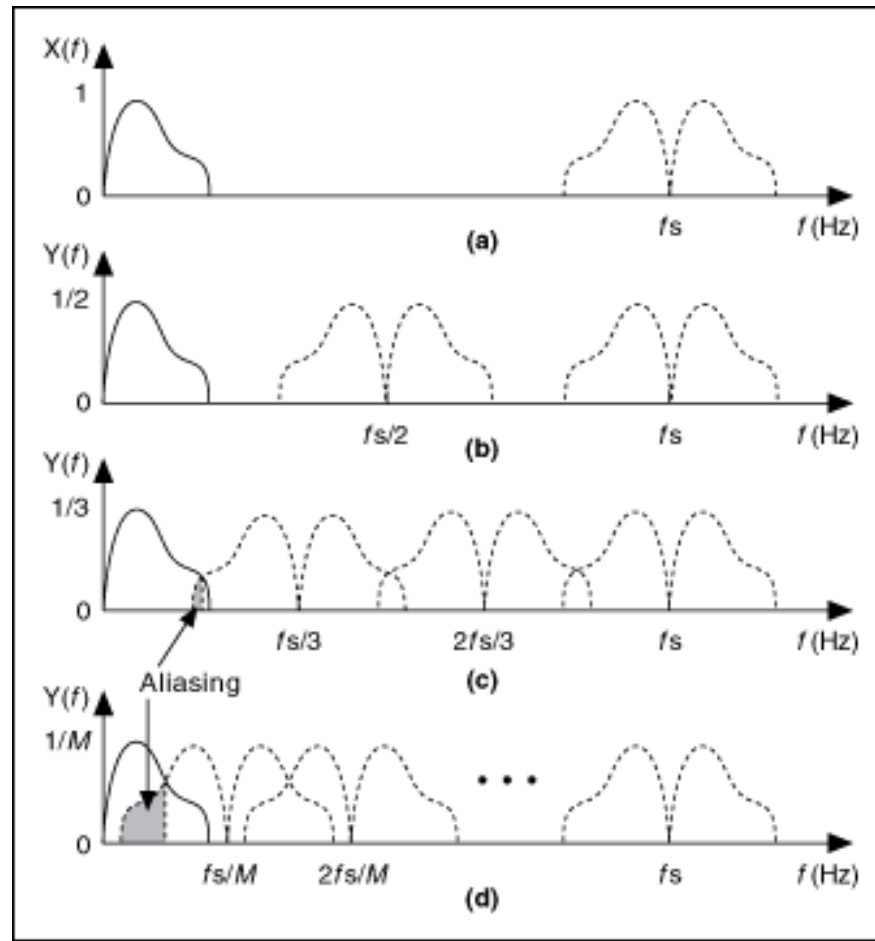
Vyříznutí jednoho spektra a následná inverzní FT

Odpovídá interpolaci v obrazové oblasti





Vzorkování s nedostatečnou frekvencí



Netradiční vzorkování

Nepravoúhlý rastr (rovnoběžník, hexagon, ...)

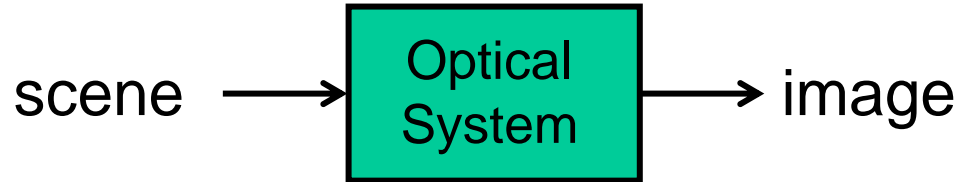
- co nejlépe pokrýt rovinu (u,v) pomocí $\text{supp}(F)$

Adaptivní vzorkování – proměnná frekvence dle charakteru obrazu

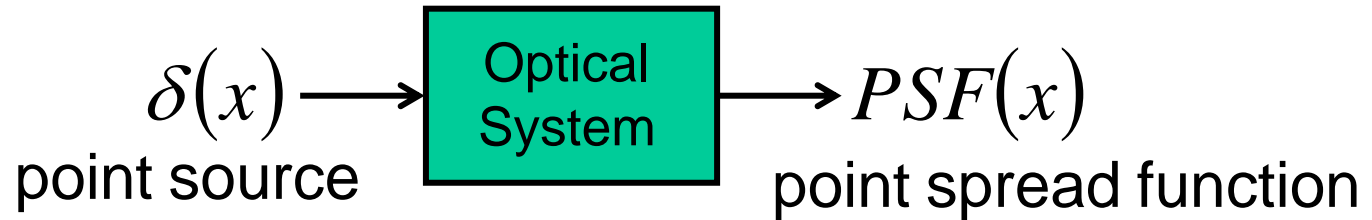
Compressive sensing

Běžné kamery a scannery neumožňují ani jedno

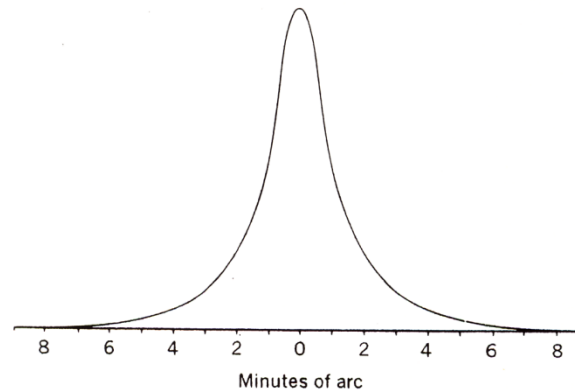
PSF - Point Spread Function



- teoreticky Dirac delta function.
- realita jiná



- PSF lidského oka



Kvantování obrazu

Kvantování – diskretizace oboru hodnot signálu

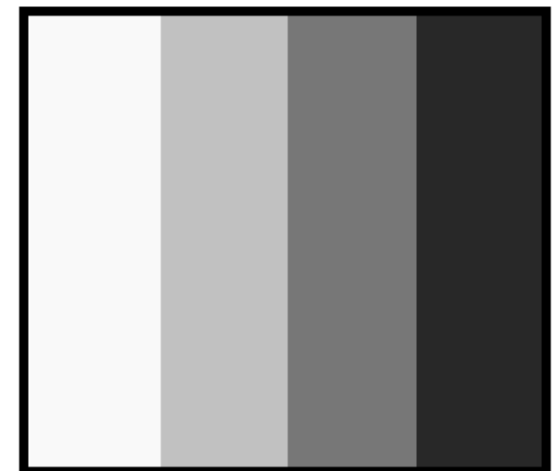
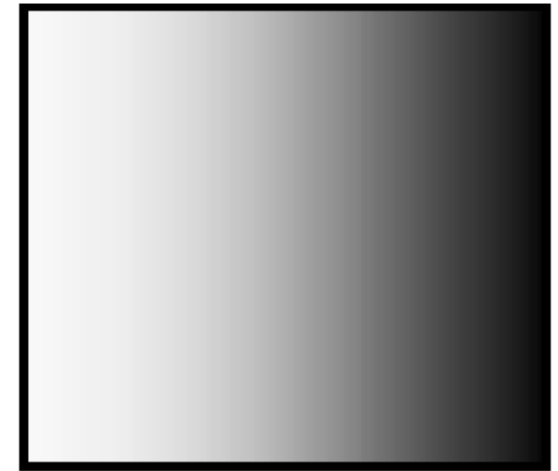
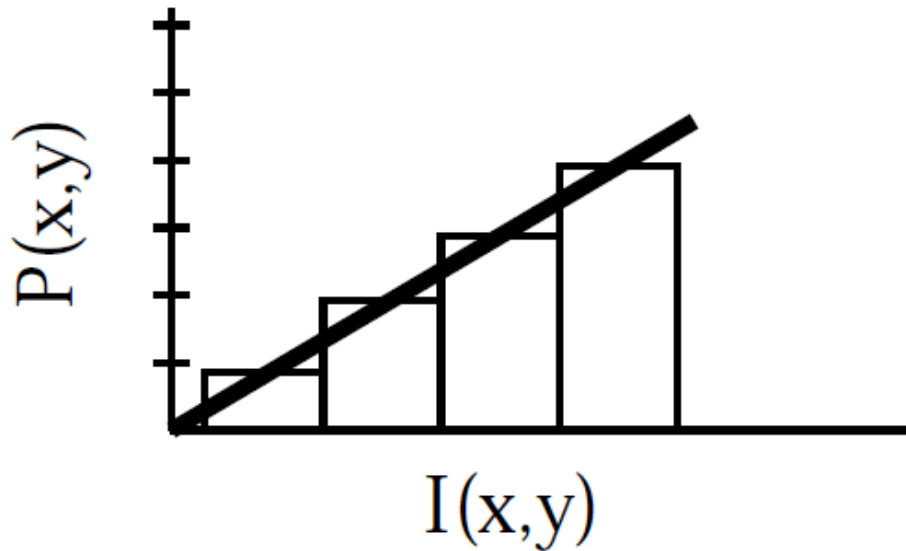
-- vždy ztrátové

Kvantizér $Q: R \sim L$

$$L = \{0, 1, \dots, k\} \quad (k = 255)$$

Kvantizér

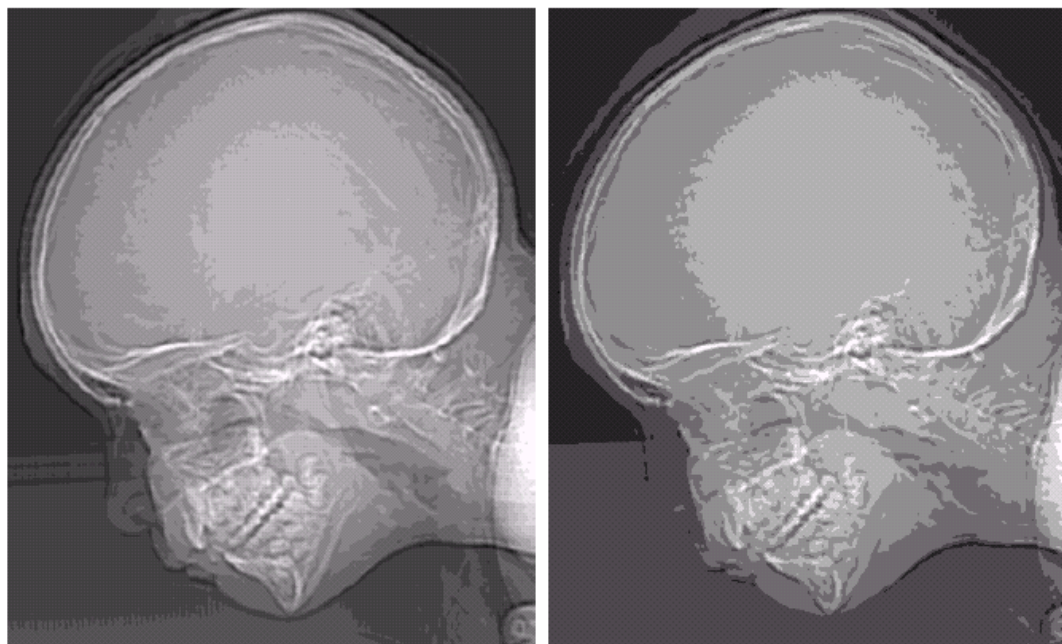
$$P(x, y) = \text{trunc}(I(x, y) + 0.5)$$



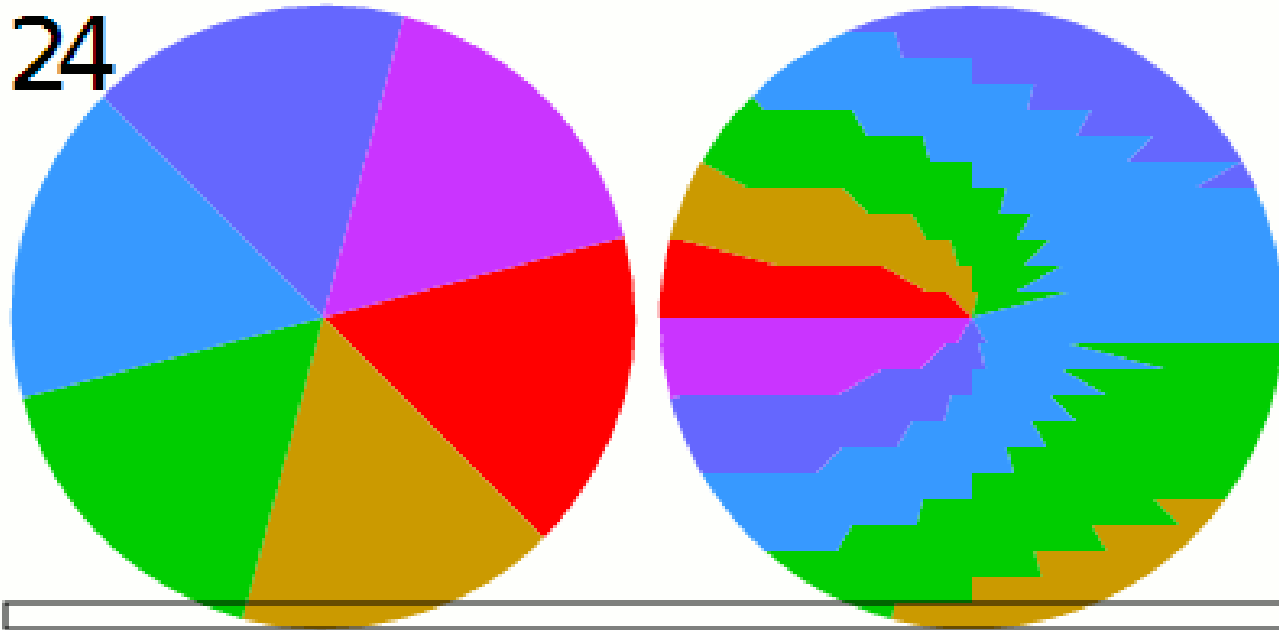
$P(x, y)$
(4 bits per pixel)

Vznik falešných kvantizačních hran

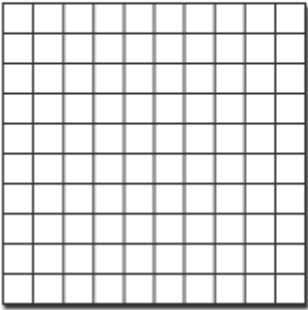
(Kvantizační šum)



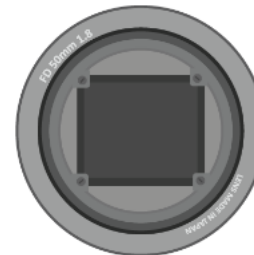
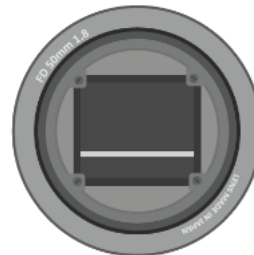
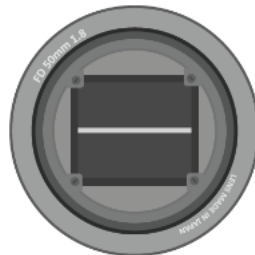
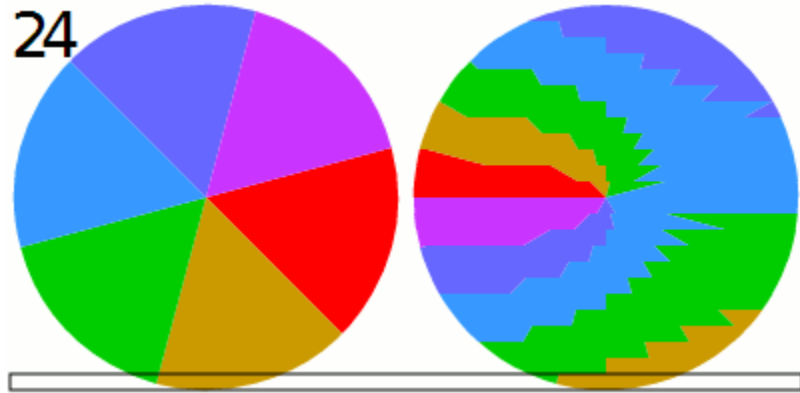
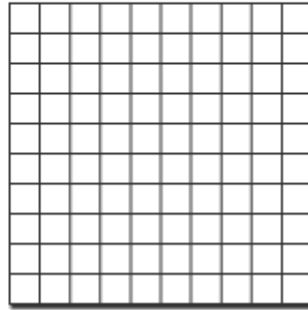
Rolling shutter



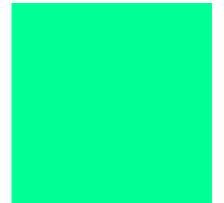
Rolling Shutter

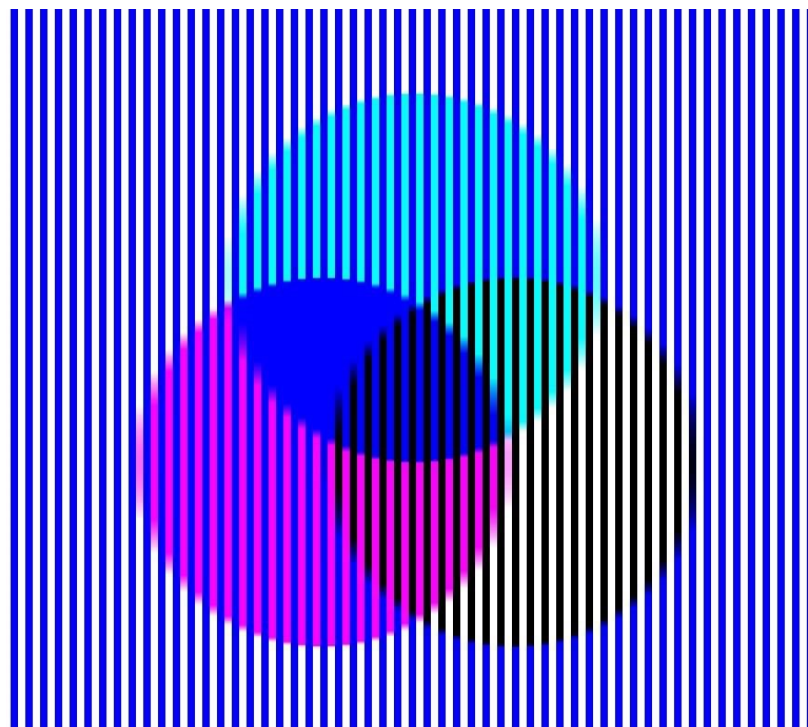


Total Shutter



Barva



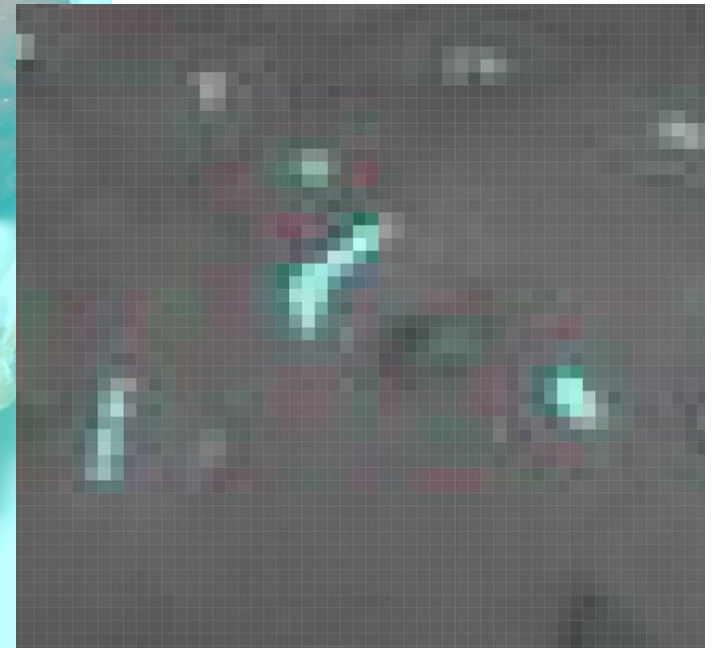
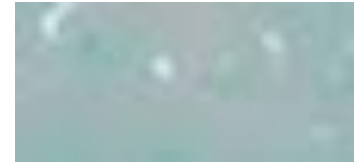




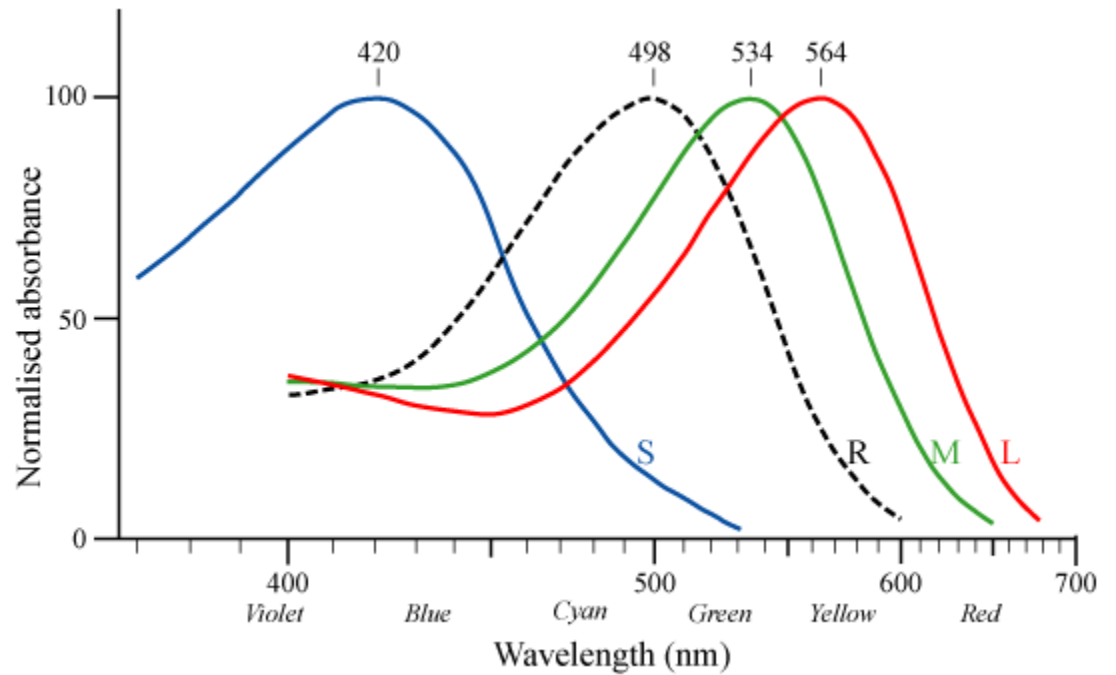
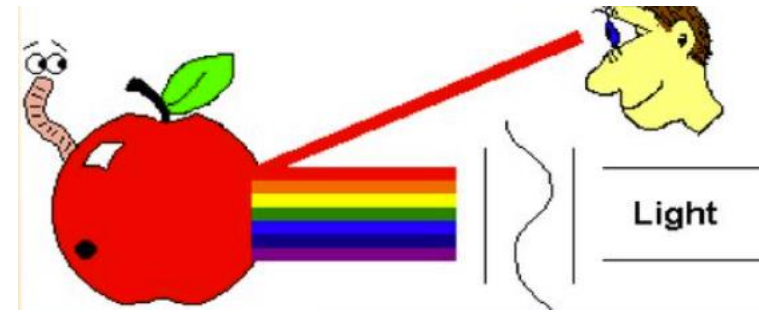
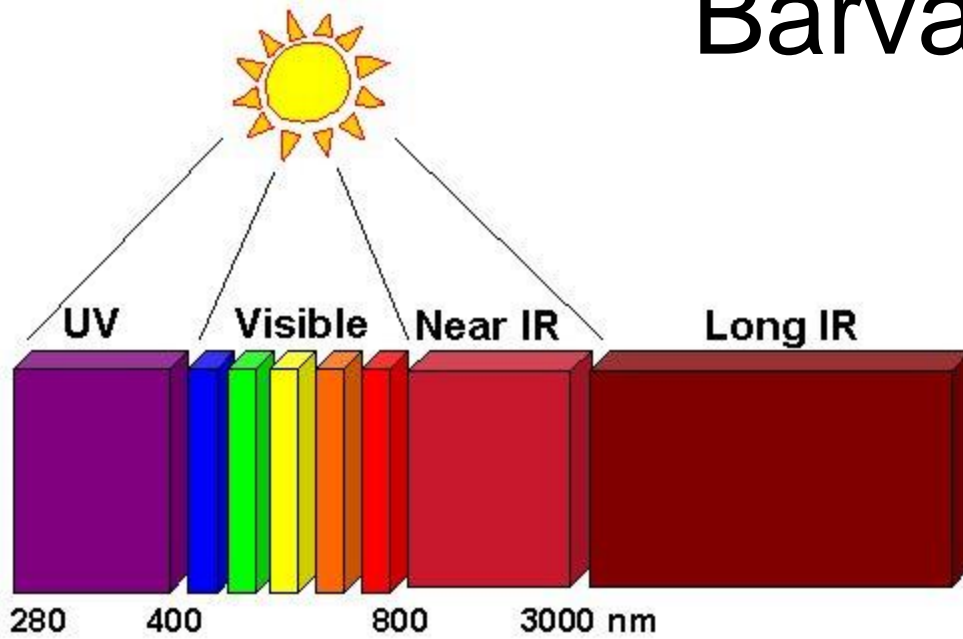
Barva



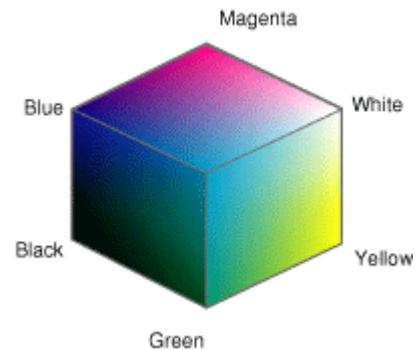
Barva



Barva



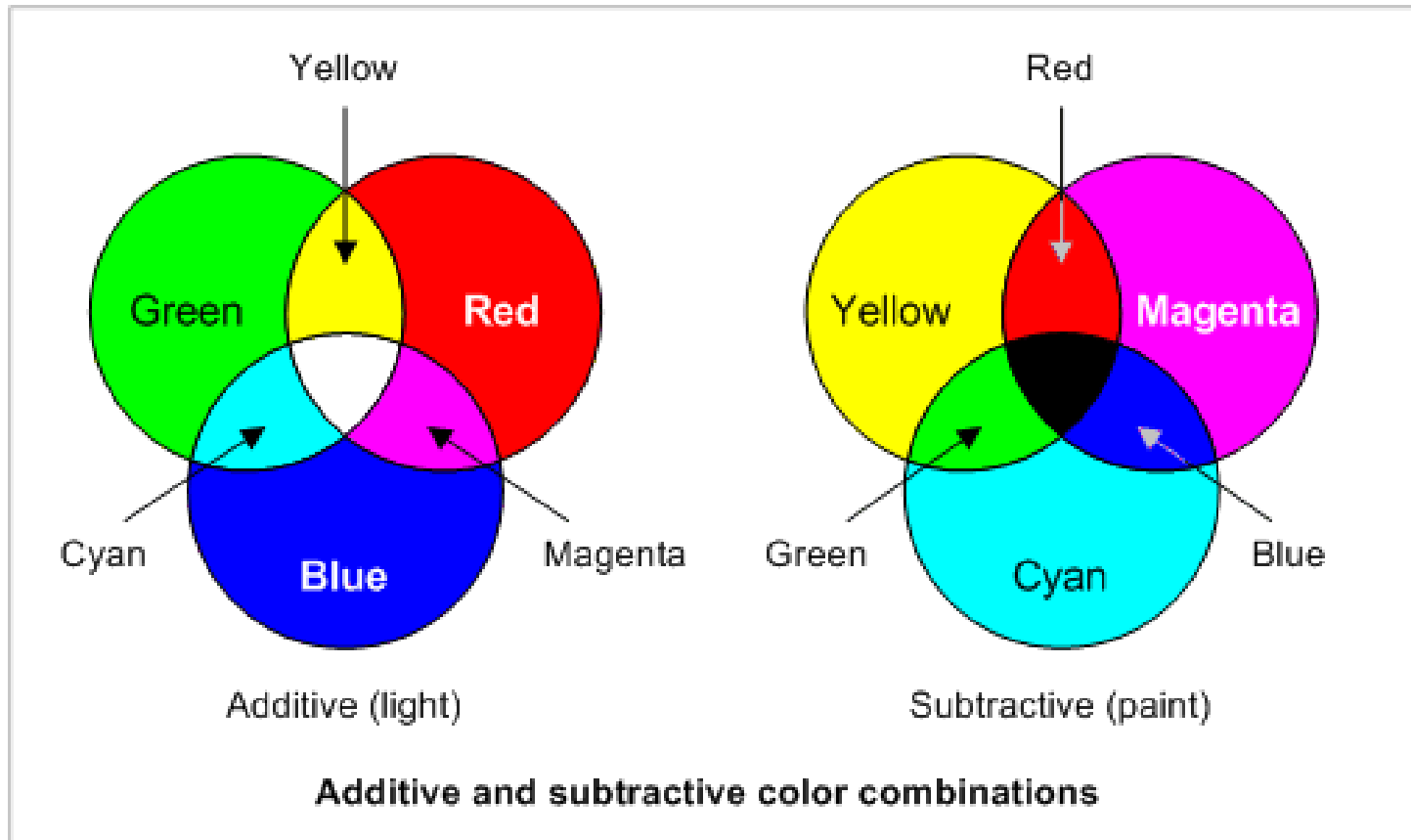
Barevné prostory



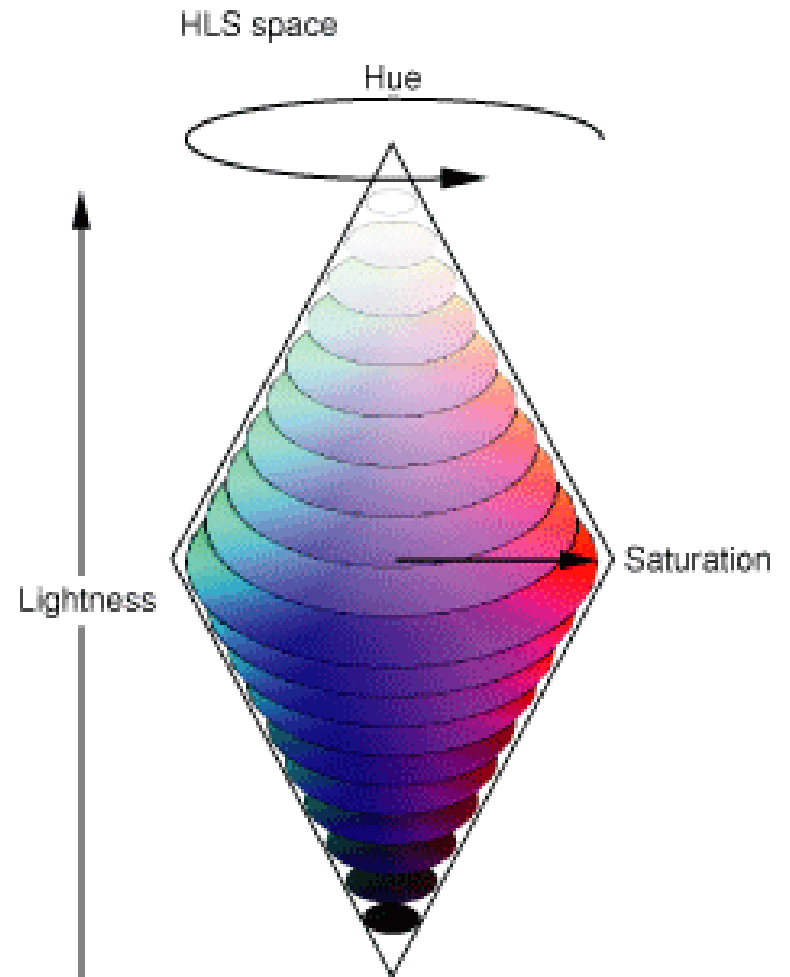
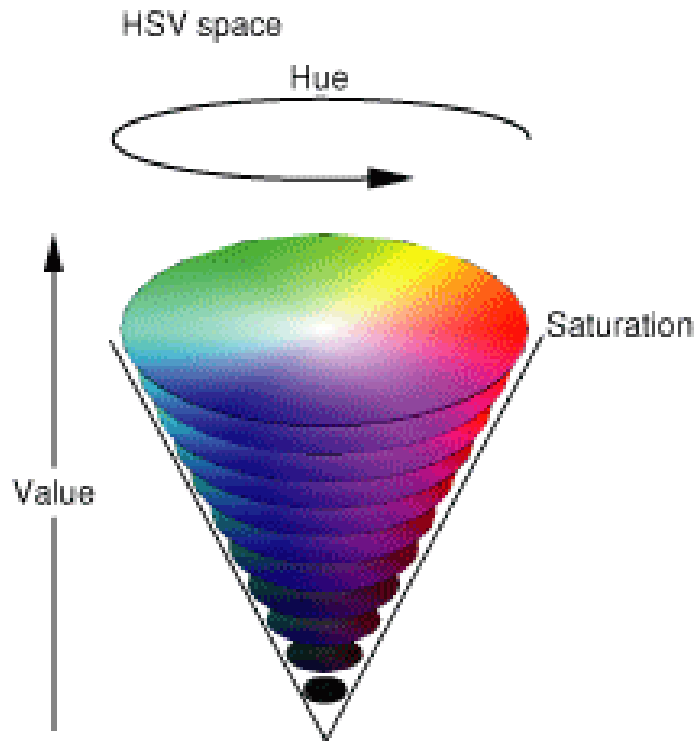
- RGB aditivní
- CMYK subtraktivní (tisk)
- „ostatní“ ... HSV, HSL,
YIQ, YUV (NTSC, PAL)

- na zařízení nezávislé prostory
(sRGB ... CRT)
- profile connection space (PCS)
CIE XYZ nebo CIELAB

Barevné prostory



Barevné prostory



HSV ... hue, saturation, value

HLS ... hue, lightness, saturation.

Barevné prostory

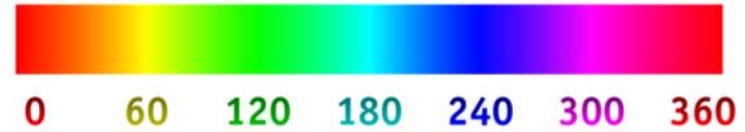


YIQ ...
NTSC

Y intenzita, I, Q barevnost

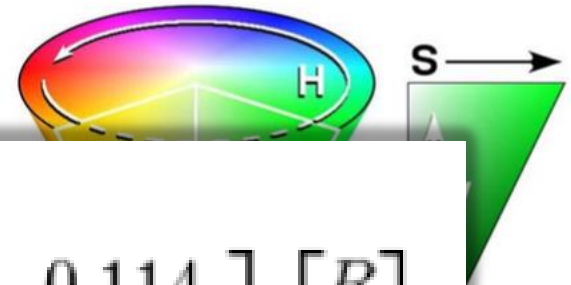
Barevné prostory

hue



- colour cone

- $H = \text{hue / colour in degrees} \in [0, 360]$
- $S = \text{saturation} \in [0, 1]$
- $V = \text{value} \in [0, 1]$



- conversion

- $V = \max =$
- $S = (\max -$

From RGB to YIQ [\[edit\]](#)

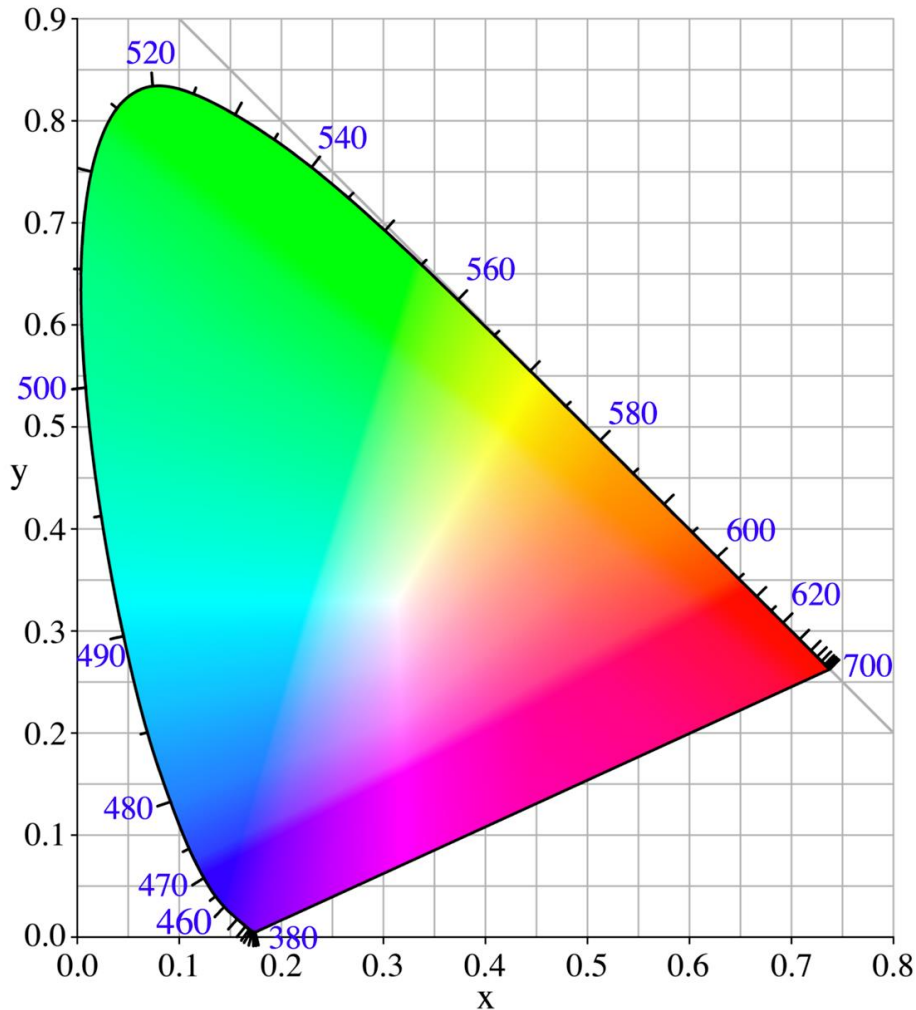
$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- $H = 60 \times \begin{cases} 0 + (G - B) / (\max - \min), & \text{if } \max = R \\ 2 + (B - R) / (\max - \min), & \text{if } \max = G \\ 4 + (R - G) / (\max - \min), & \text{if } \max = B \end{cases}$

$H = H + 360, \text{ if } H < 0$

CIE XYZ

CIE 1931 XYZ



ICC standard

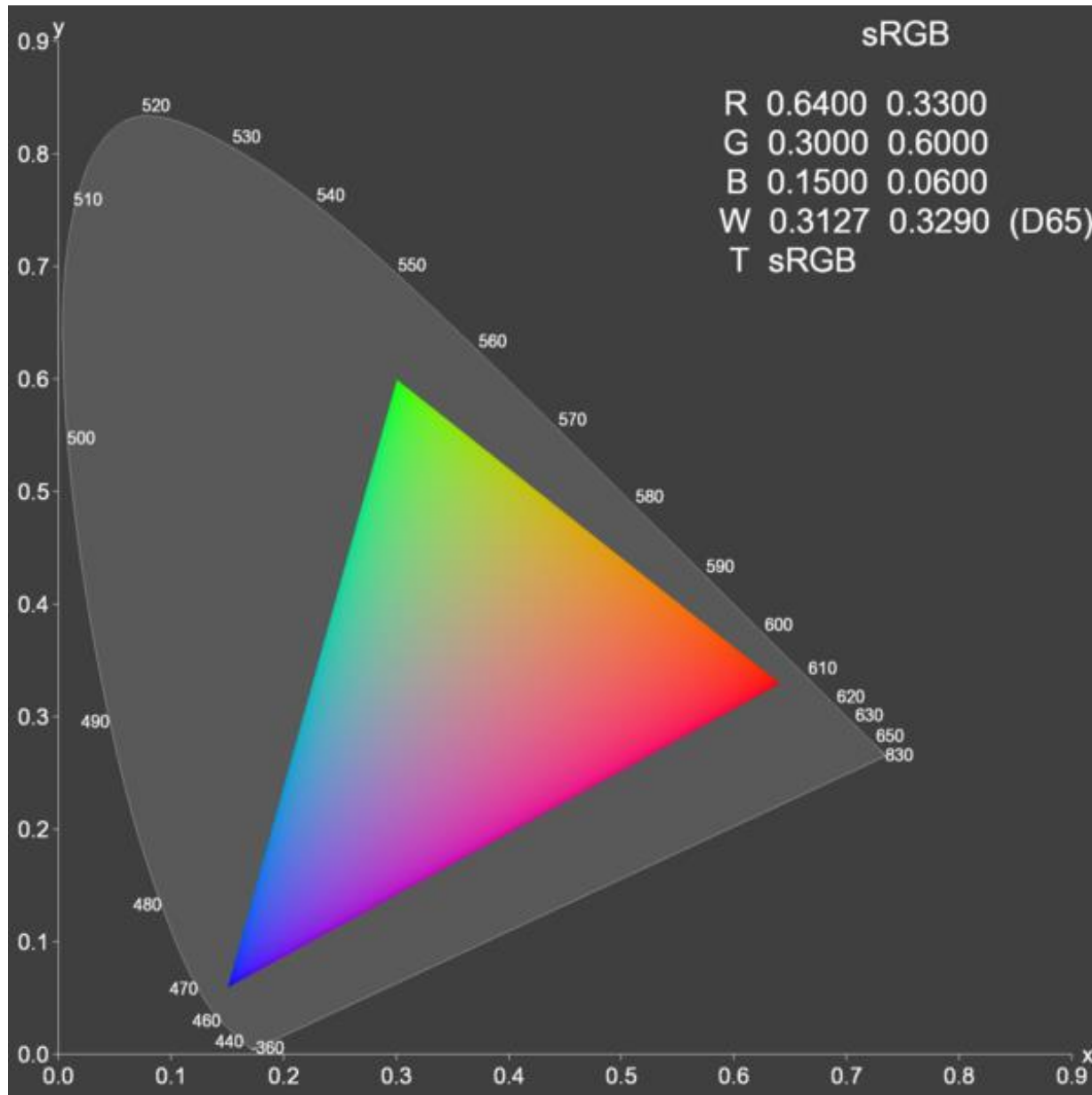
předloha odrážející světlo
nasvícené pod úhlem 45 stupňů
pozorovaném ve směru kolmém
při osvětlení o síle 500 luxů
standardní iluminant D509
(denní světlo

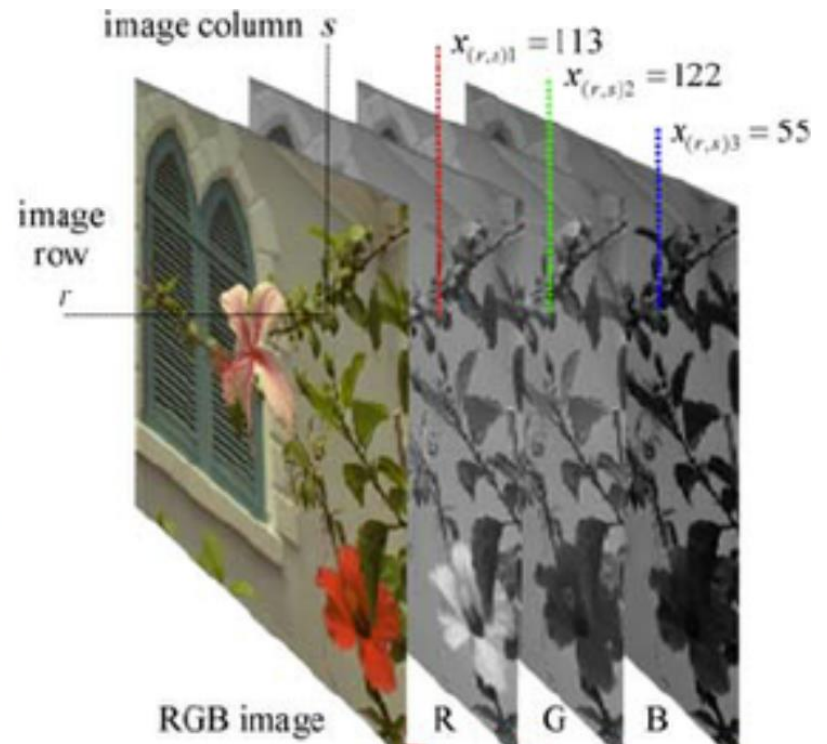
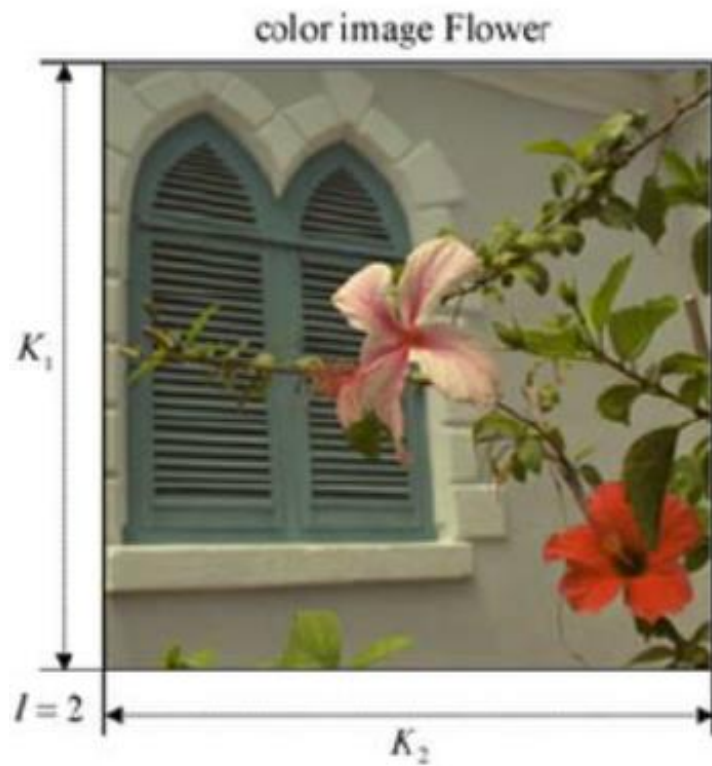
o barevné teplotě 5000 K)

1920s,

W. D. Wright and John
Guild

Barva - gamut





| | | | | | | | | | | | | |
|---|------------|-----|-----|-----|------------|-----|-----|-----|------------|-----|-----|-----|
| | | | | | | | | | | | | |
| R | 255 | 255 | 255 | 125 | 0 | 0 | 0 | 0 | 0 | 125 | 255 | 255 |
| G | 0 | 125 | 255 | 255 | 255 | 255 | 255 | 125 | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 0 | 0 | 0 | 125 | 255 | 255 | 255 | 255 | 255 | 125 |

RGB (K. Plataniotis)



RGB



HSV

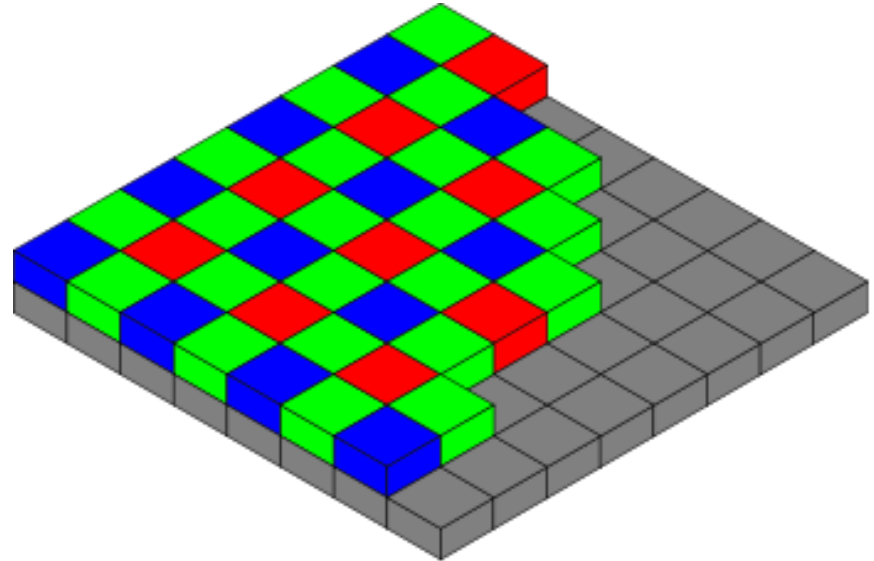


YUV

Barevný senzor

Bayer pattern (Eastman Kodak)

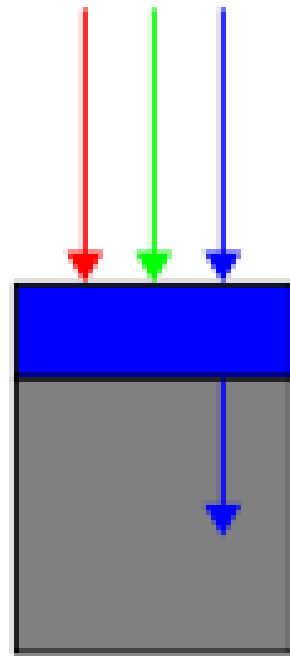
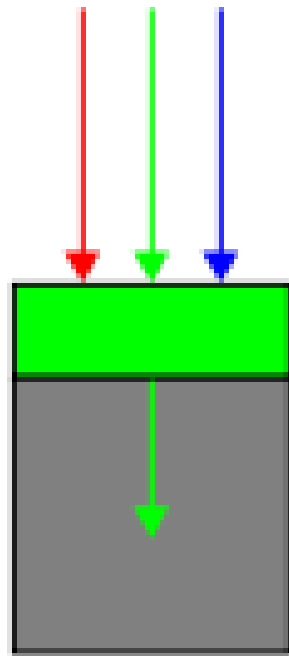
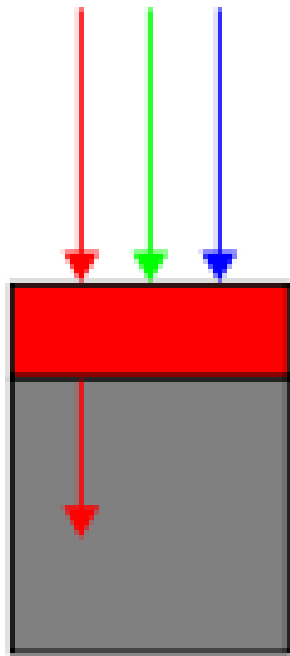
color filter array (CFA)



RGB barevné filtry na fotosenzoru

50% zelená, 25% červená, 25% modrá

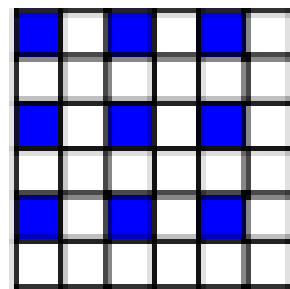
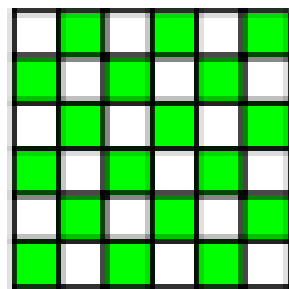
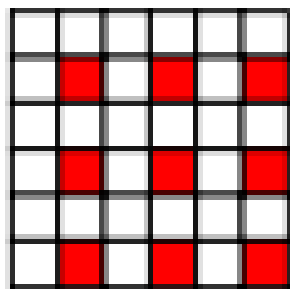
RGBG, GRGB, RGGB



Incoming light

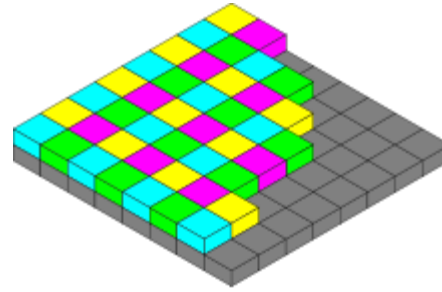
Filter layer

Sensor array

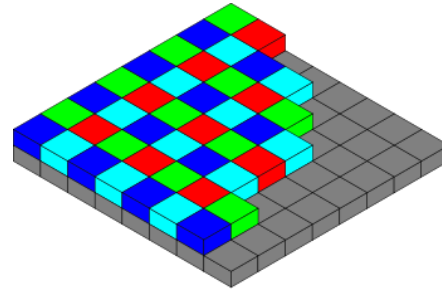


Resulting pattern

CYGM



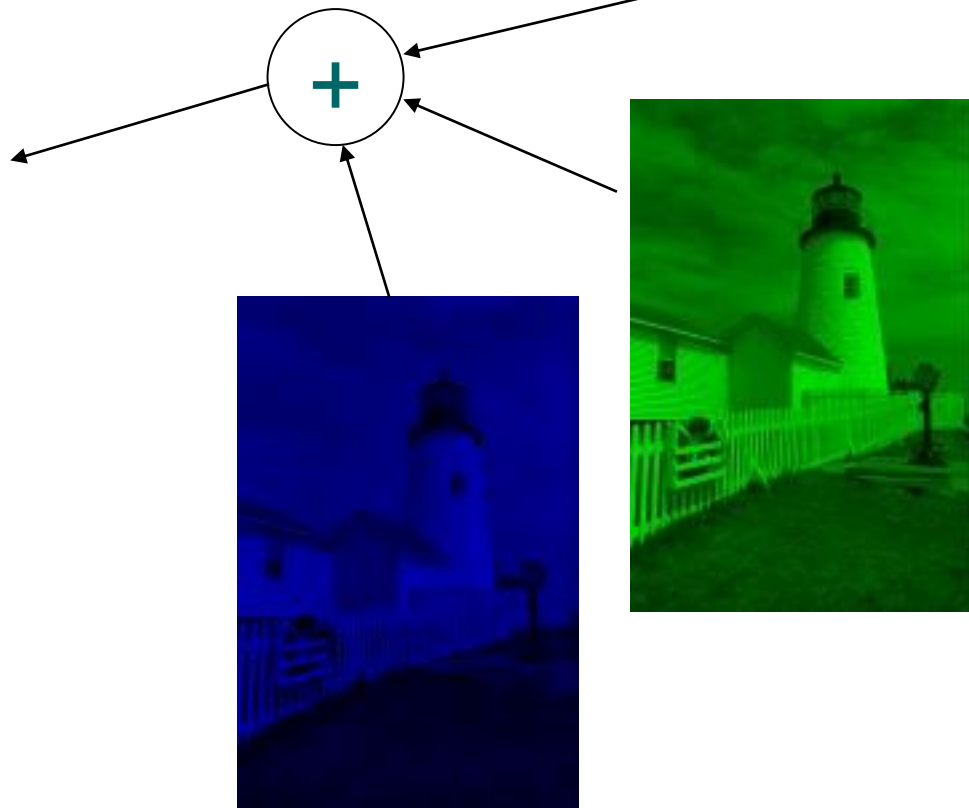
RGBE



Foveon X3 sensor

3 CCDs

Barvy



E. Dubois



**Lighthouse
original**



**Lighthouse
red original**

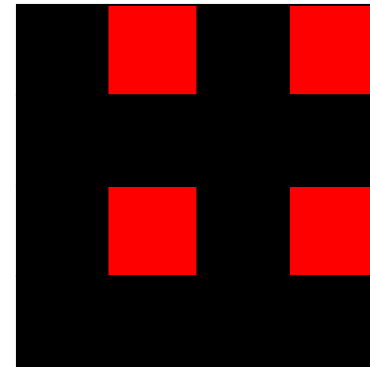
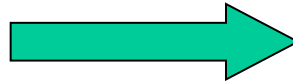
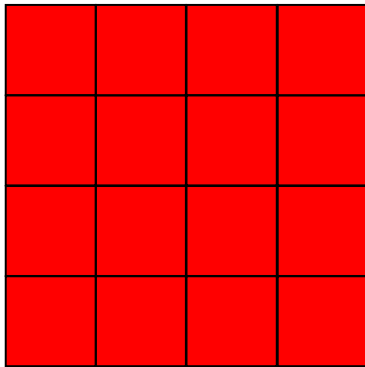
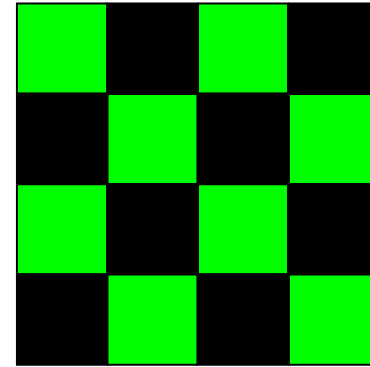
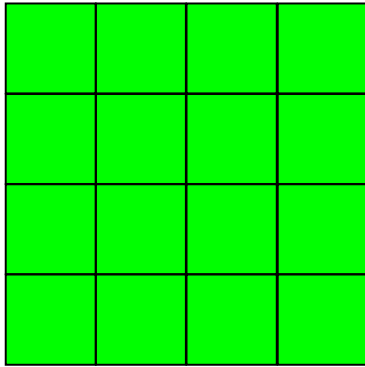


**Lighthouse
green original**



**Lighthouse
blue original**

Formation of Color planes

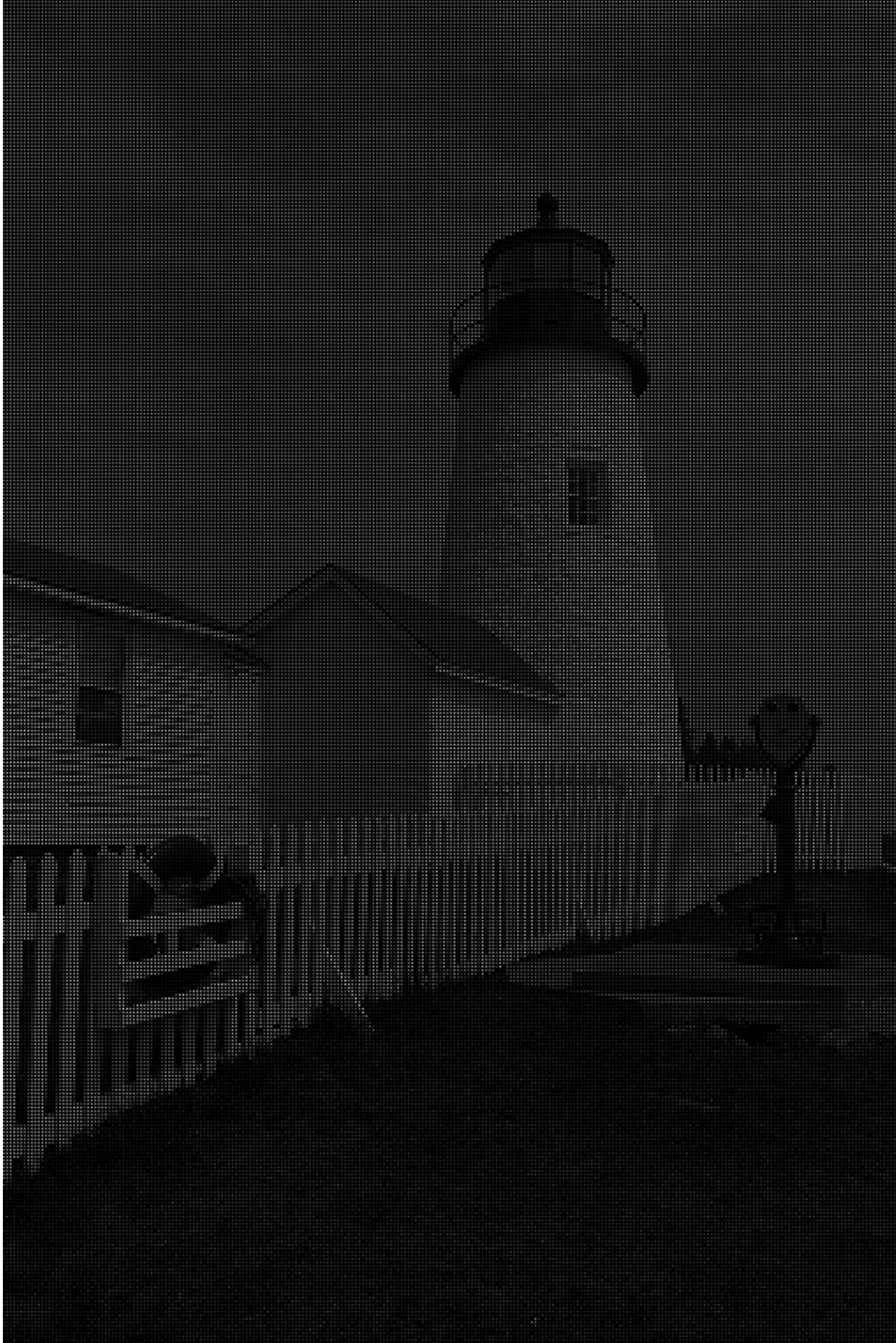




Lighthouse
red subsampled



**Lighthouse
green subsampled**



Lighthouse
blue subsampled

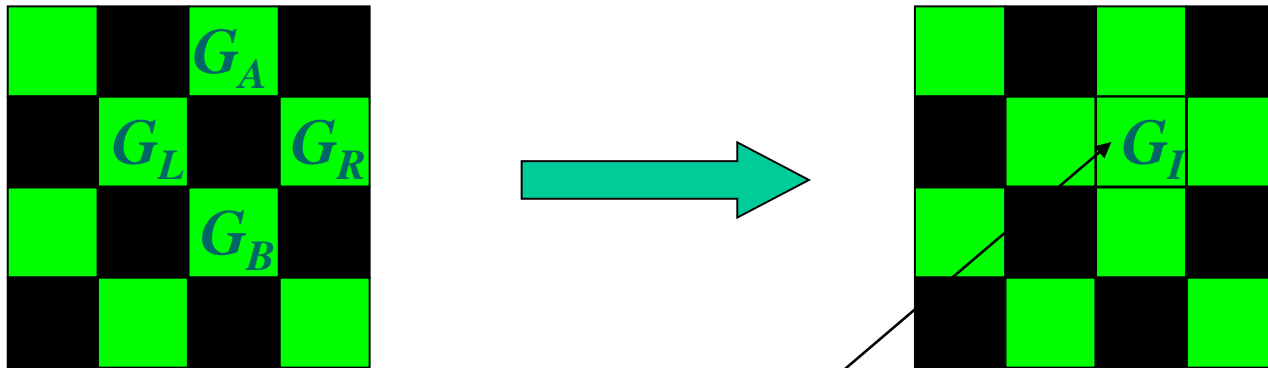


Lighthouse

Bayer CFA image

Interpolace

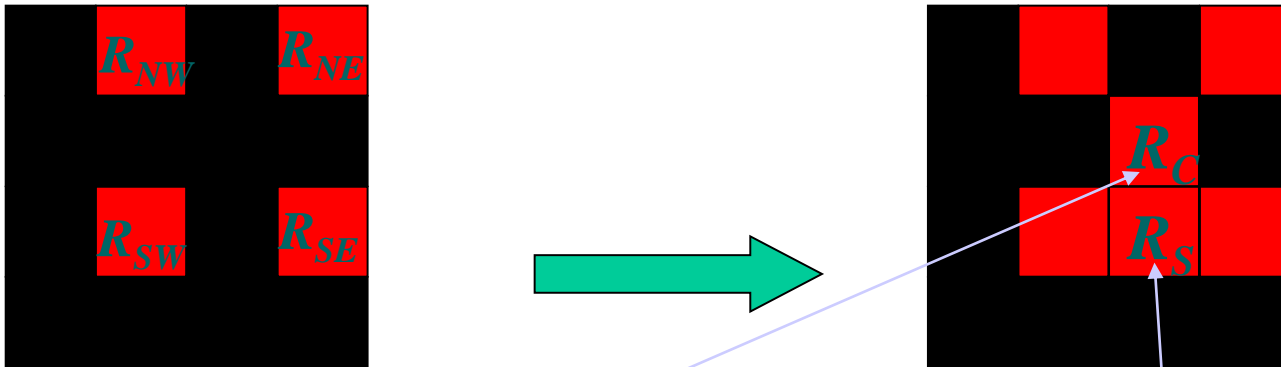
bilinear interpolation



$$G_I = \frac{1}{4}(G_L + G_R + G_B + G_A)$$

Interpolace

bilinear interpolation



$$R_C = \frac{1}{4} (R_{NW} + R_{NE} + R_{SW} + R_{SE})$$

$$R_S = \frac{1}{2} (R_{SW} + R_{SE})$$



Lighthouse

red interpolated



Lighthouse

green interpolated



Lighthouse

blue interpolated



Lighthouse
Interpolated
color image



**Lighthouse
original**



Lighthouse

red

interpolated

with bilinear

interpolator



Lighthouse

red

interpolated

with bicubic

interpolator

Vylepšení

- Interpolace ☹️
- Předfiltrování 😊



Lighthouse

red

interpolated

with bilinear

interpolator



Lighthouse

**prefiltered red
interpolated**

**with bilinear
interpolator**



Lighthouse
Interpolated
color image



Lighthouse
Prefiltered
Interpolated
color image



**Lighthouse
original**

Předzpracování obrazu

- Image enhancement (subjective)
- Image restoration (objective)

Image enhancement

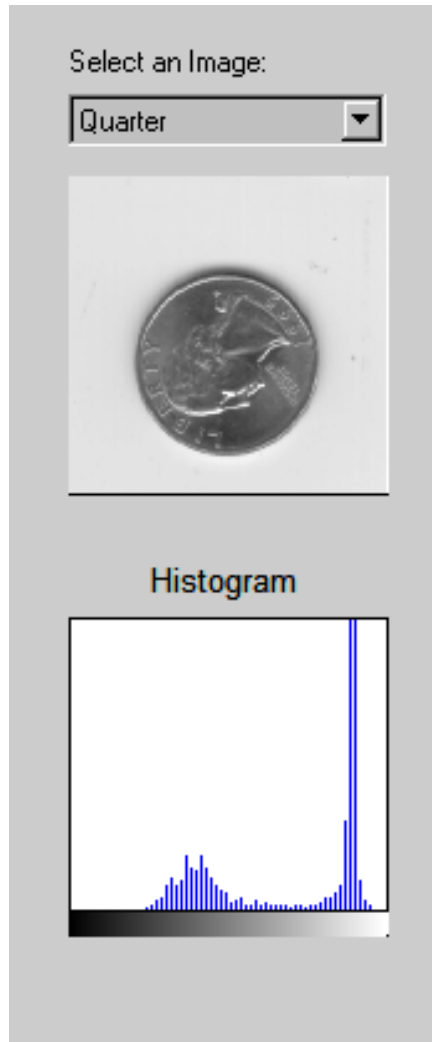
- Změny kontrastu a jasu
- Potlačení šumu
- Detekce a zvýraznění hran



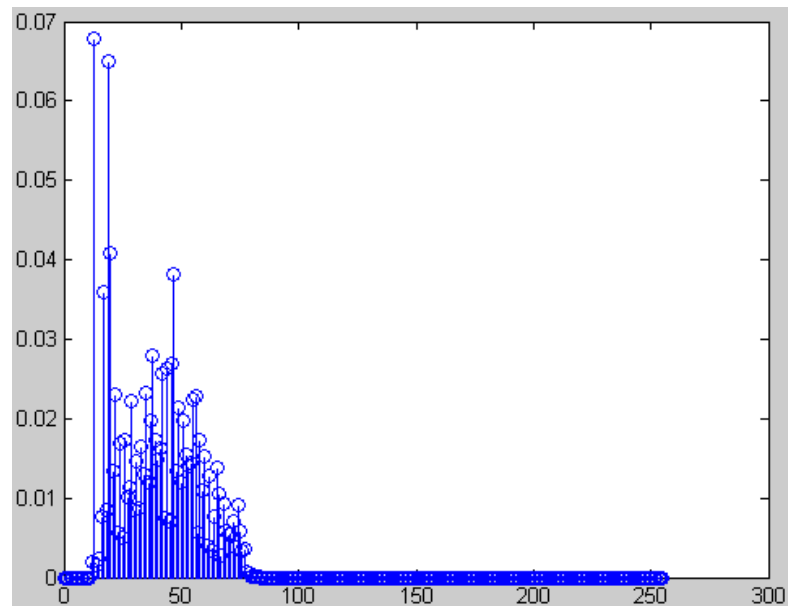
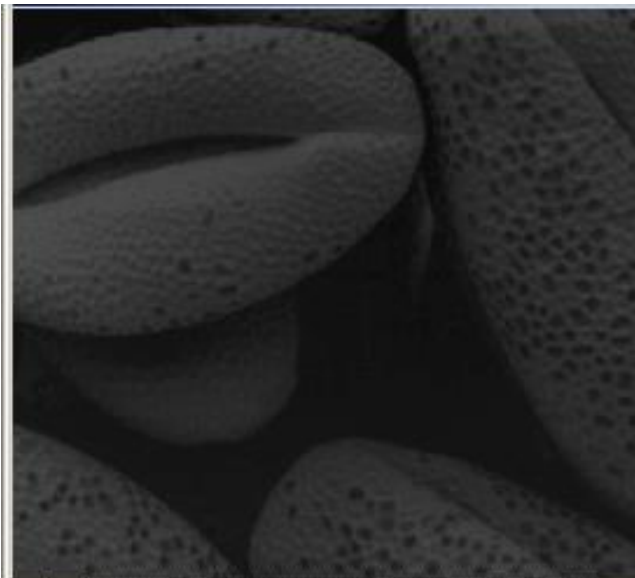
Změny kontrastu a jasu

- Histogram
- Lineární změny kontrastu a jasu
- Nelineární změny kontrastu a jasu, ekvalizace
- Lokální operace

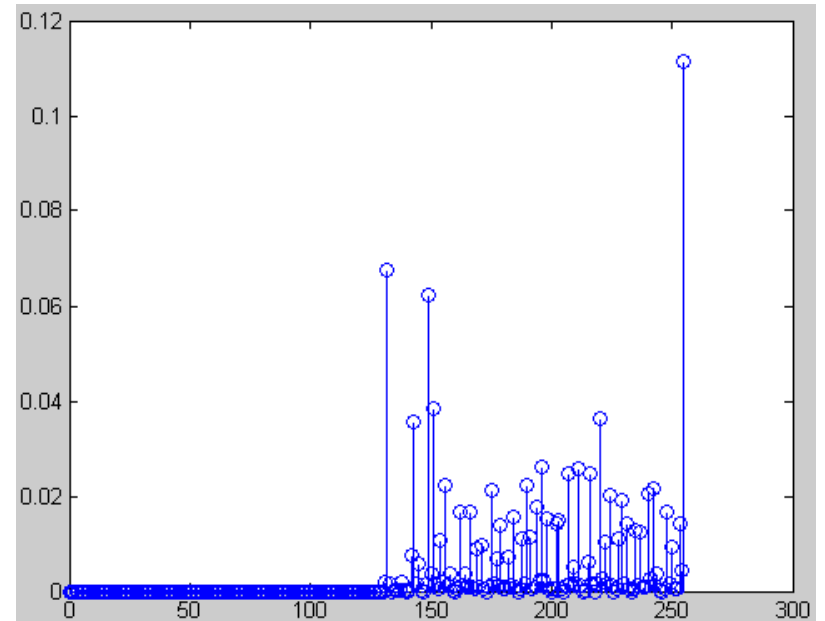
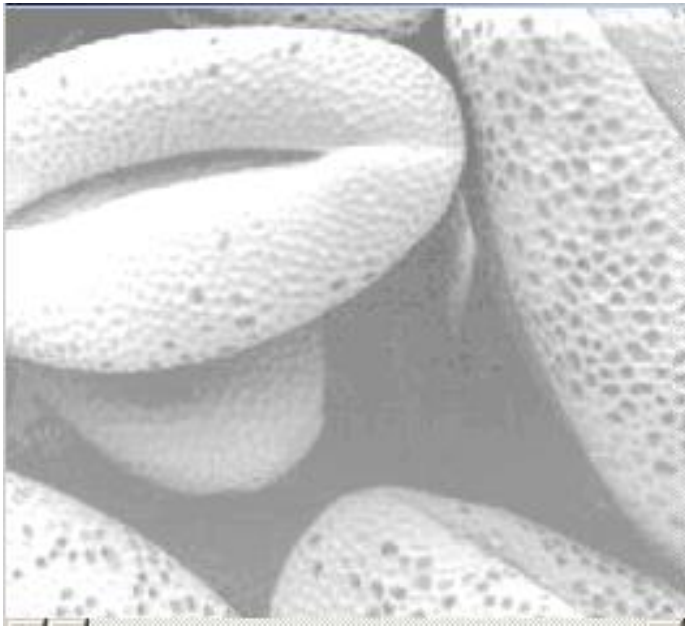
Histogram



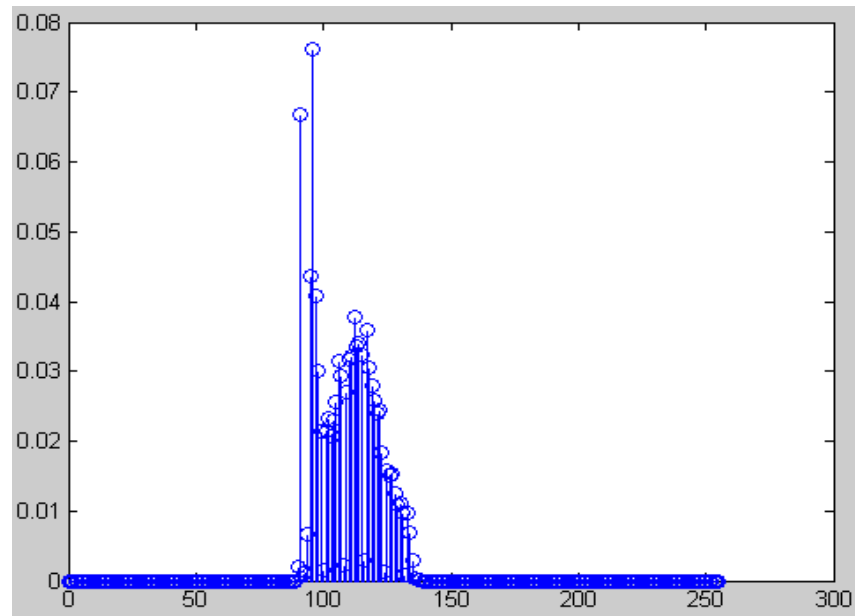
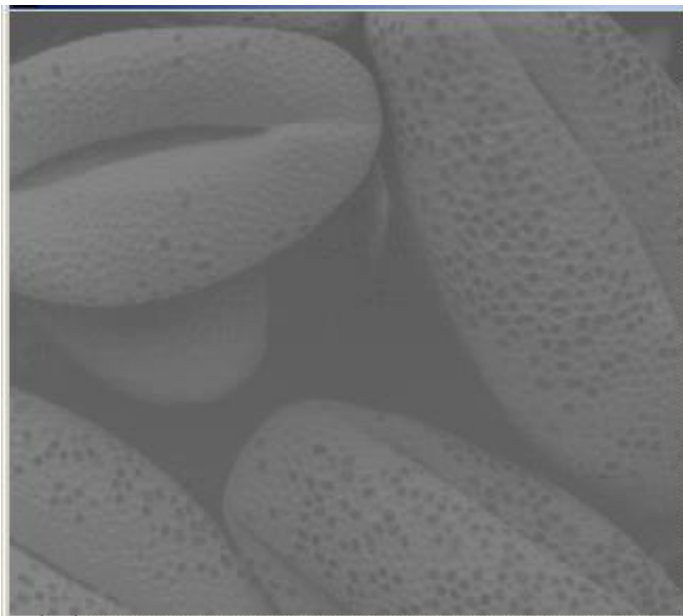
Histogram - tmavý



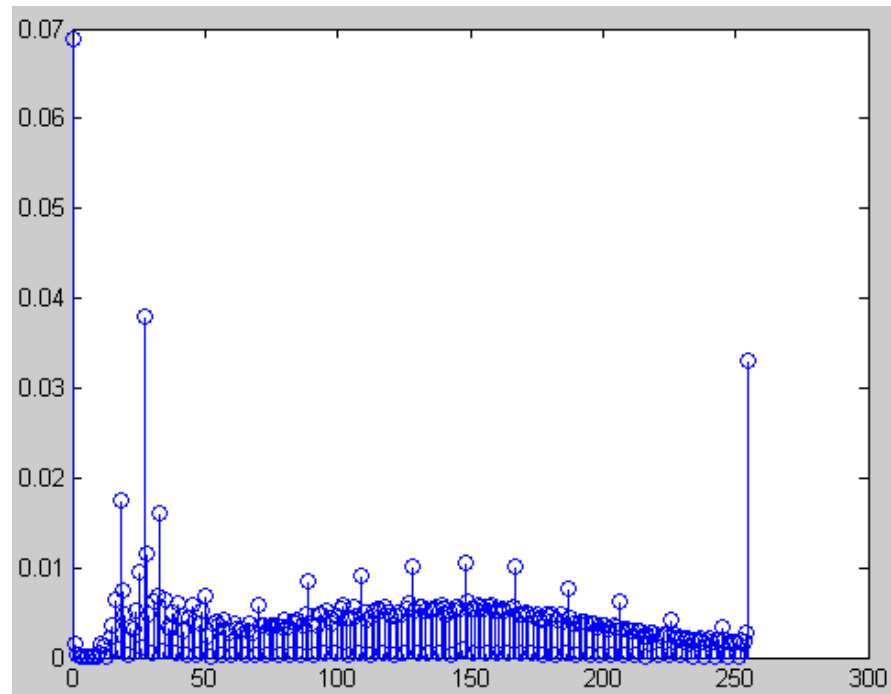
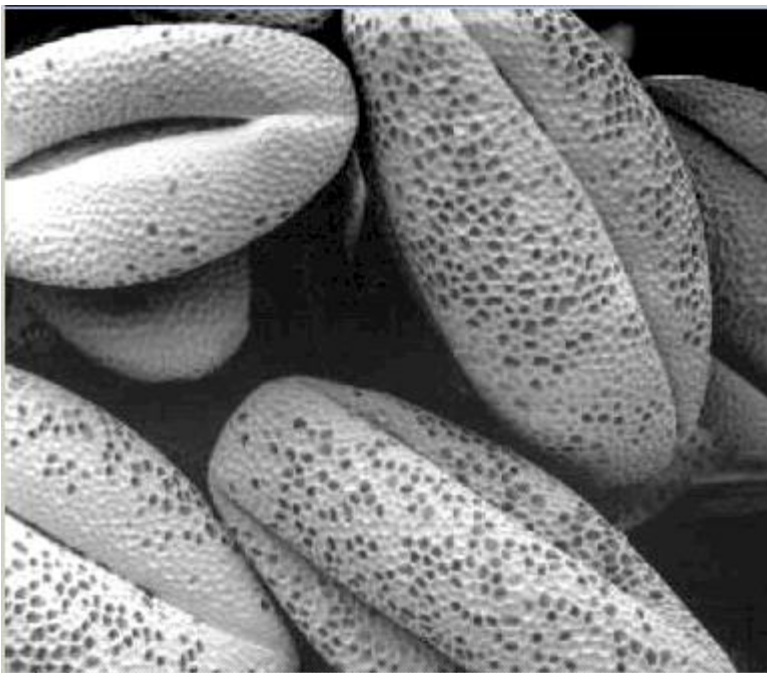
Histogram - světlý



Histogram – nízký kontrast



Histogram – vysoký kontrast



Histogram

histogram obrázku s hodnotami šedi r_0, r_1, \dots, r_{L-1}

$$p(r_k) = \frac{n_k}{n}$$

n_k : počet pixlů s hodnotou intenzity r_k

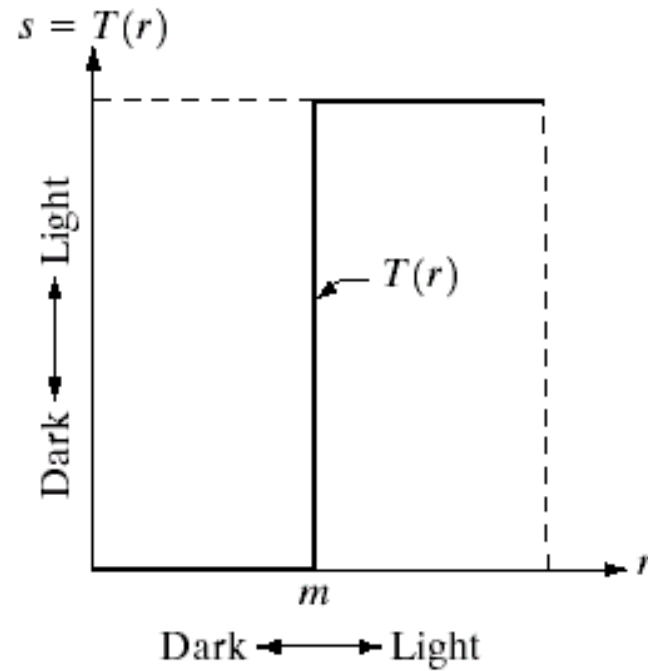
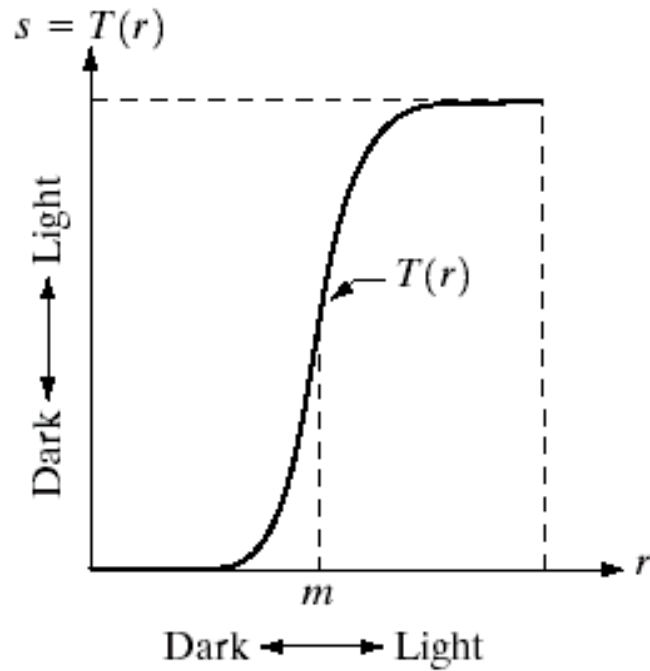
n : celkový počet pixlů obrázku

intenzita – realizace náhodné veličiny R s nějakou hustotou pravděpodobnosti

histogram – odhad hustoty pravděpodobnosti

$$\Pr(R = r_k) \approx p(r_k)$$

Změny kontrastu a jasu

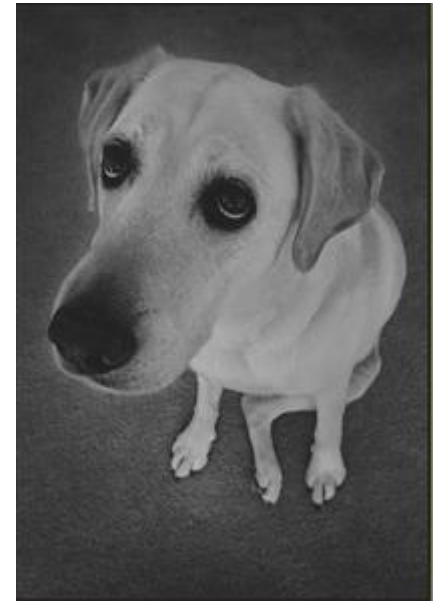




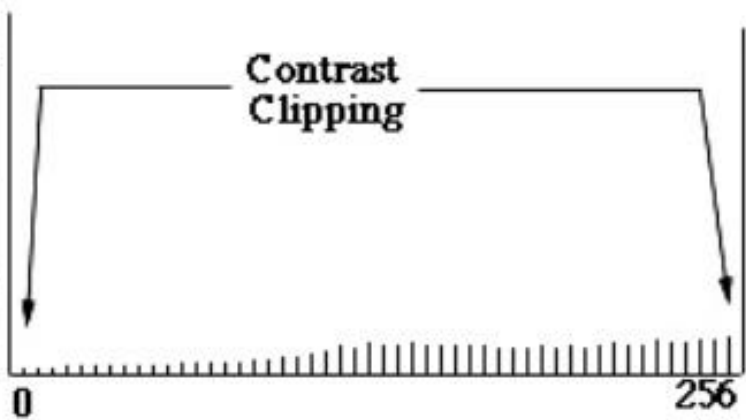
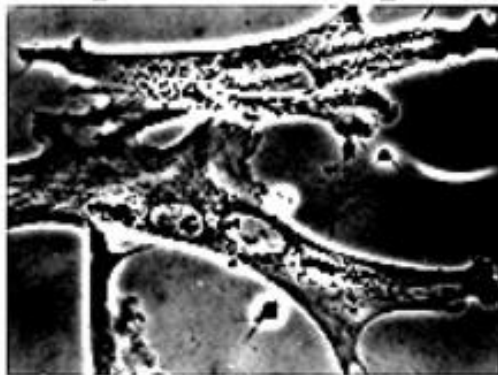
jas



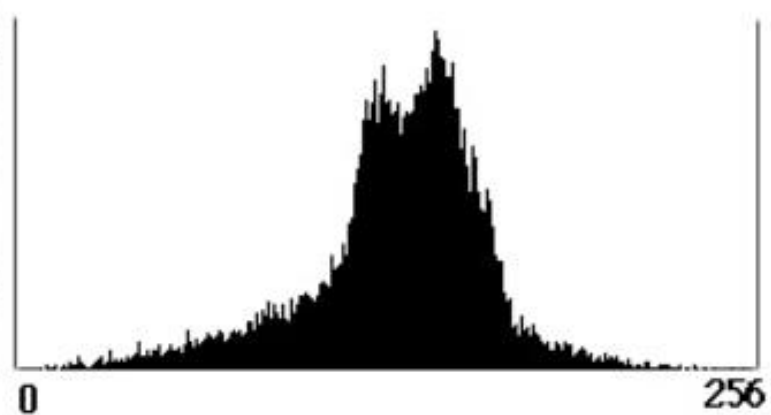
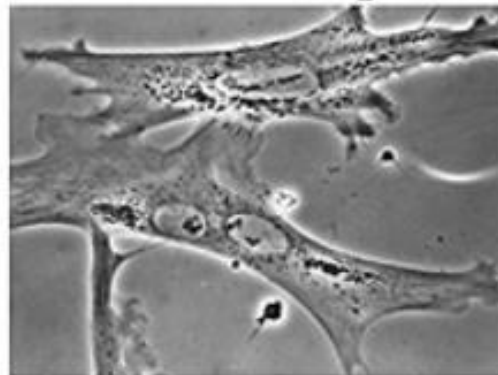
kontrast



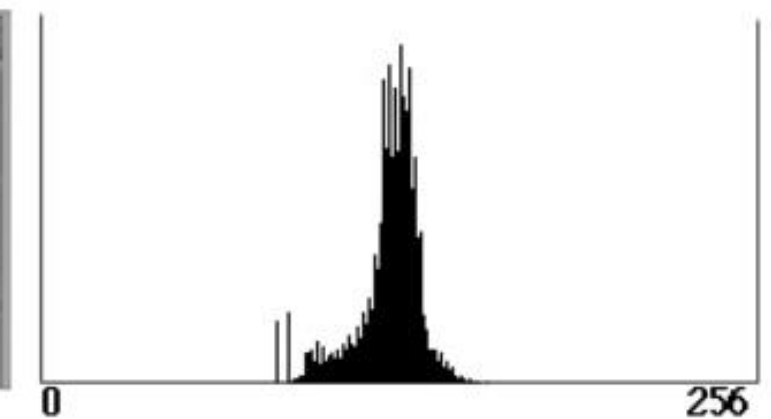
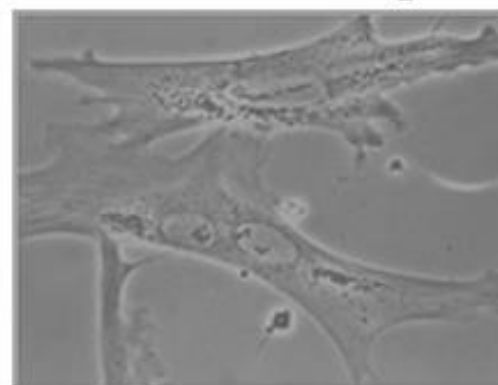
High Contrast Image



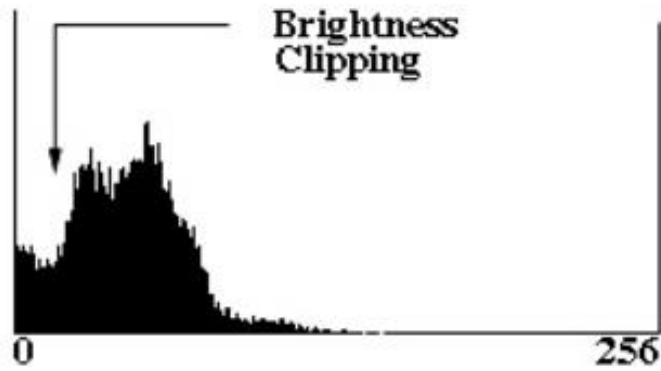
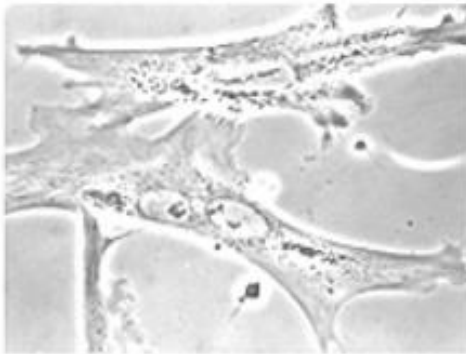
Full Contrast Range



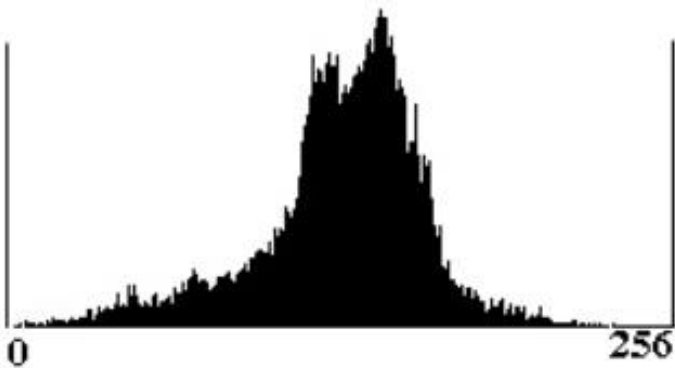
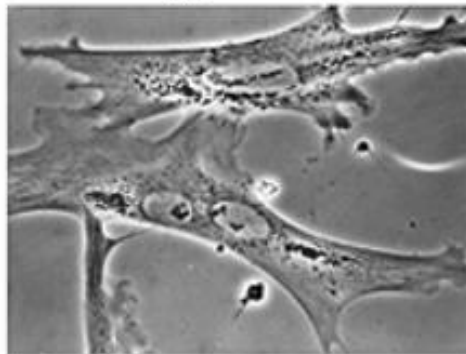
Low Contrast Image



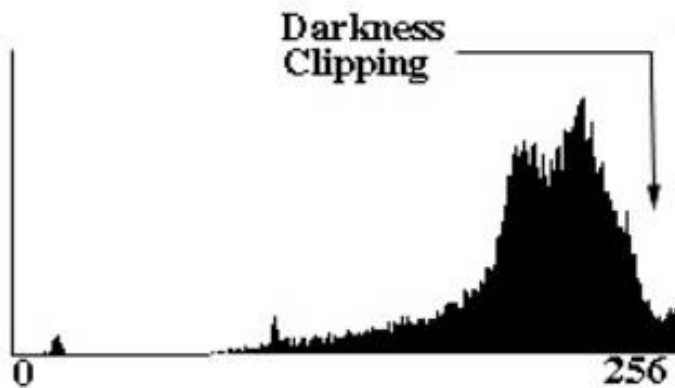
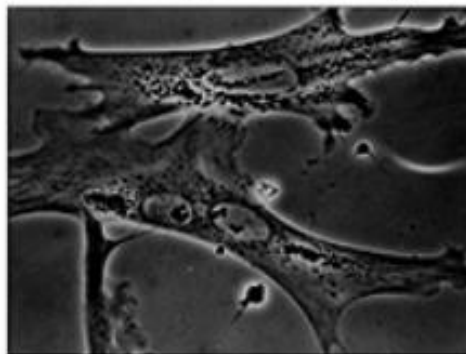
Bright Grey Level Image



Unclipped Image

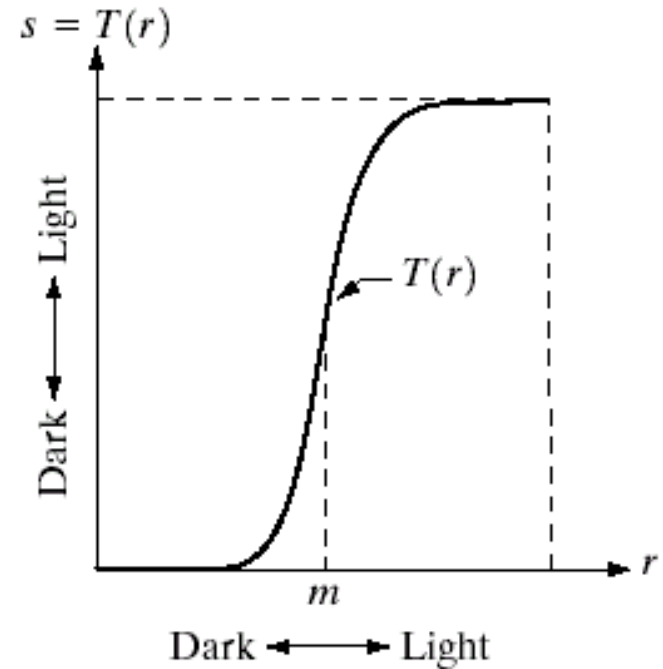


Dark Grey Level Image



Categories of illumination changes

- Point-wise transforms





- Global x local
- Linear x nonlinear

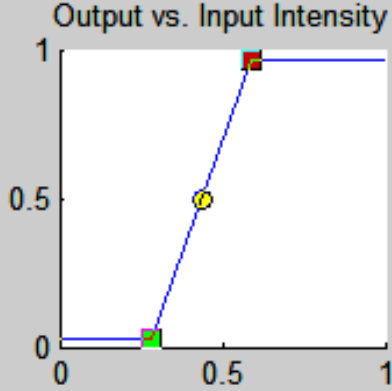
Změny kontrastu a jasu – lineární

Select an Image:
Pout

Adjusted Image

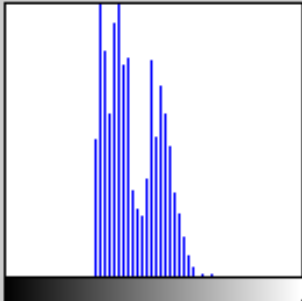


Output vs. Input Intensity

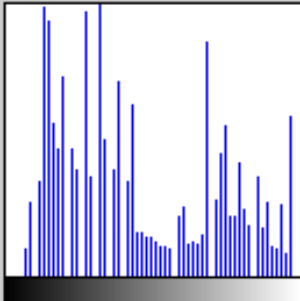


Gamma: 1

Histogram



Histogram



Operations:

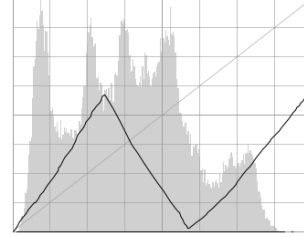
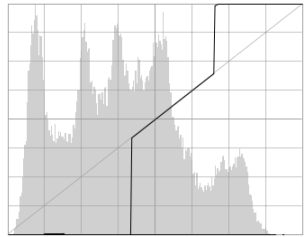
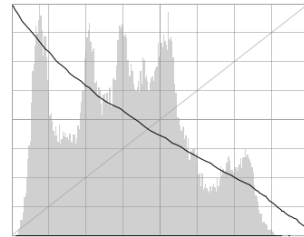
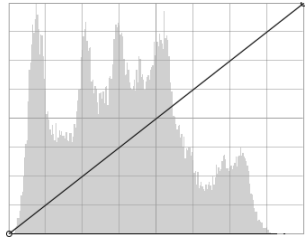
Intensity Adjustment

+ Brightness - Brightness

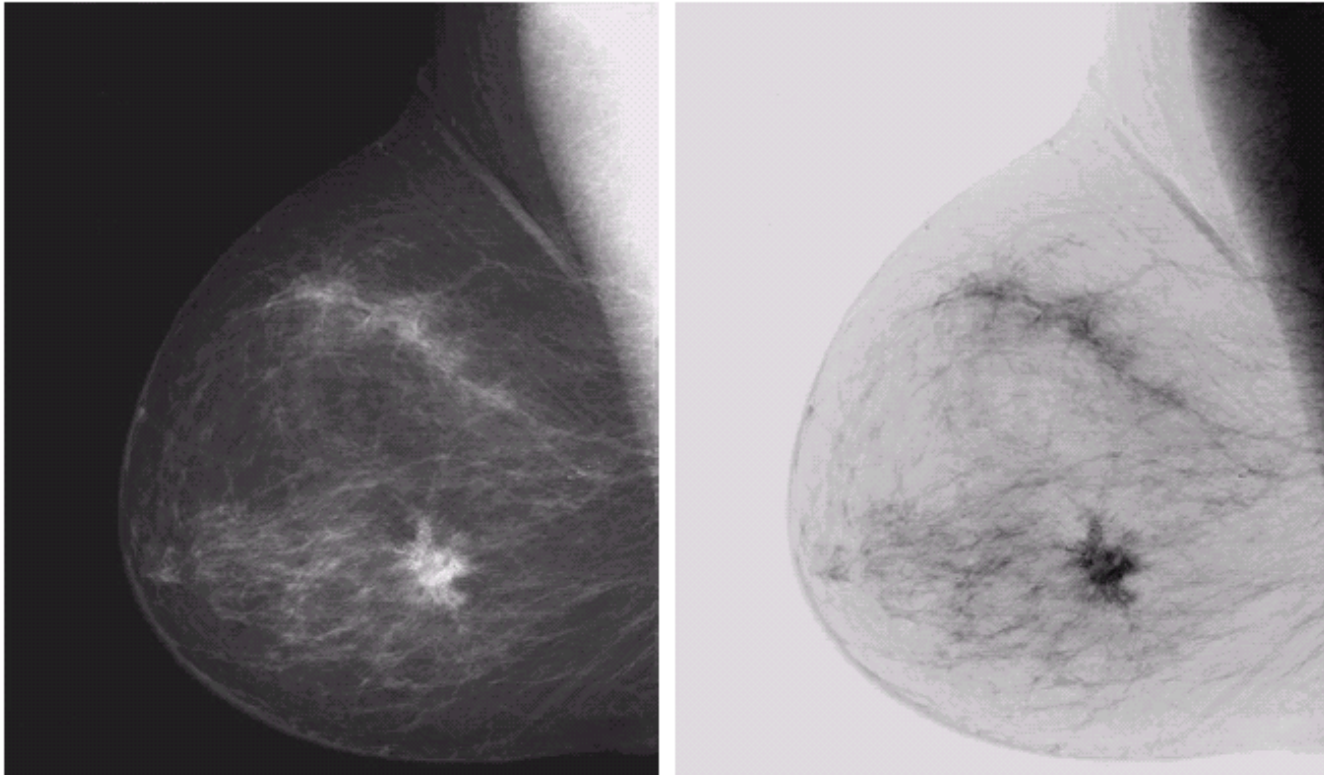
+ Contrast - Contrast

+ Gamma - Gamma

Info Close

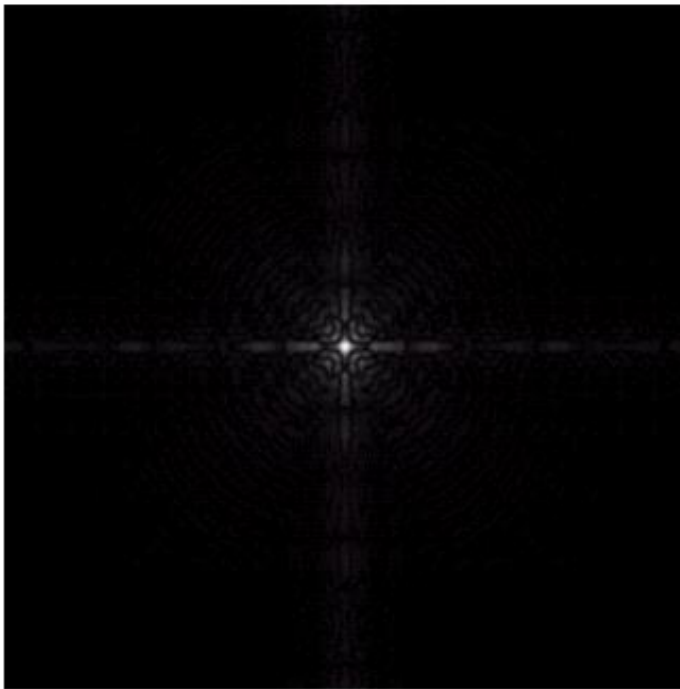


Negativ

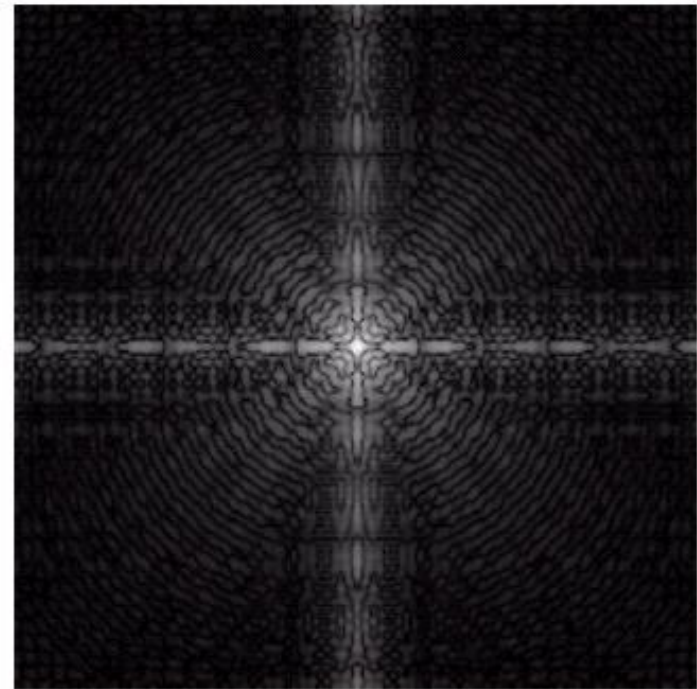


Mamogram

Nelineární transformace šedi



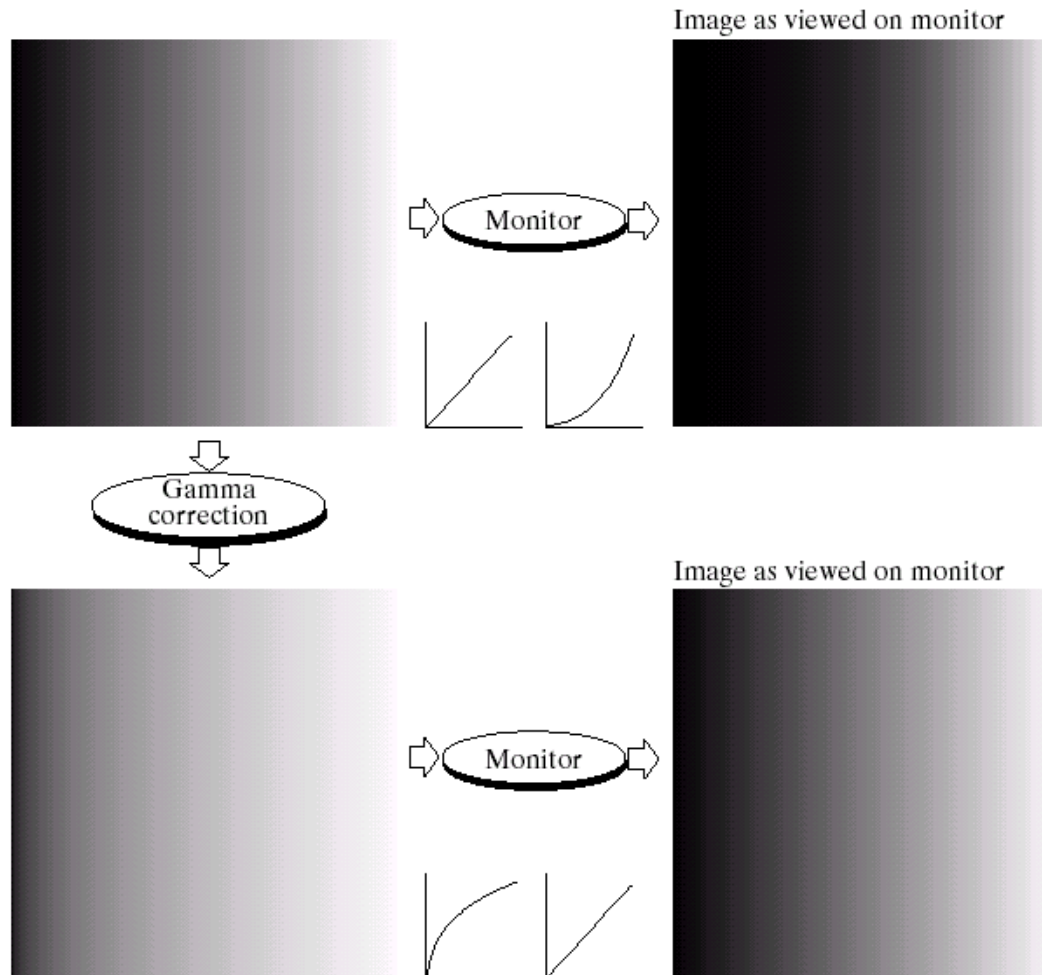
ampl



$\log(\text{ampl} + 1)$

Gama korekce

$$\text{Output} = c (\text{input})^{\text{gama}}$$



2.5

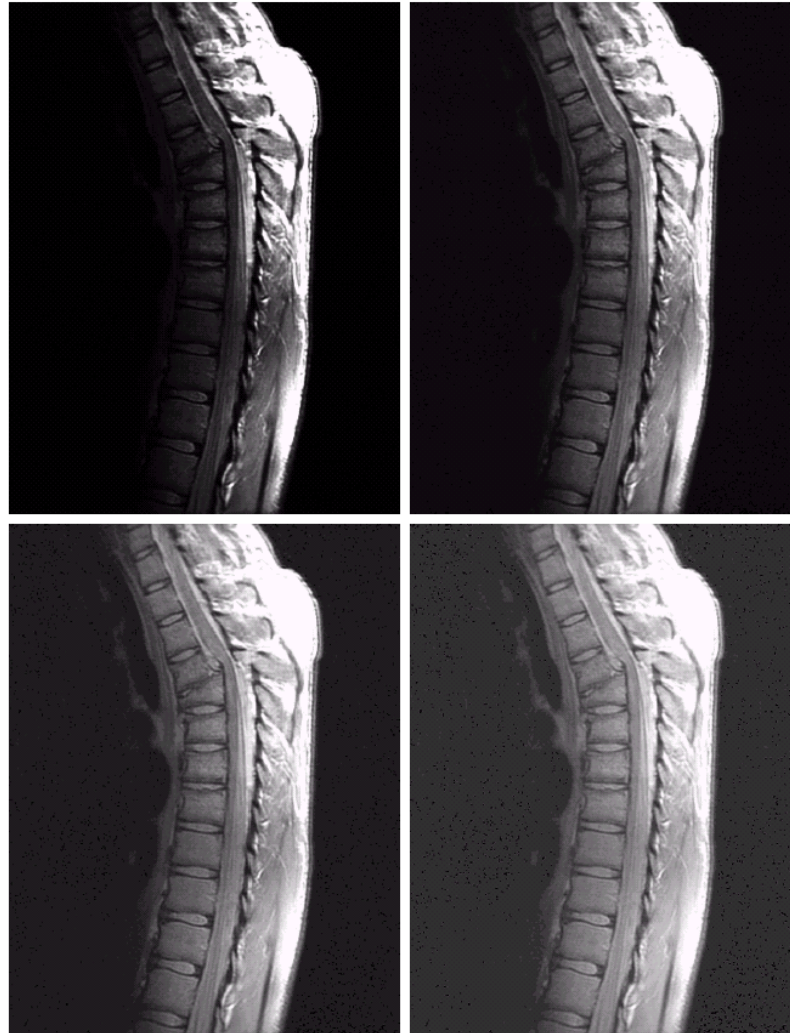
Gama korekce

Output = (input)^{gama} gama = 3, 4, 5



Gama korekce

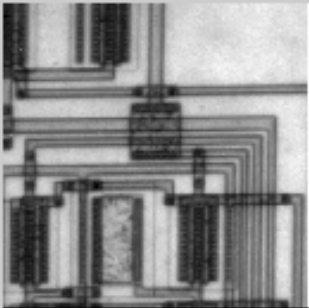
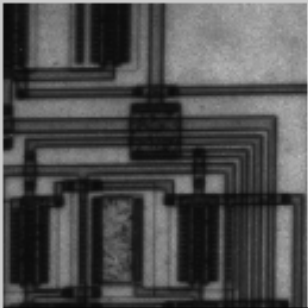
Output = (input)^{gama} gama = 0.6, 0.4, 0.3



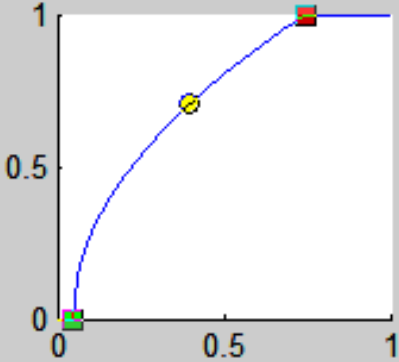
Gama korekce

Select an Image:
Circuit

Adjusted Image

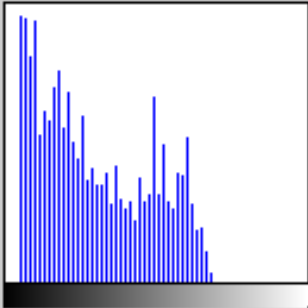


Output vs. Input Intensity

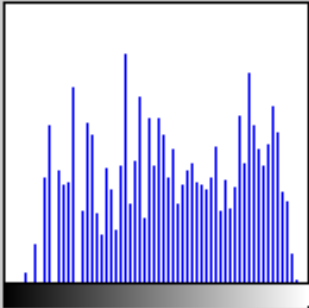


Gamma: 0.50493

Histogram



Histogram



Operations:
Intensity Adjustment

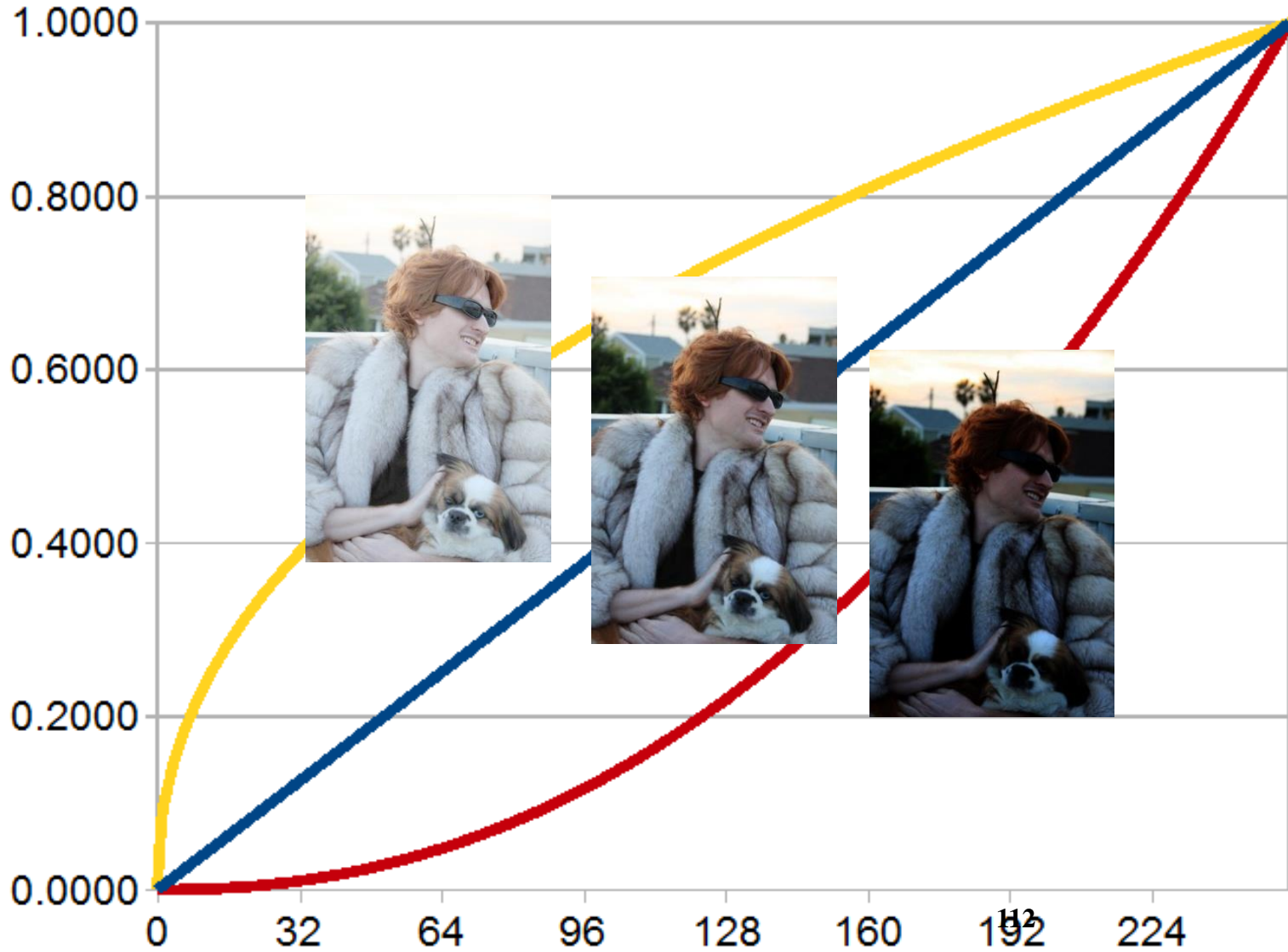
+ Brightness - Brightness

+ Contrast - Contrast

+ Gamma - Gamma

Info Close



Gama korekce



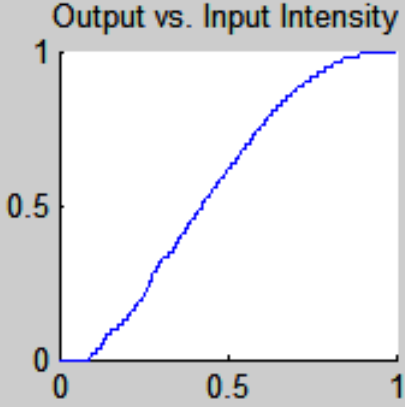
Ekvalizace histogramu

Select an Image:
Flower

Equalized Image

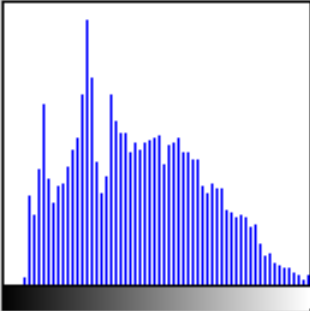


Output vs. Input Intensity

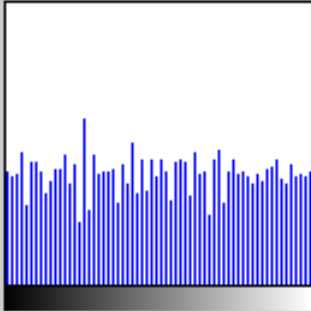


Gamma: 1

Histogram



Histogram



Operations:

Histogram Equalization

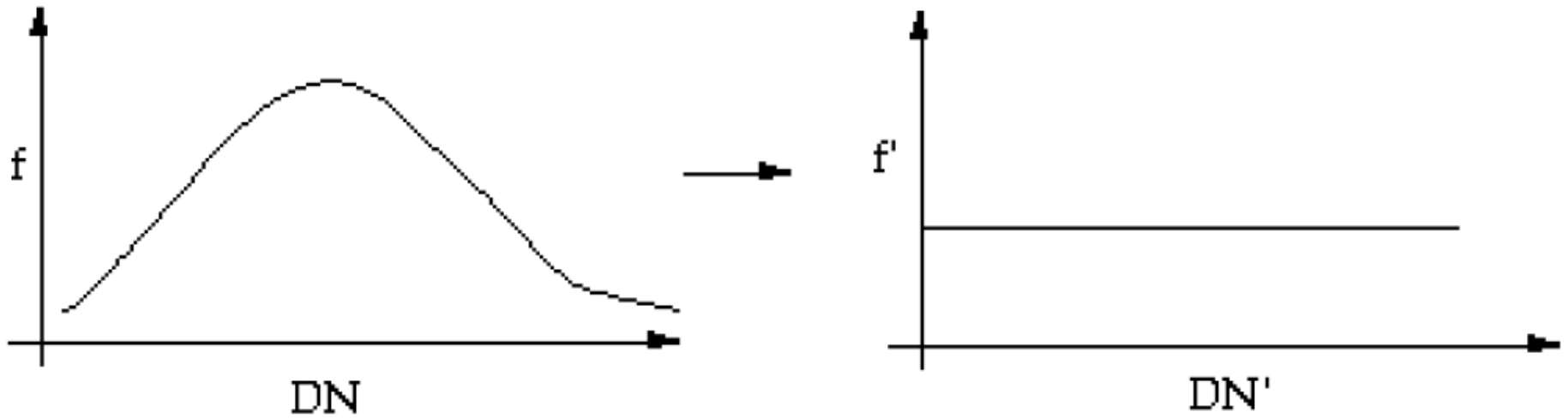
+ Brightness - Brightness

+ Contrast - Contrast

+ Gamma - Gamma

Info Close

Ekvalizace histogramu



řešení -> kumulativní histogram

Histogram - ekvalizace

motivace – spojité hodnoty intenzit

hledáme $s = T(r)$

(1) $T(r)$ monotónně rostoucí pro $0 \leq r \leq 1$

(zachovává pořadí černá \rightarrow bílá)

(2) $T(r): [0,1] \rightarrow [0,1]$ (zachovává rozsah hodnot)

Totéž pro inverzní $r = T^{-1}(s)$

$p_{in}(r)$, $p_{out}(s)$ hustoty pravděpodobnosti intenzit

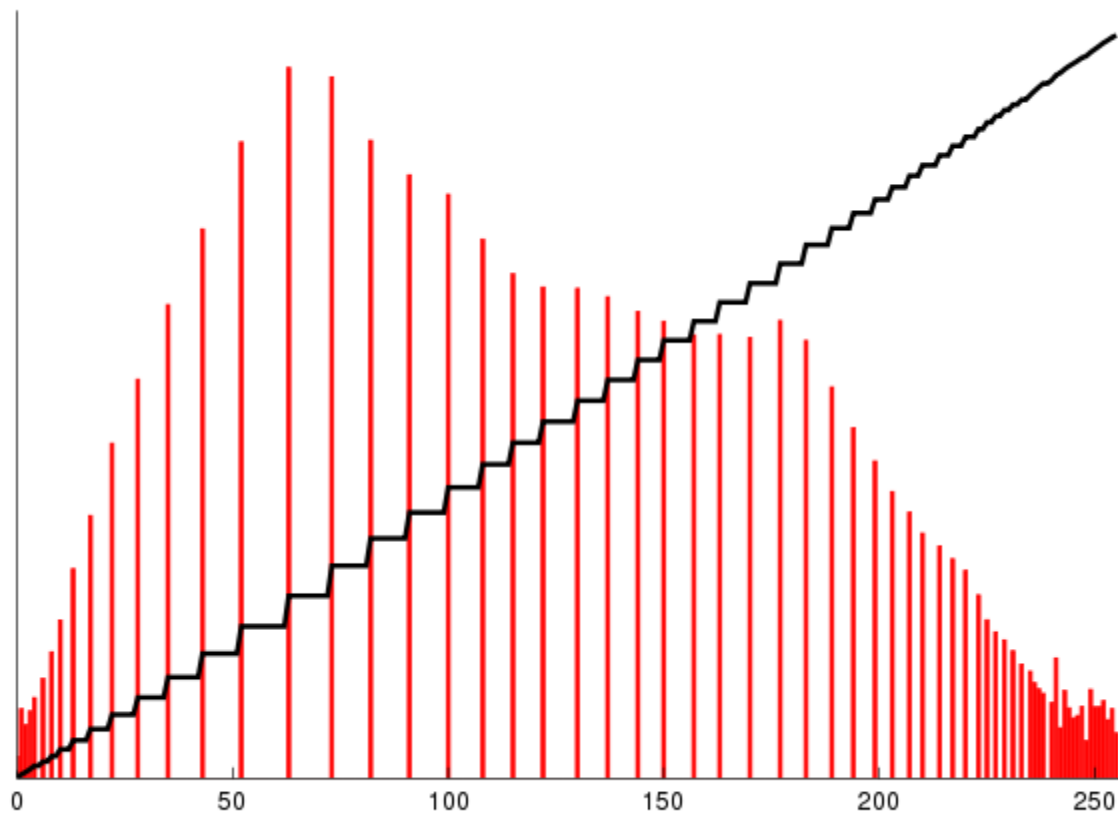
Histogram - ekvalizace

předpokládejme

$$s = T(r) = \int_0^r p_{in}(w)dw, \quad 0 \leq r \leq 1$$

(kumulativní) distribuční funkce (CDF) pro $p_{in}(r)$,
splňuje podmínky

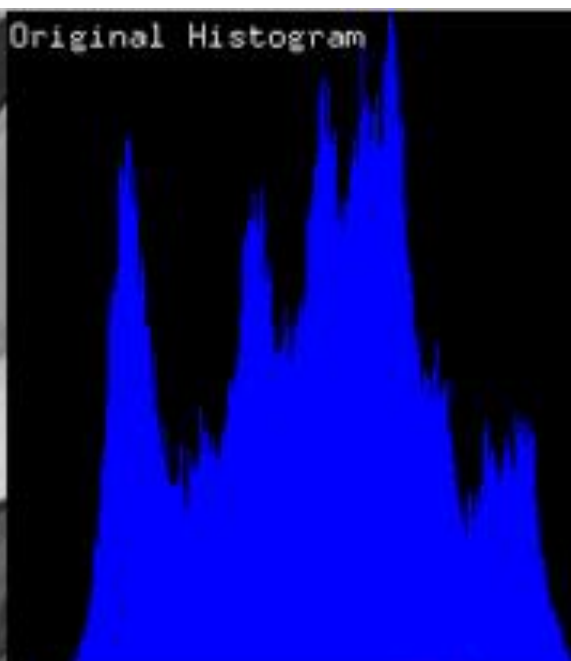
Ekvalizace histogramu



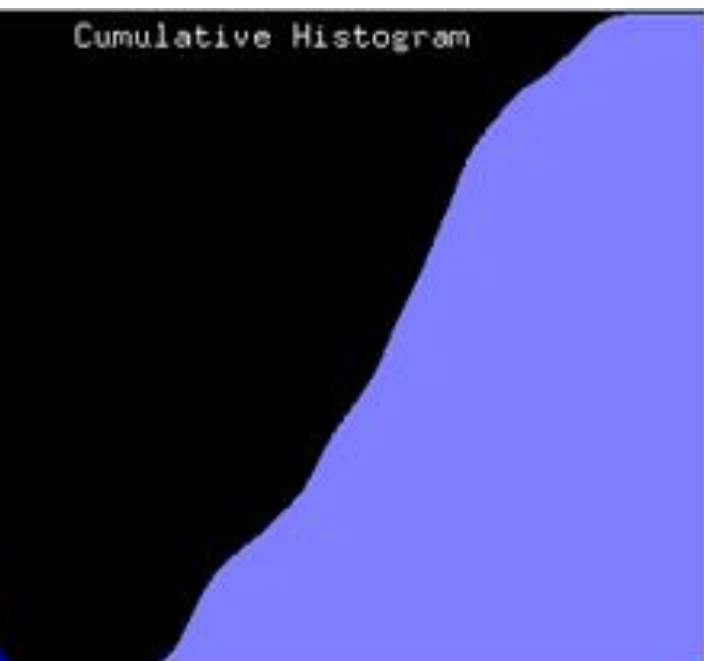
Original Image



Original Histogram



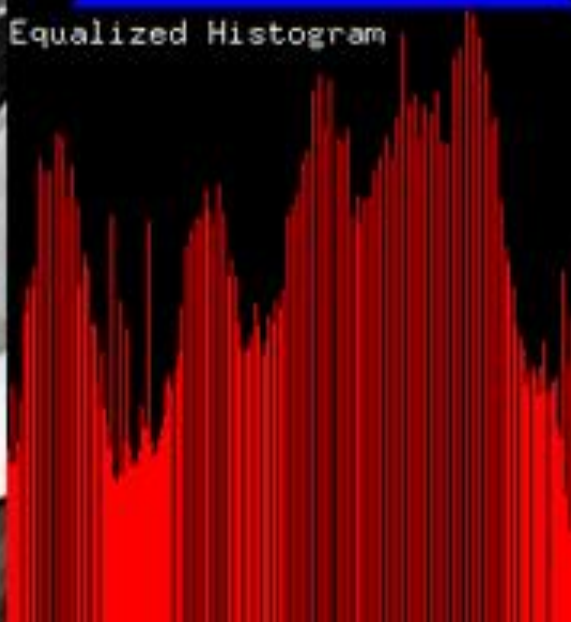
Cumulative Histogram



Enhanced Image



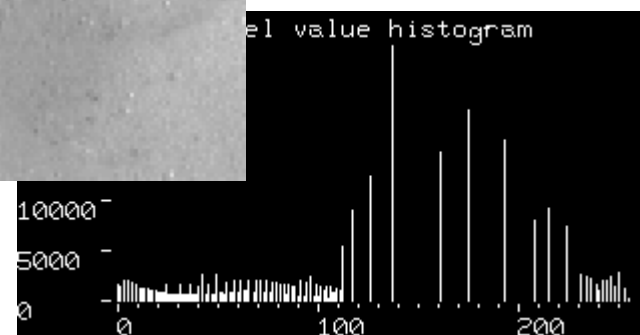
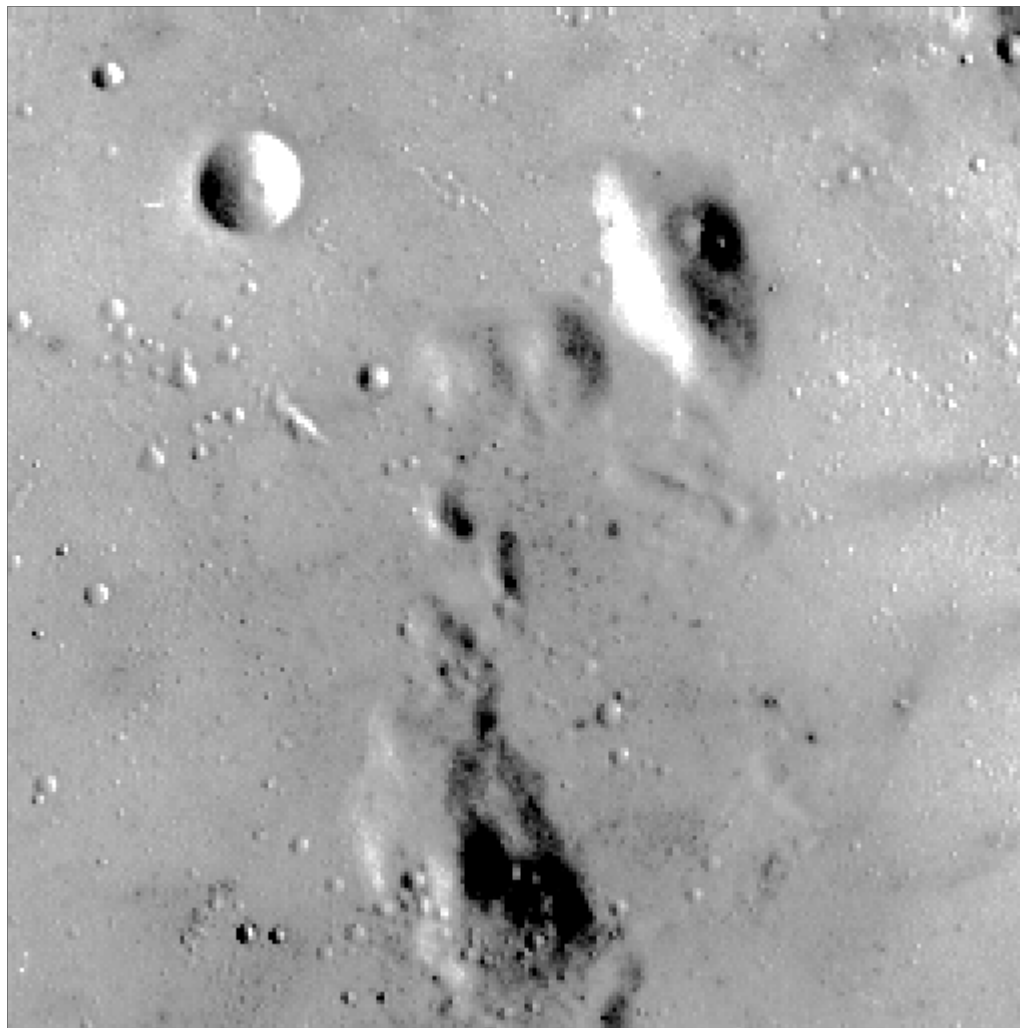
Equalized Histogram

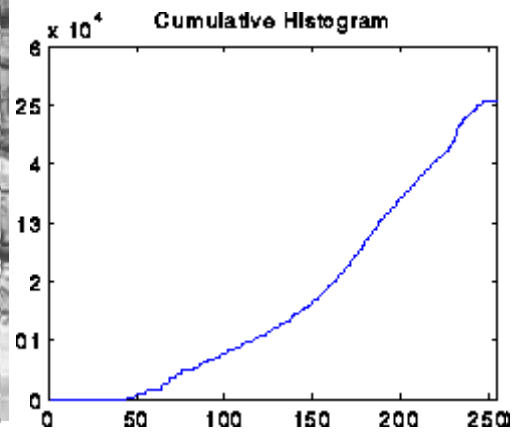


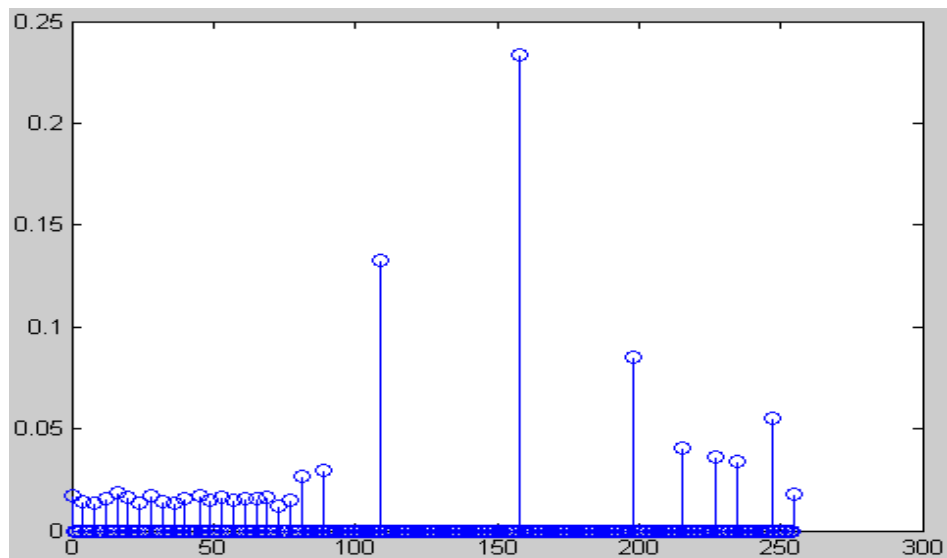
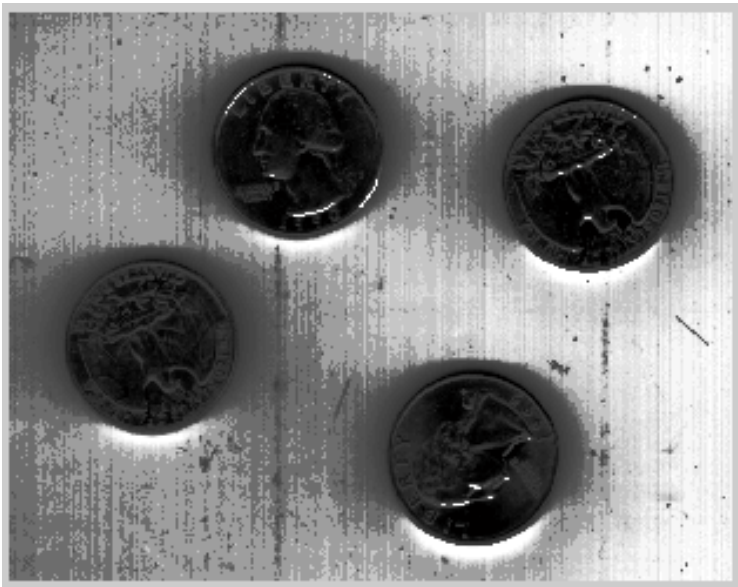
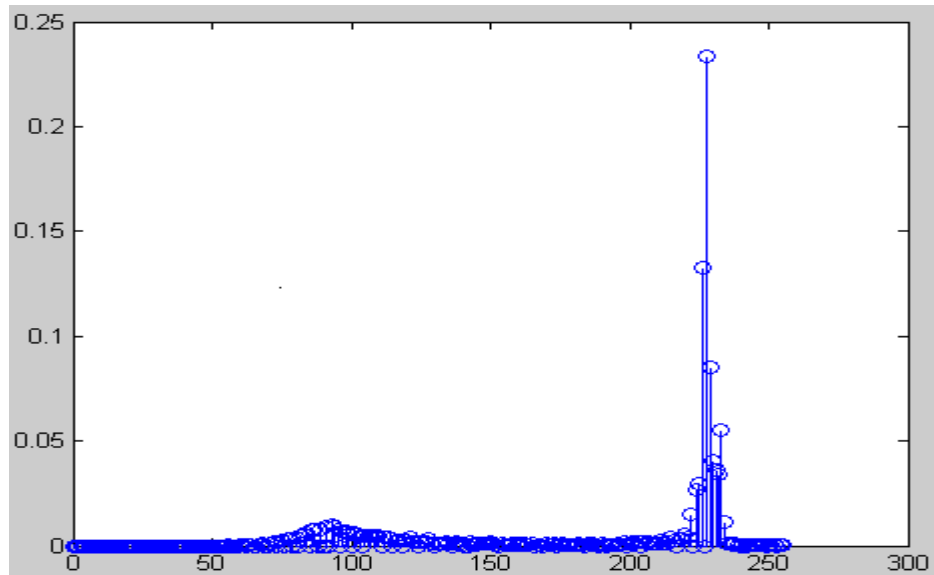
Cumulative Equalized Histogram



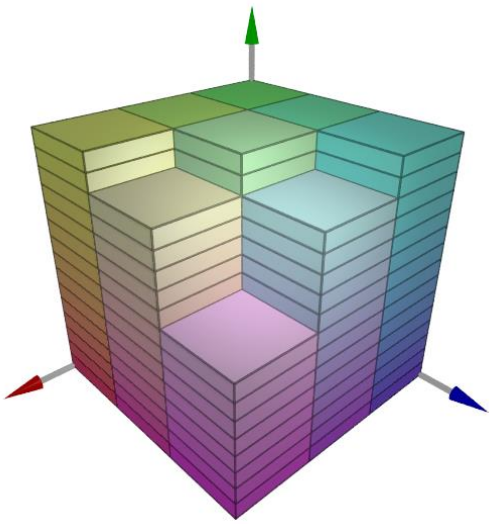
Ekvalizace histogramu



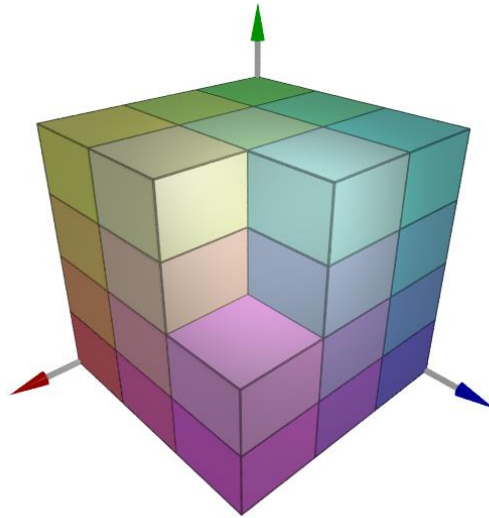




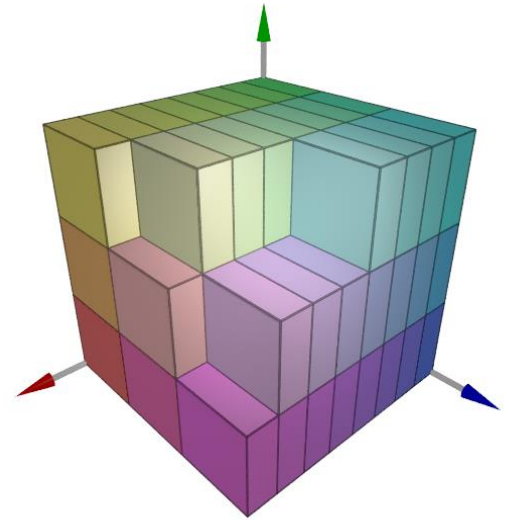
Barevné histogramy



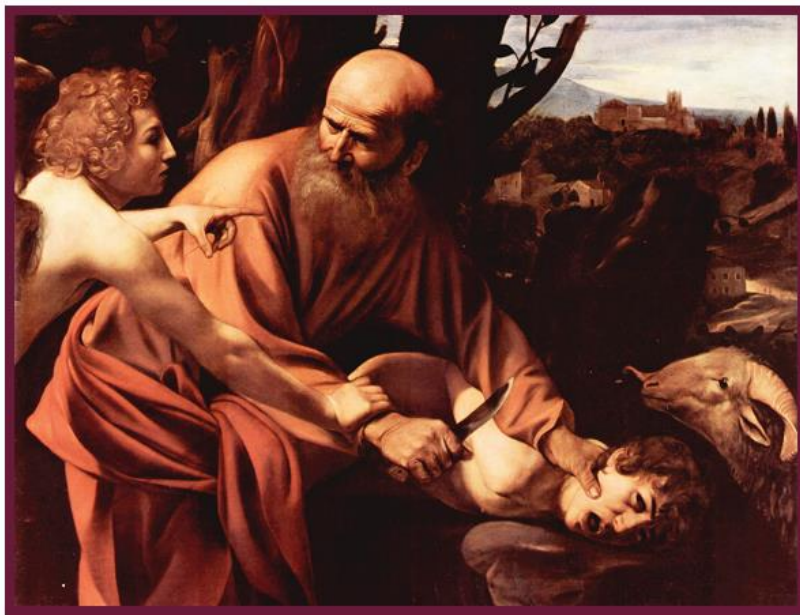
3x15x3



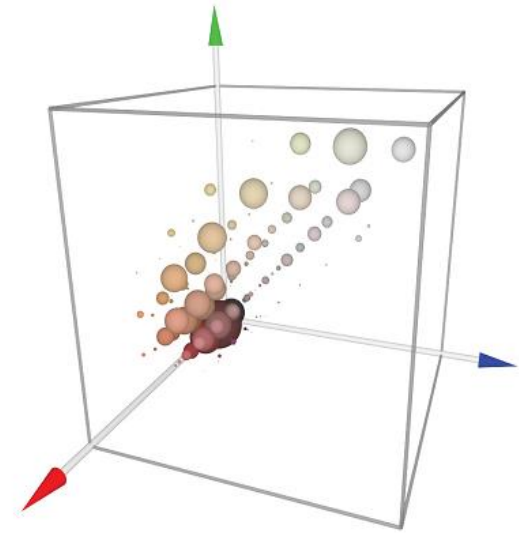
3x4x3



8x3x3



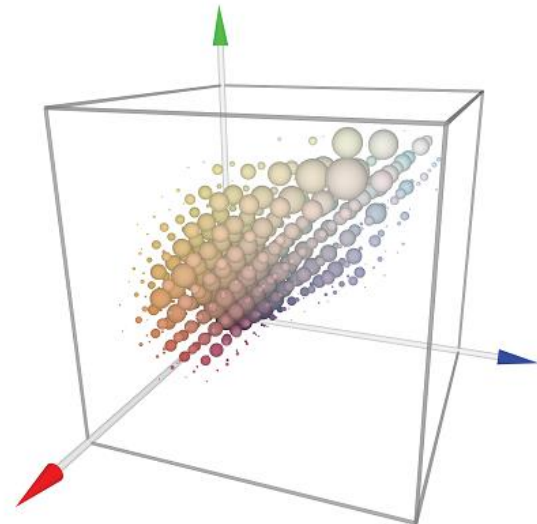
(a) Source image.



(b) RGB color histogram.

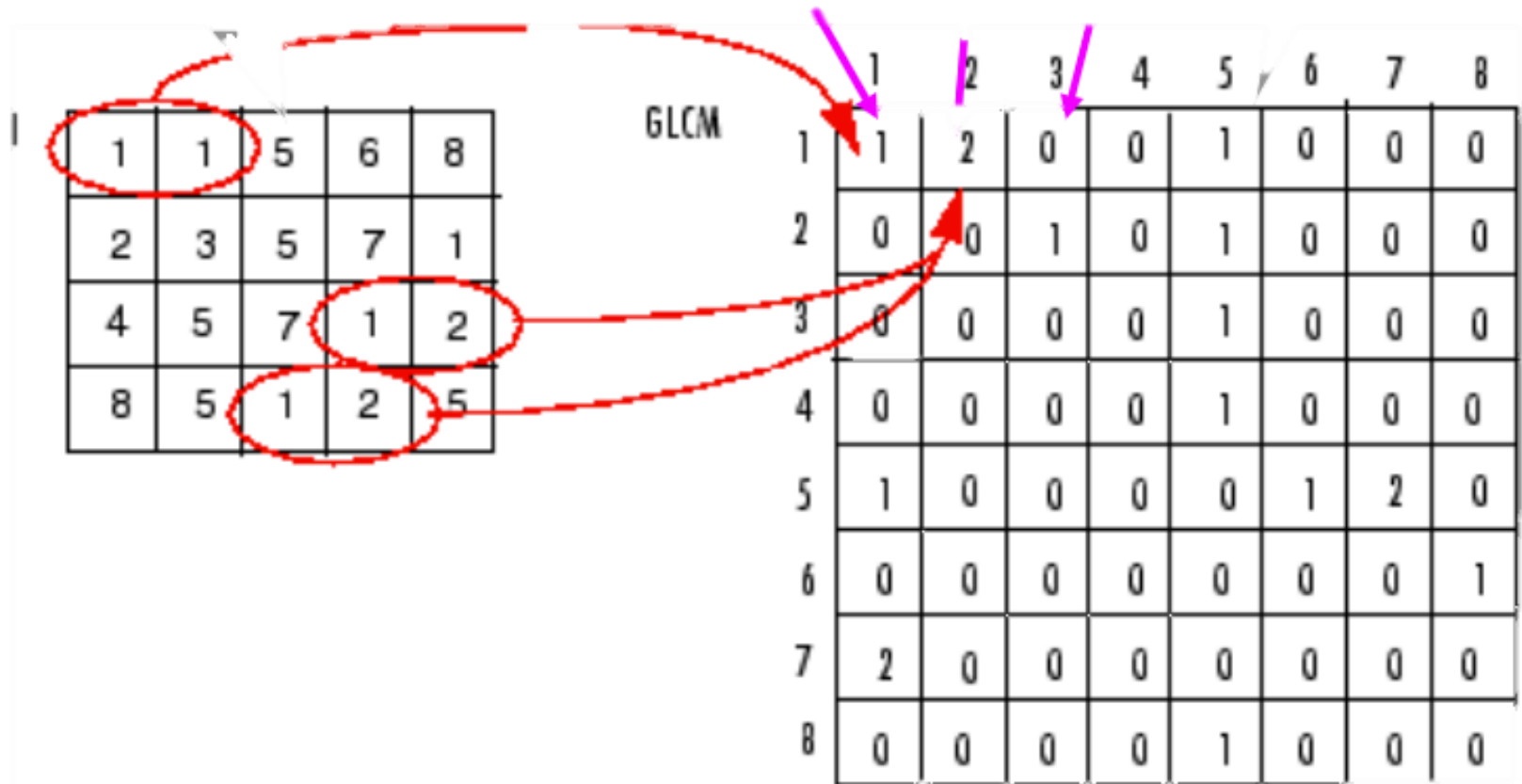


(c) Source image.



(d) RGB color histogram.

Co-occurrence matrices

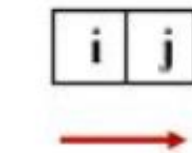


Co-occurrence matrices

| | | | |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 0 | 0 | 2 | 2 |
| 0 | 0 | 2 | 2 |

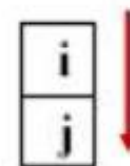
Image I

| | | | | |
|---|---|---|---|---|
| | | j | | |
| | | 0 | 1 | 2 |
| i | 0 | 4 | 0 | 2 |
| | 1 | 2 | 2 | 0 |
| | 2 | 0 | 0 | 2 |



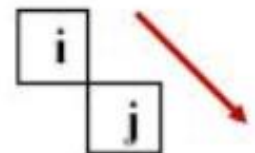
Horizontal

| | | | | |
|---|---|---|---|---|
| | | j | | |
| | | 0 | 1 | 2 |
| i | 0 | 4 | 0 | 2 |
| | 1 | 2 | 2 | 0 |
| | 2 | 0 | 0 | 2 |



Vertical

| | | | | |
|---|---|---|---|---|
| | | j | | |
| | | 0 | 1 | 2 |
| i | 0 | 2 | 0 | 1 |
| | 1 | 2 | 1 | 1 |
| | 2 | 0 | 0 | 1 |



Diagonal

Co-occurrence matrices – Haralick descriptors

| Descriptor | Equation | Meaning |
|------------------------------|---|------------------------------------|
| Variance | $\sum_i \sum_j (i - j)^2 P(i, j)$ | level of contrast of the image |
| Entropy | $\sum_i \sum_j P(i, j) \log P(i, j)$ | suavity of the image |
| Energy | $\sum_i \sum_j P^2(i, j)$ | uniformity of the image |
| Homogeneity | $\sum_i \sum_j P(i, j) / (1 + i - j)$ | homogeneity of pixels distribution |
| 3 rd Order Moment | $\sum_i \sum_j (i - j)^3 P(i, j)$ | level of distortion |
| Inverse Variance | $\sum_i \sum_j P(i, j) / (i - j)^2$ | inverse level of contrast |

Barevné histogramy

charakterizace obrazu

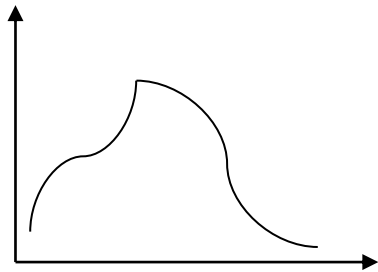
CBIR

Lokální úpravy kontrastu



Histogram - specifikace

histogram₁

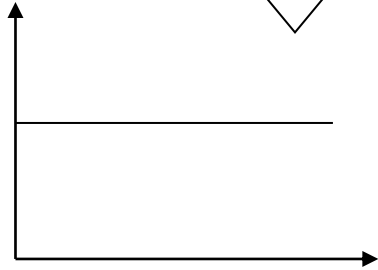


$S^{-1} * T$

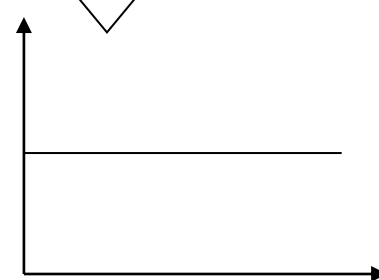
histogram₂



T

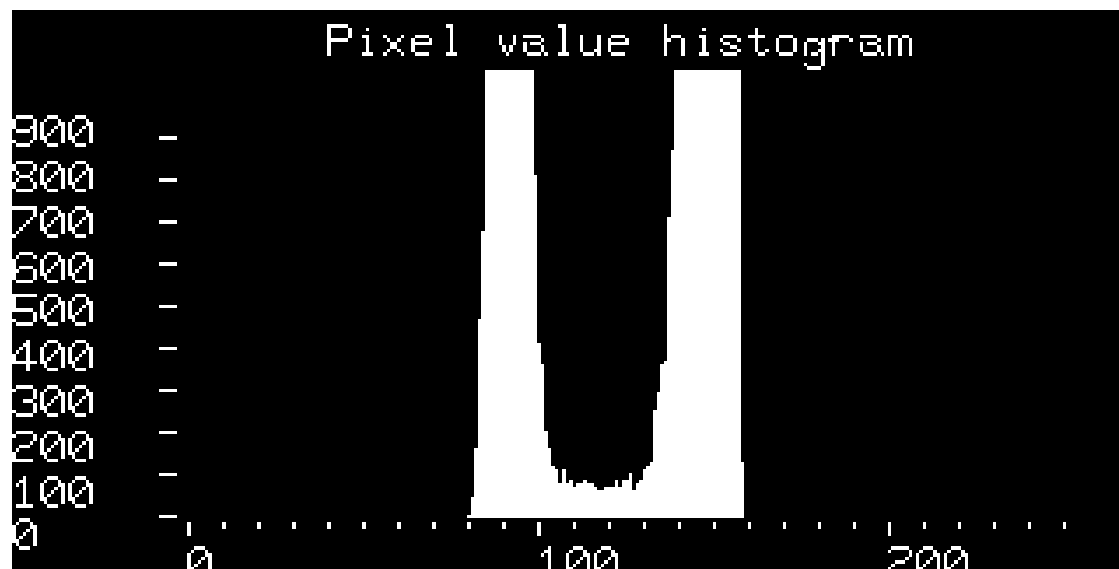
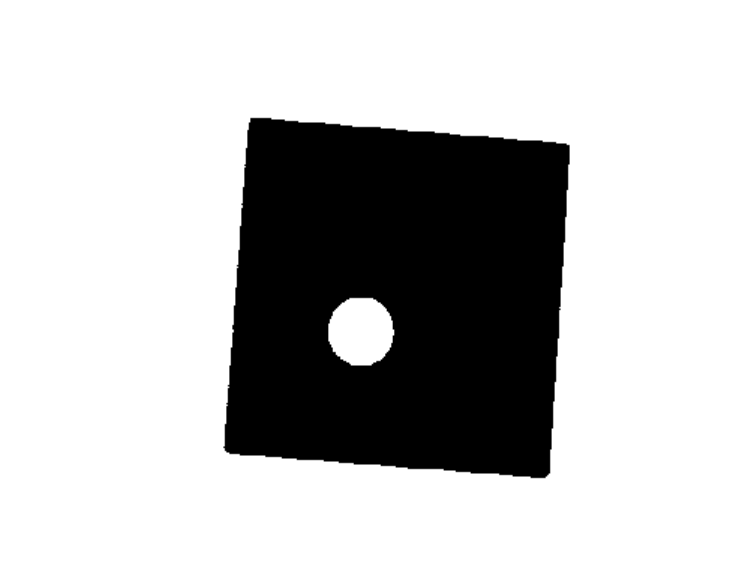
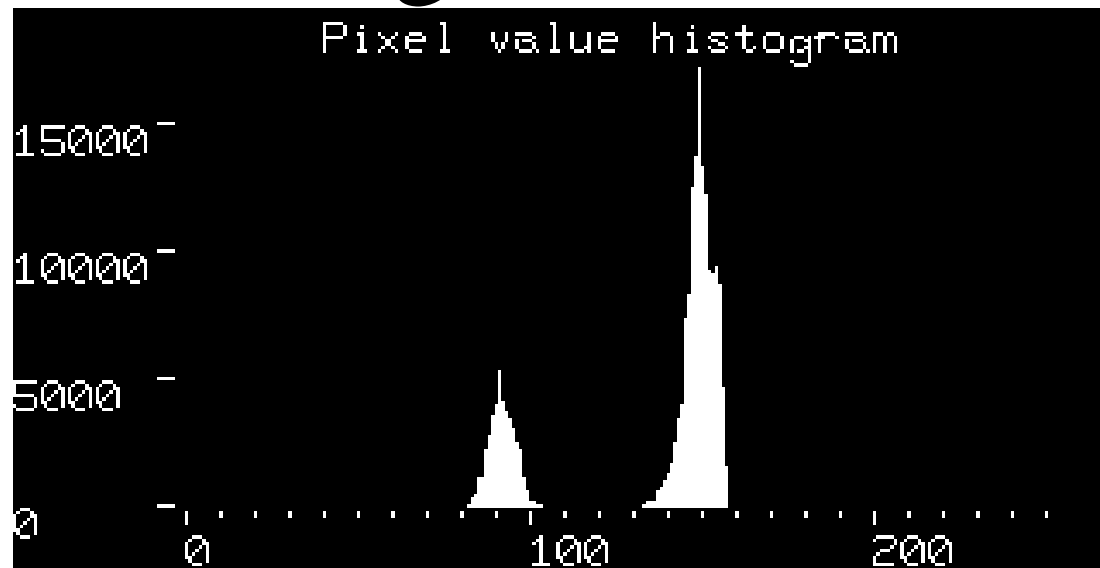
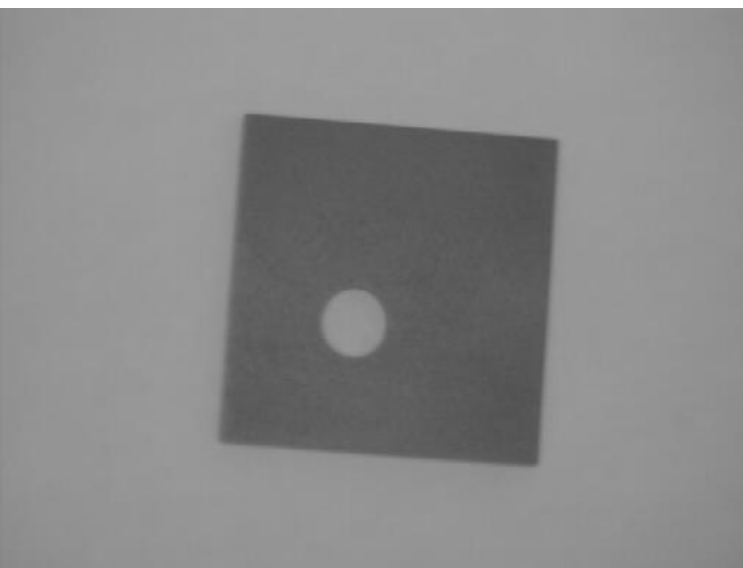


S

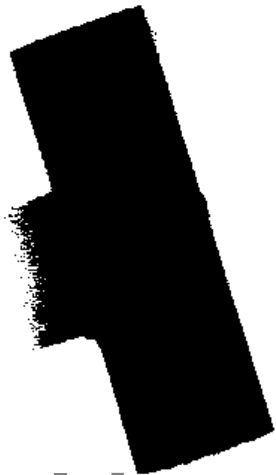
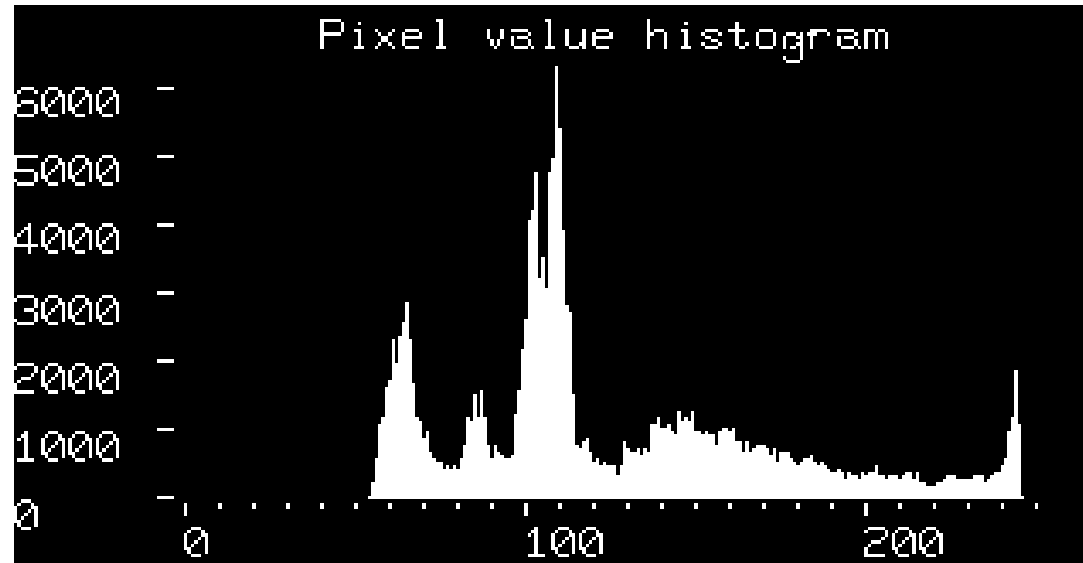
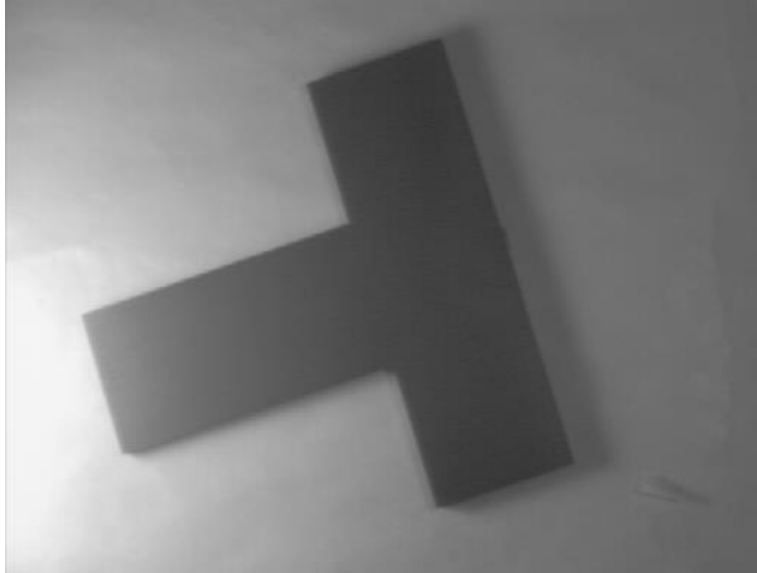


?

Thresholding



Thresholding II

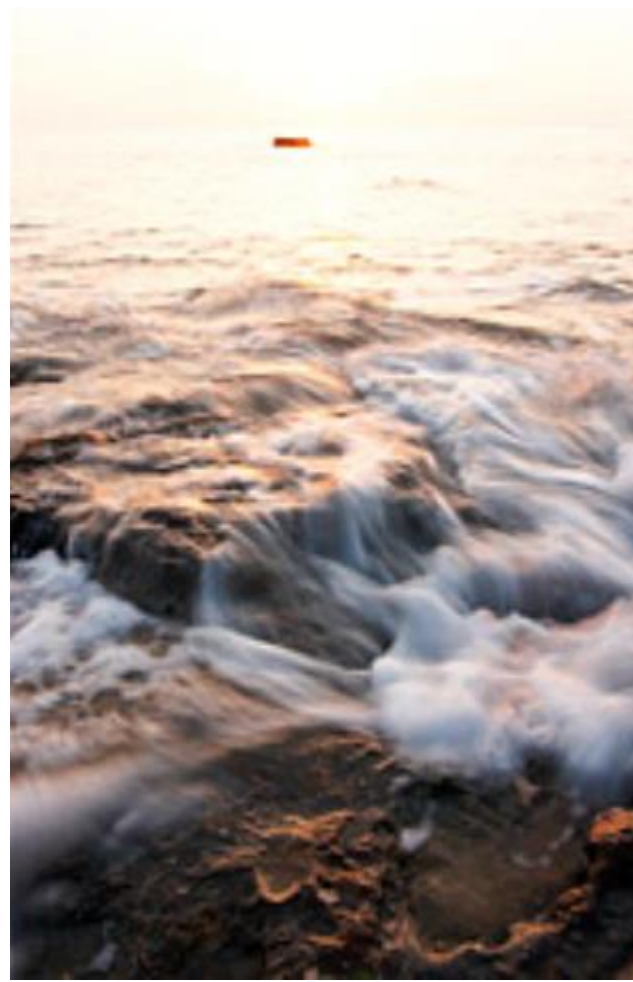
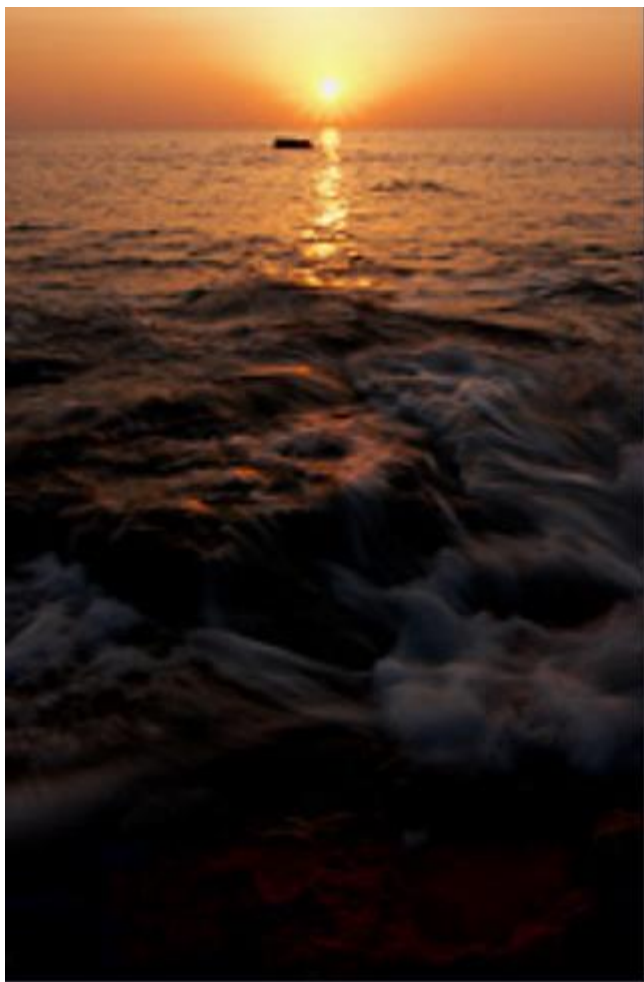


threshold: 80

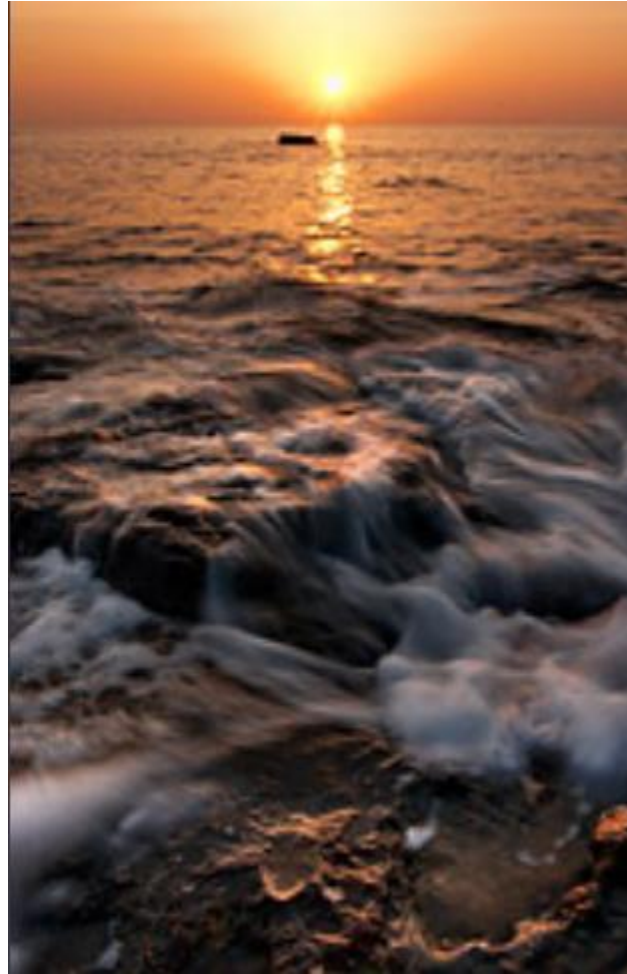
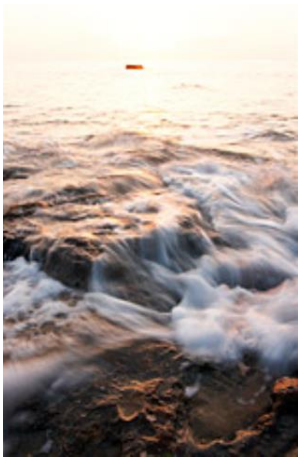
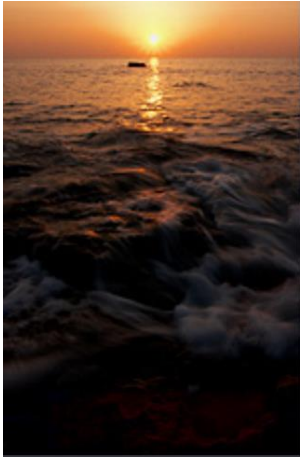


threshold: 120

High dynamic range images



High dynamic range images





Šum



thermal imaging



electrical interference



ultrasound imaging

Potlačení šumu v obraze

Modely šumu

Lineární filtrace

Nelineární metody

Modely šumu

Aditivní náhodný šum

$$g = f + n$$

„tepelný, fotografický šum, kvantizační šum“

jednoduchý model

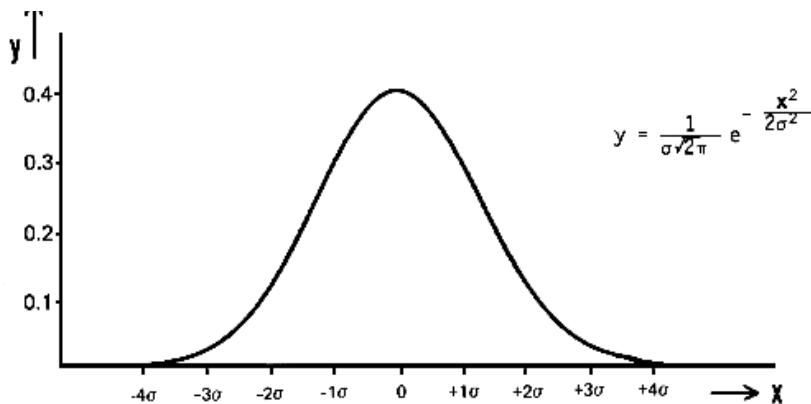
Modely šumu



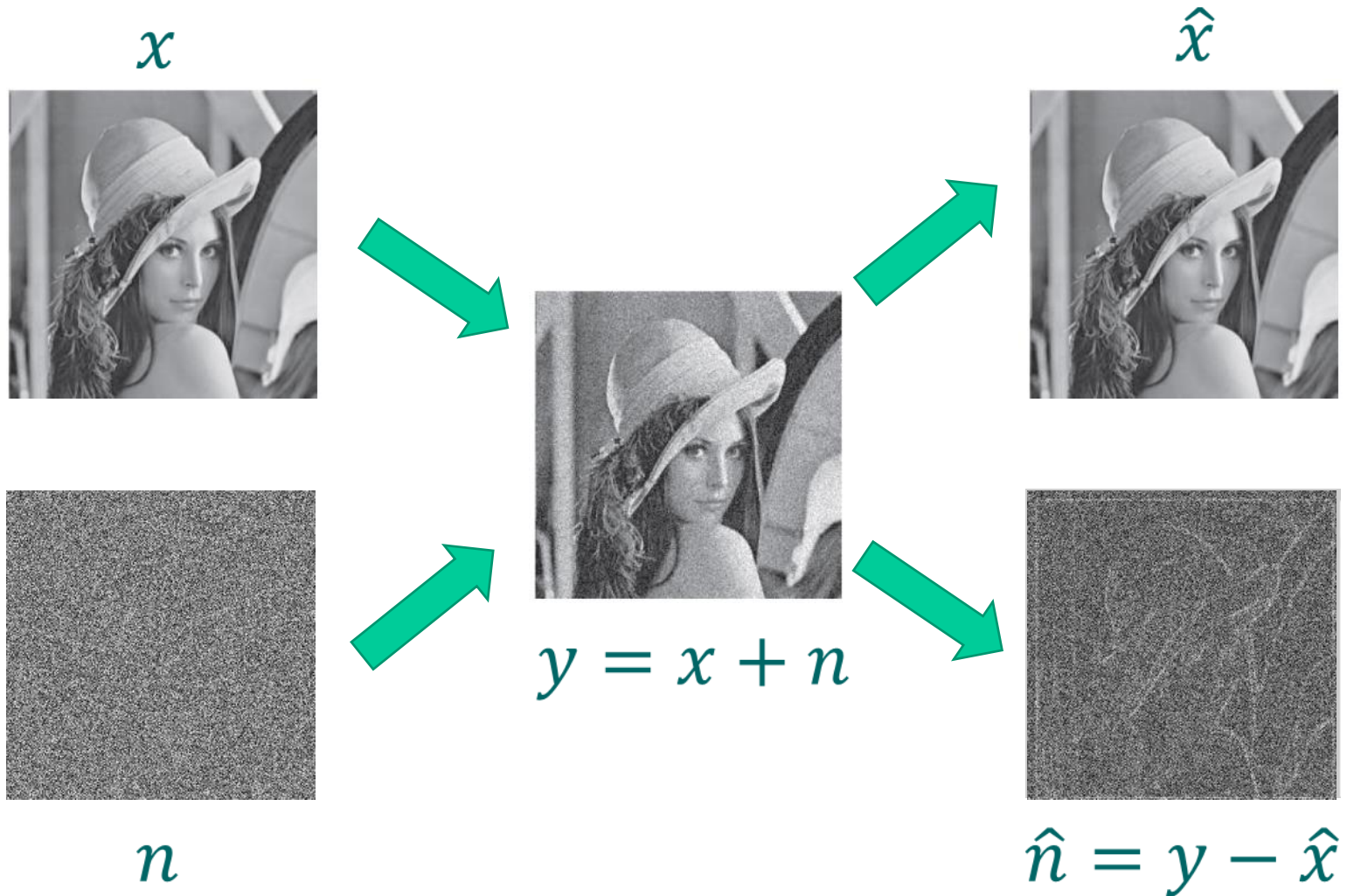
Gaussovský bílý šum (AGWN)

- White – $n(i,j)$ independent in both space and time
- Zero-mean – $\bar{I}(i, j) = 0$
- Gaussian - $n(i,j)$ is random variable with distribution

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$



Aditivní šum



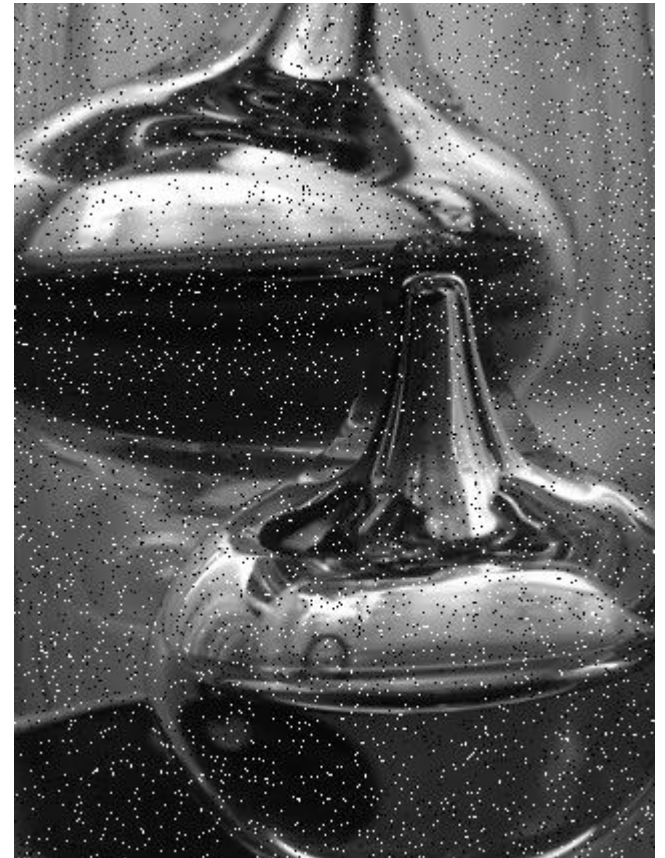
Modely šumu

Impulsní šum (sůl a pepř)

$$P(f) = 1 - K$$

$$P(1) = K/2$$

$$P(0) = K/2$$





Original



Salt and pepper noise



Impulse noise



Gaussian noise

Míra šumu v obraze

Rozptyl

Signal-to-noise ratio (SNR)

$$\text{SNR} = 10 \log (D(f)/D(n)) \quad [\text{dB}]$$

$$\frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x, y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - \hat{f}(x, y)]^2}$$

Gaussovský bílý šum

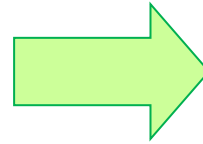


20 dB

10 dB

0 dB

Průměrování v čase



Průměrování v čase

4

8

šum



16

32

64

Průměrování v čase

g_1, \dots, g_N – nezávislá pozorování

$$g_k = f + n_k$$

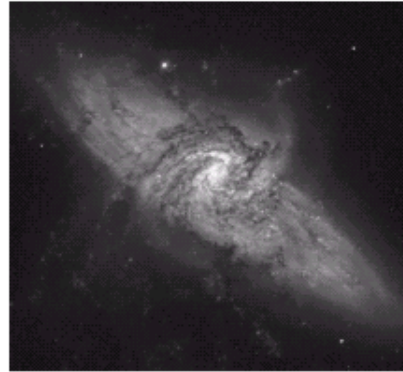
$$n_k \sim p(0, \sigma^2)$$

$$g = \frac{1}{N} \sum_{k=1}^N g_k = f + n$$

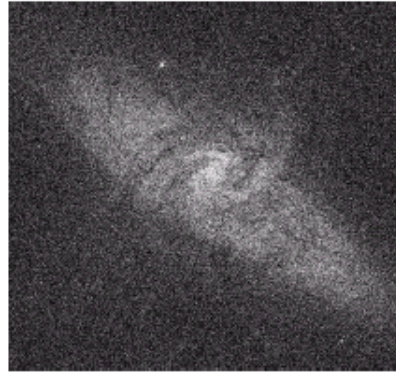
$$n \sim p(0, \sigma^2/N)$$

Průměrování v čase

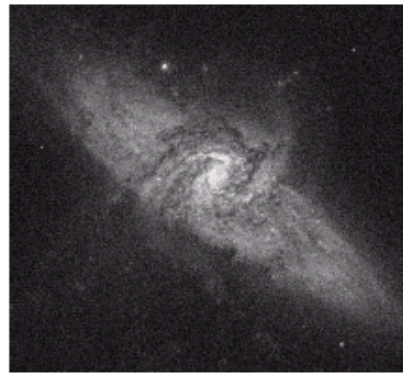
orig



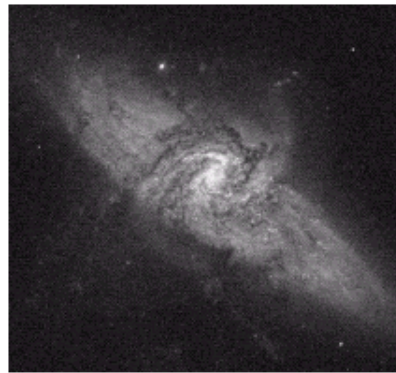
šum



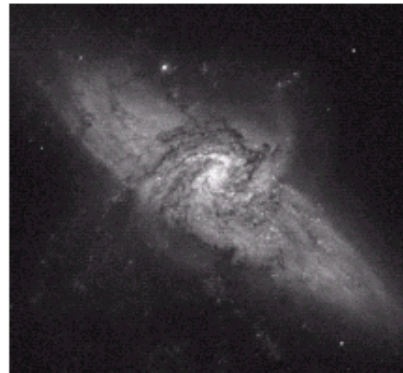
8



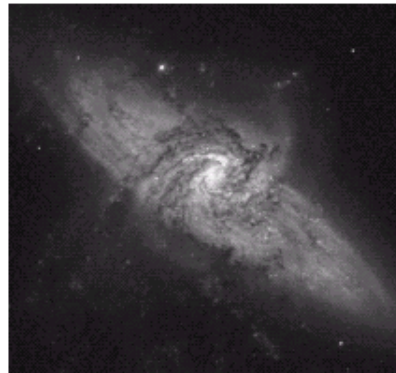
16



64

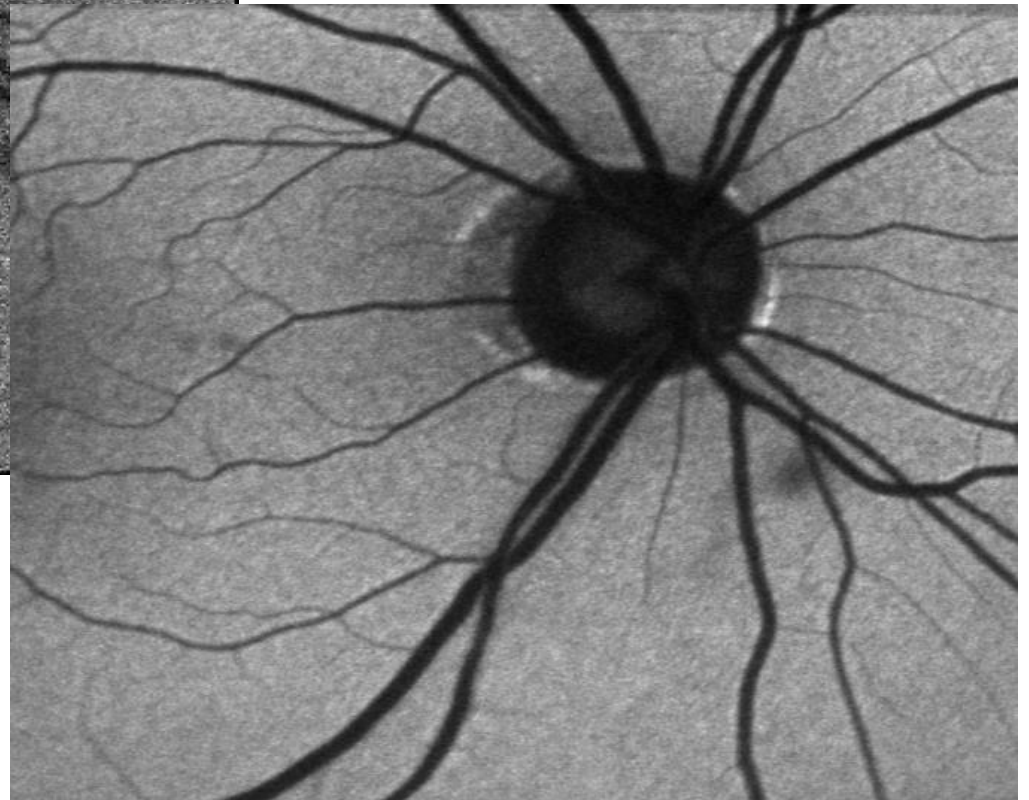
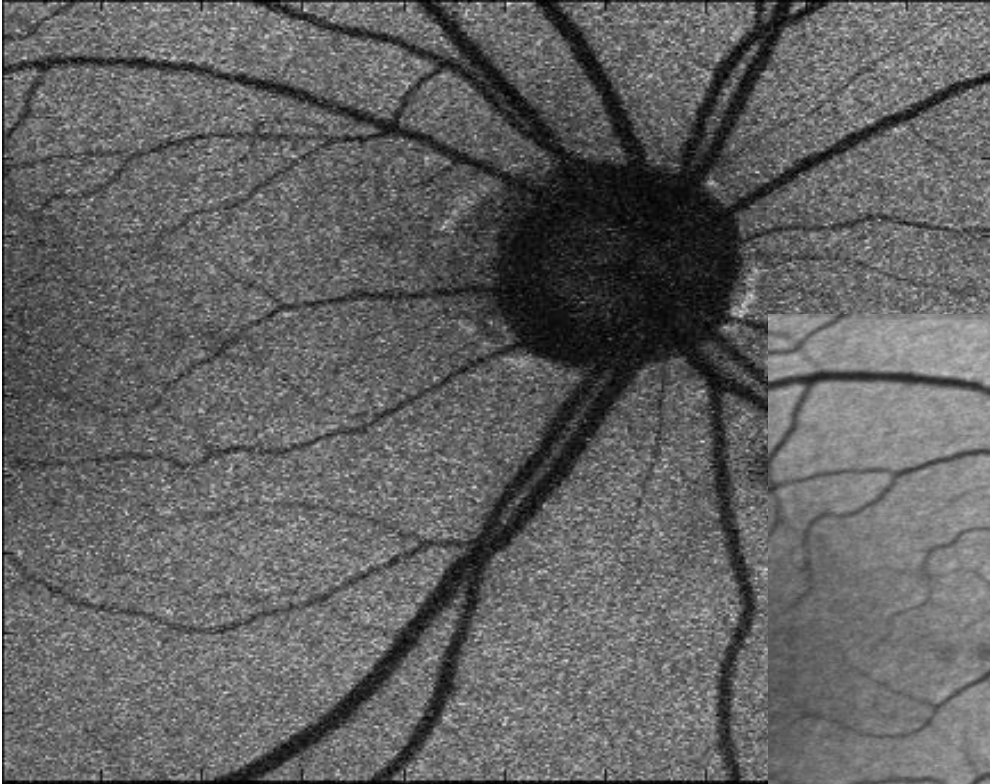


128



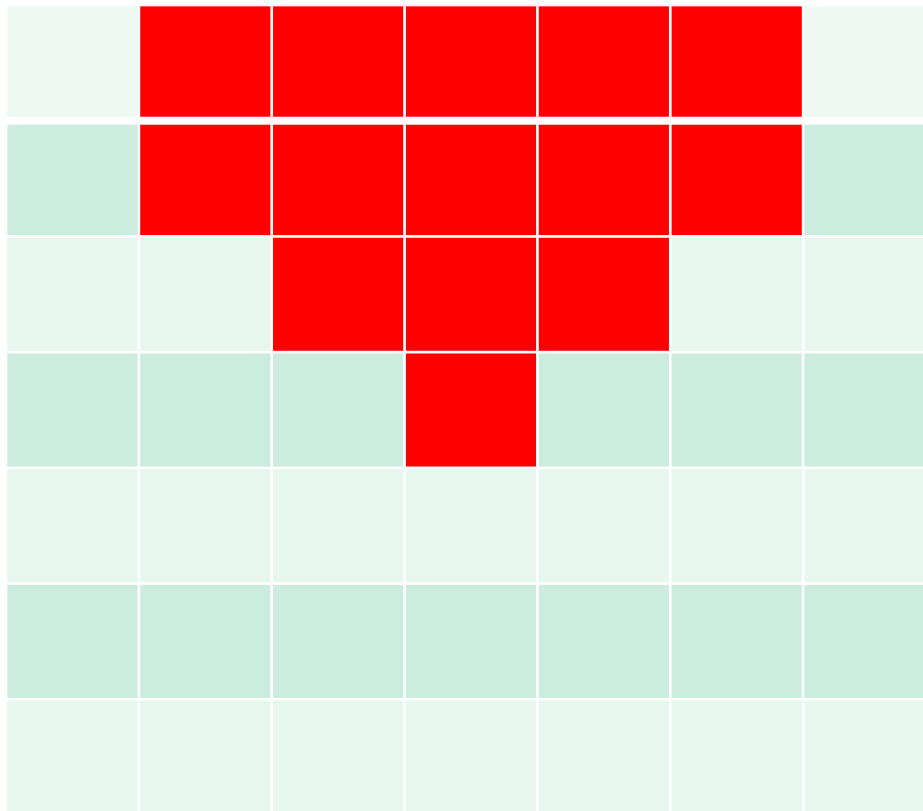


Průměrování v čase



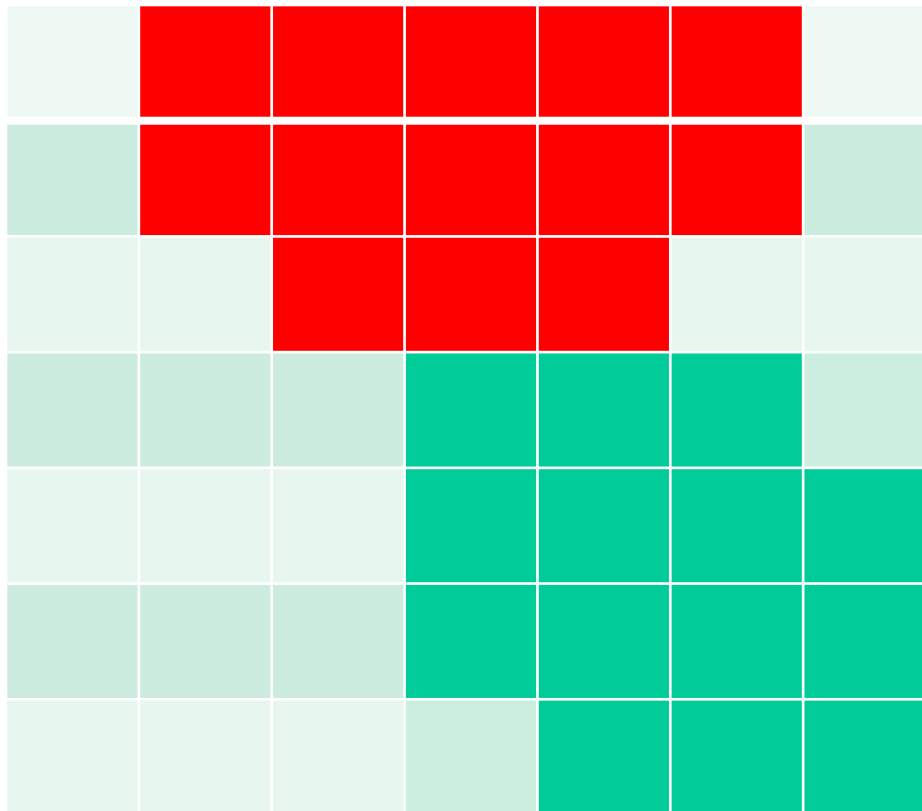
Konvoluční filtry

Rotující okno

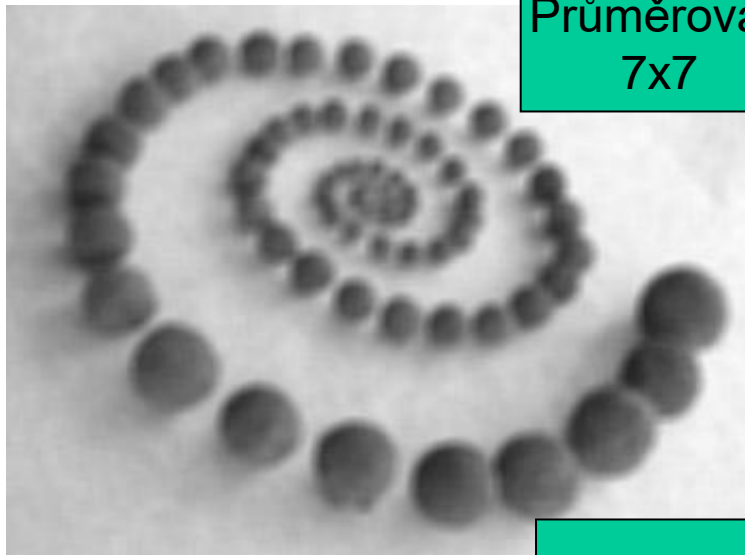


Konvoluční filtry

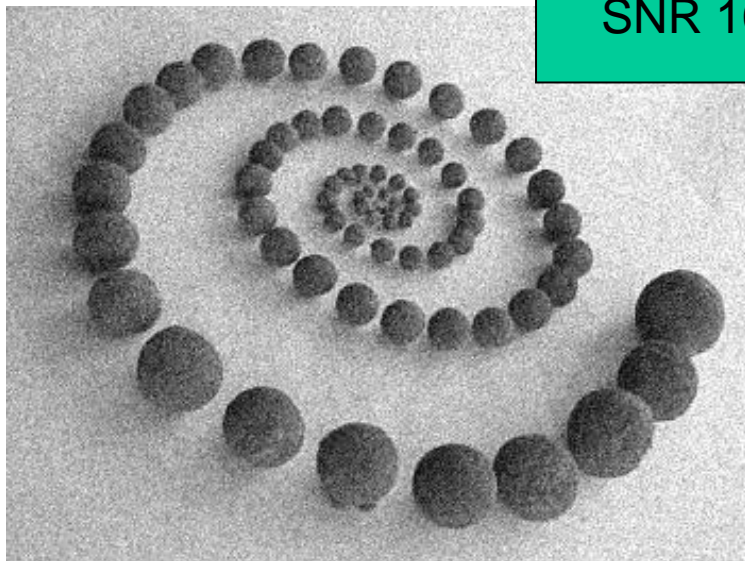
Rotující okno



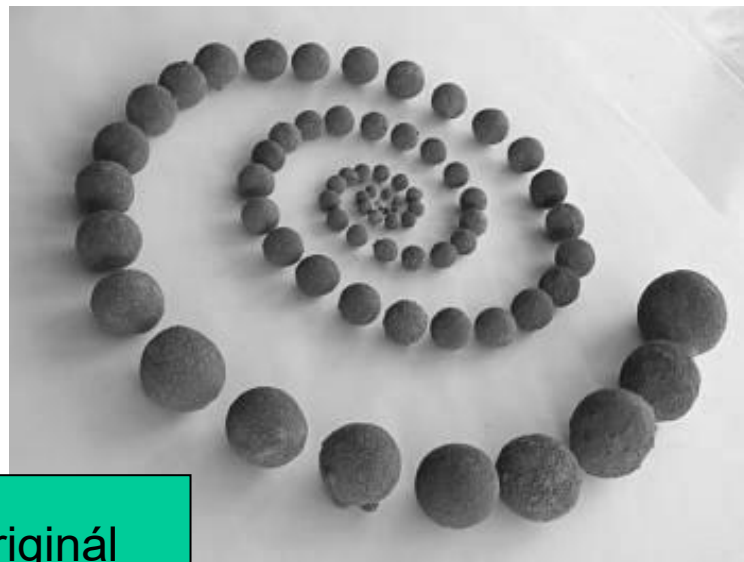
Rotující okno



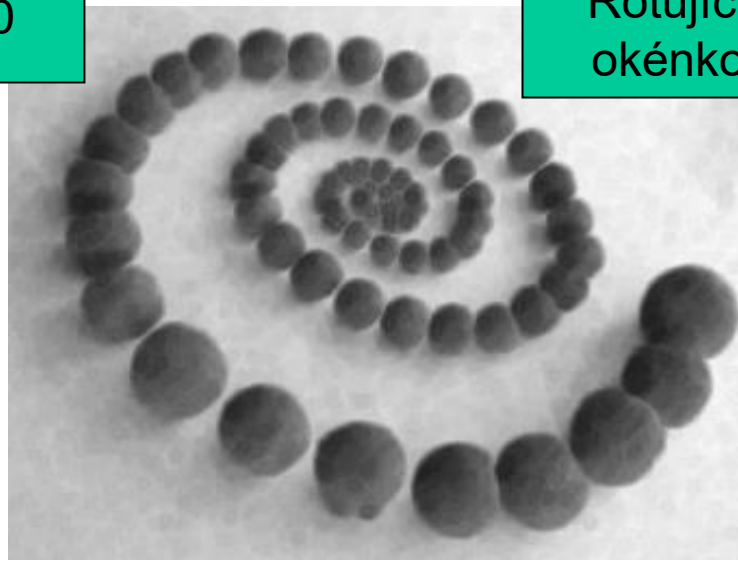
Průměrování
7x7



SNR 10



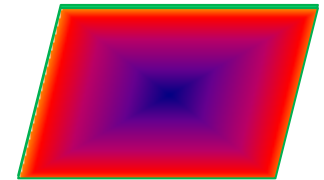
Originál



Rotující
okénko

Konvoluční filtry

Průměrování (prosté a vážené)



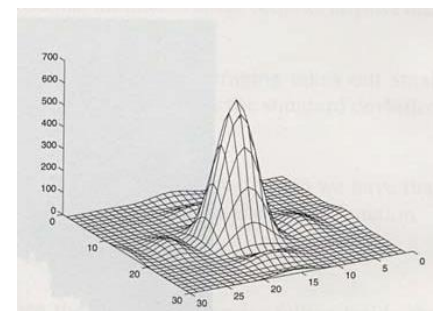
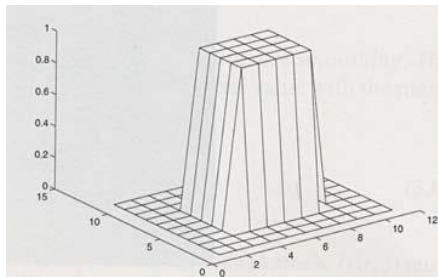
Průměrování podél hran



Rotující okno

Filtry ve frekvenční oblasti

Průměrování v obraze



šum



3x3

5x5

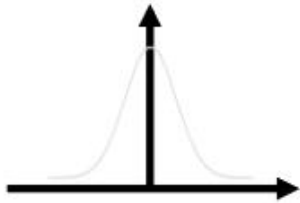


7x7

Průměrování v obraze

$$GB[I]_p = \sum_{q \in S} G(\|\mathbf{p} - \mathbf{q}\|) I_q$$

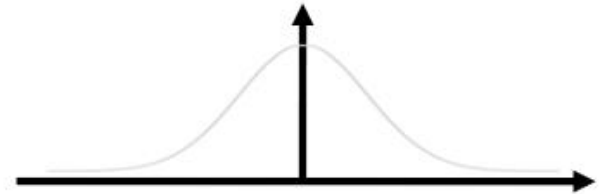
σ
size of the window



small σ



limited smoothing




large σ



strong smoothing

Průměrování v obraze

Original Image Corrupted Image Filtered Image


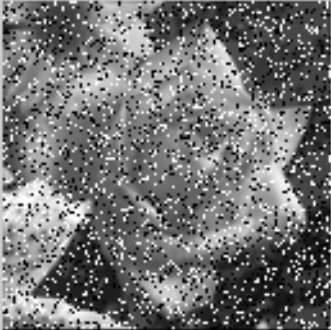
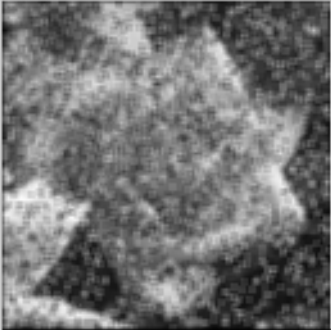


Select an Image: Image Noise Type: Noise Removal Filter:

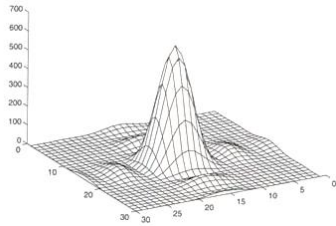
Mean: Filtering Neighborhood:

Variance:

Průměrování v obraze

| Original Image | Corrupted Image | Filtered Image |
|---|--|---|
|  |  |  |
| Select an Image: | Image Noise Type: | Noise Removal Filter: |
| Flower ▼ | Salt & Pepper ▼ | Averaging ▼ |
| | Density: <input data-bbox="971 911 1170 963" type="text" value="0.2"/> | Filtering Neighborhood: |
| | | 3-by-3 ▼ |

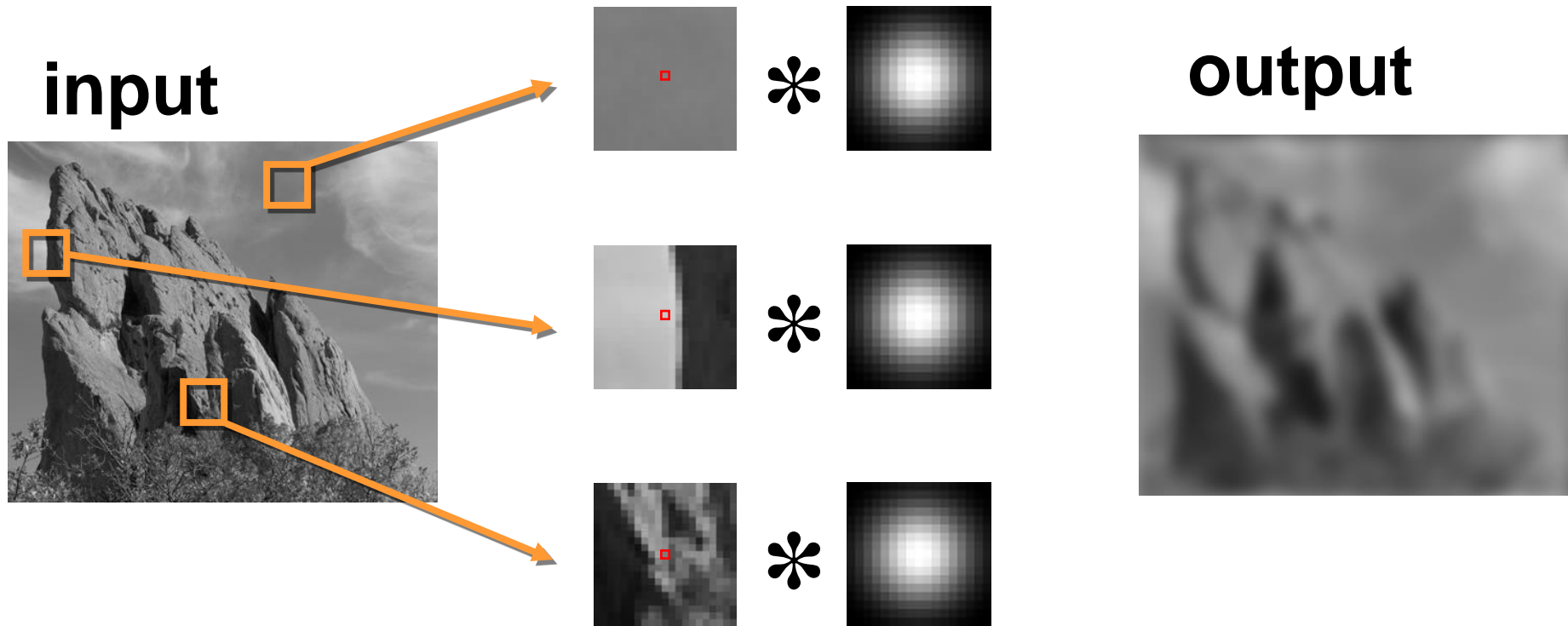
Adaptivní průměrování



Adaptivní průměrování

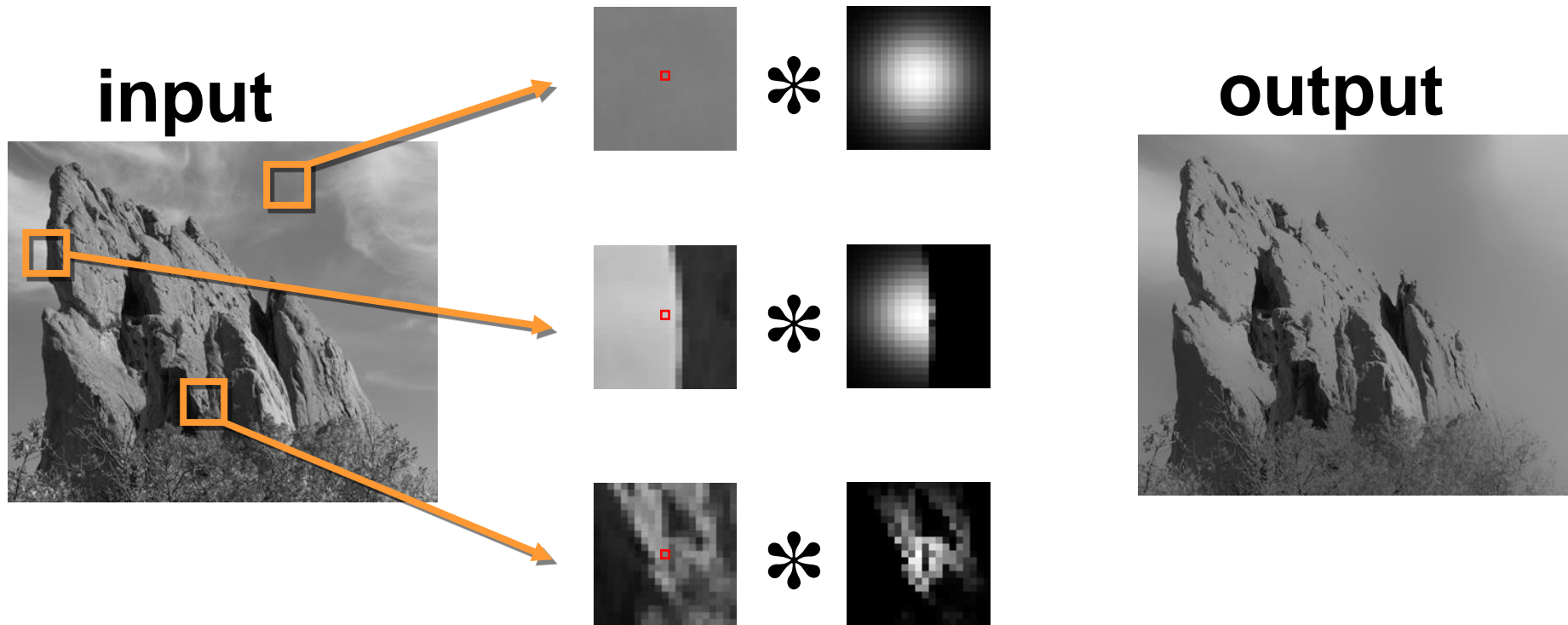


Gaussian Smoothing



Same Gaussian kernel everywhere
Averages across edges \Rightarrow blur

Bilateral Filtering



Kernel shape depends on image content

Avoids averaging across edges

Bilateral Filtering

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in \mathcal{S}} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

new


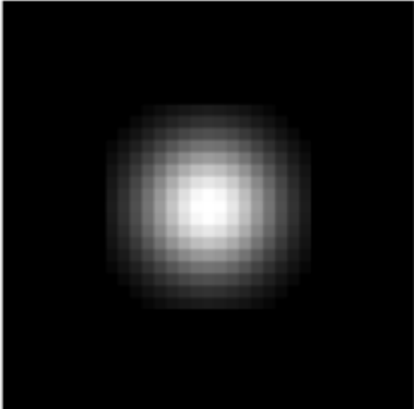
not new

new

normalization factor

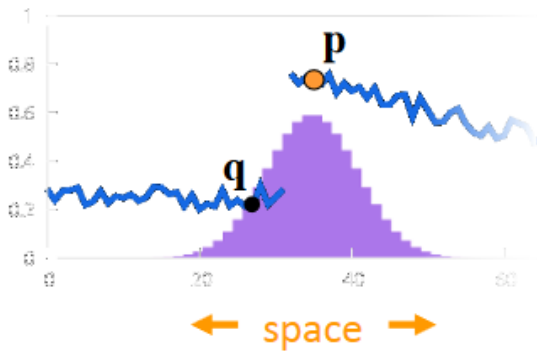
space weight

range weight



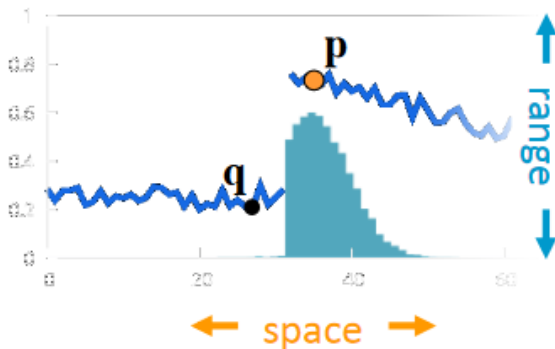
Bilateral Filtering

Gaussian blur



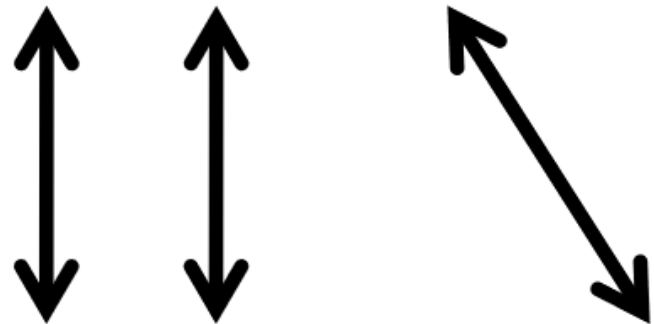
Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]



$$GB[I]_p = \sum_{q \in S} G_{\sigma}(\|p - q\|) I_q$$

space

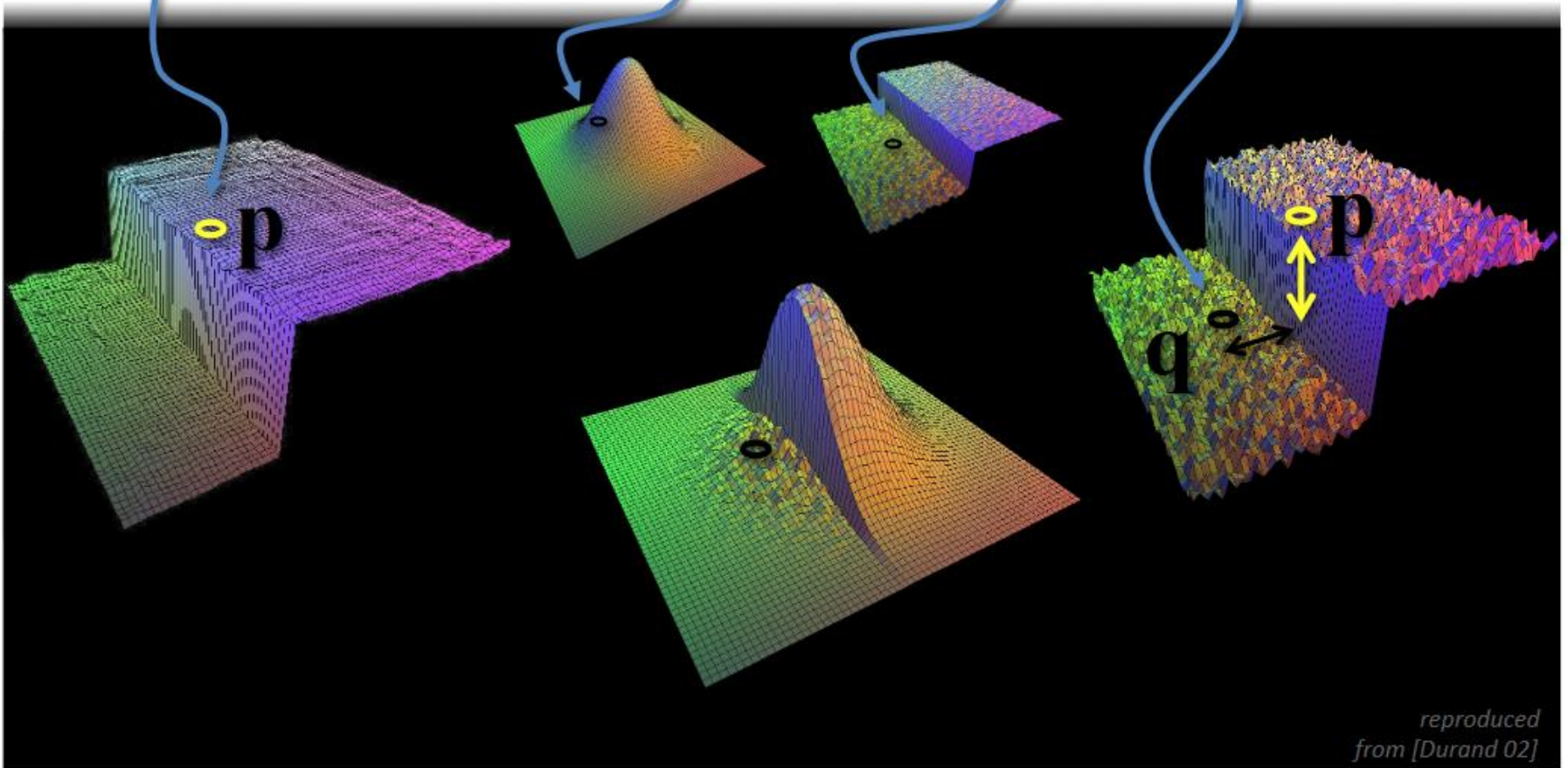


$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_s}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

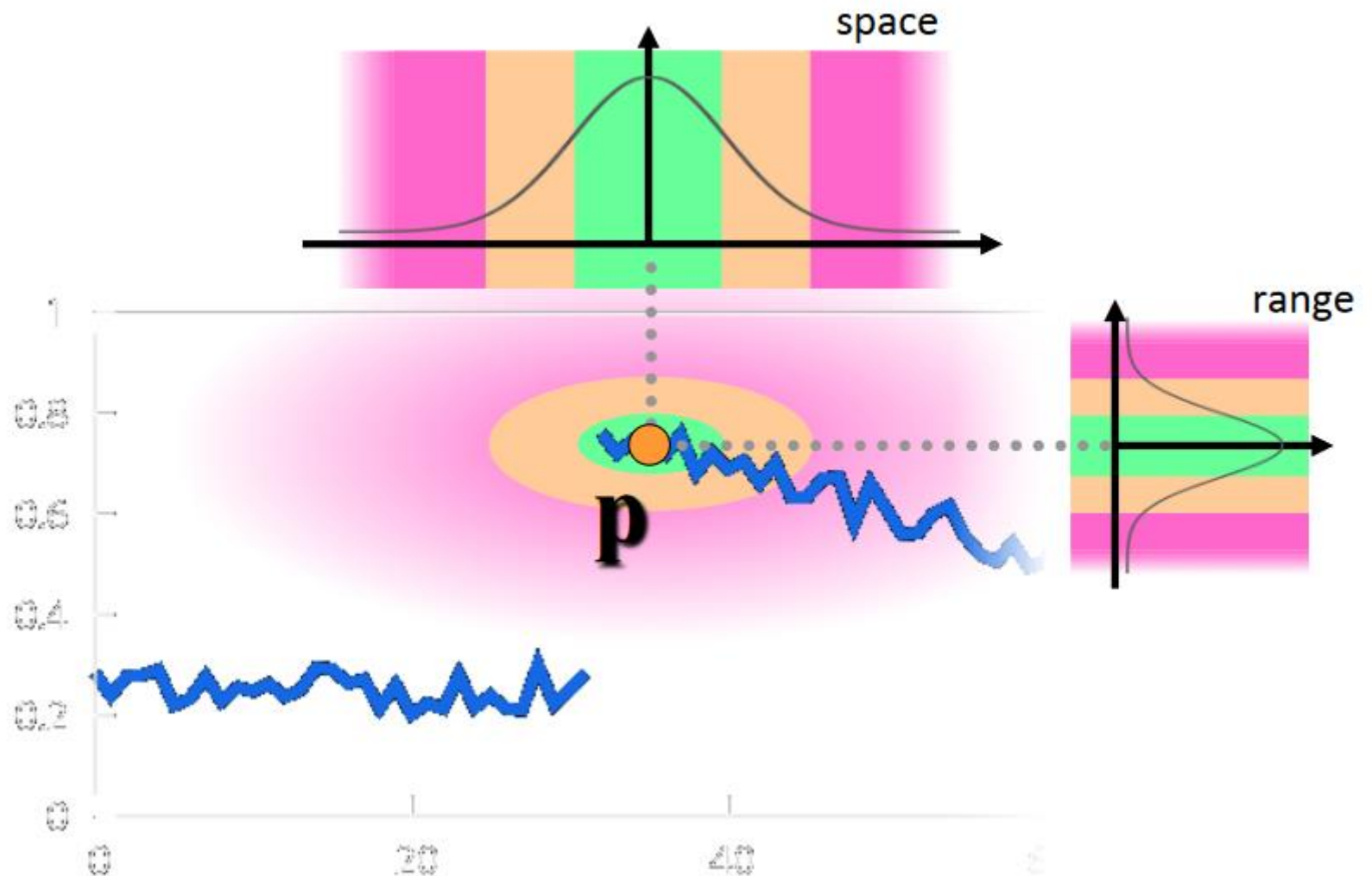
normalization space range

Bilateral Filtering

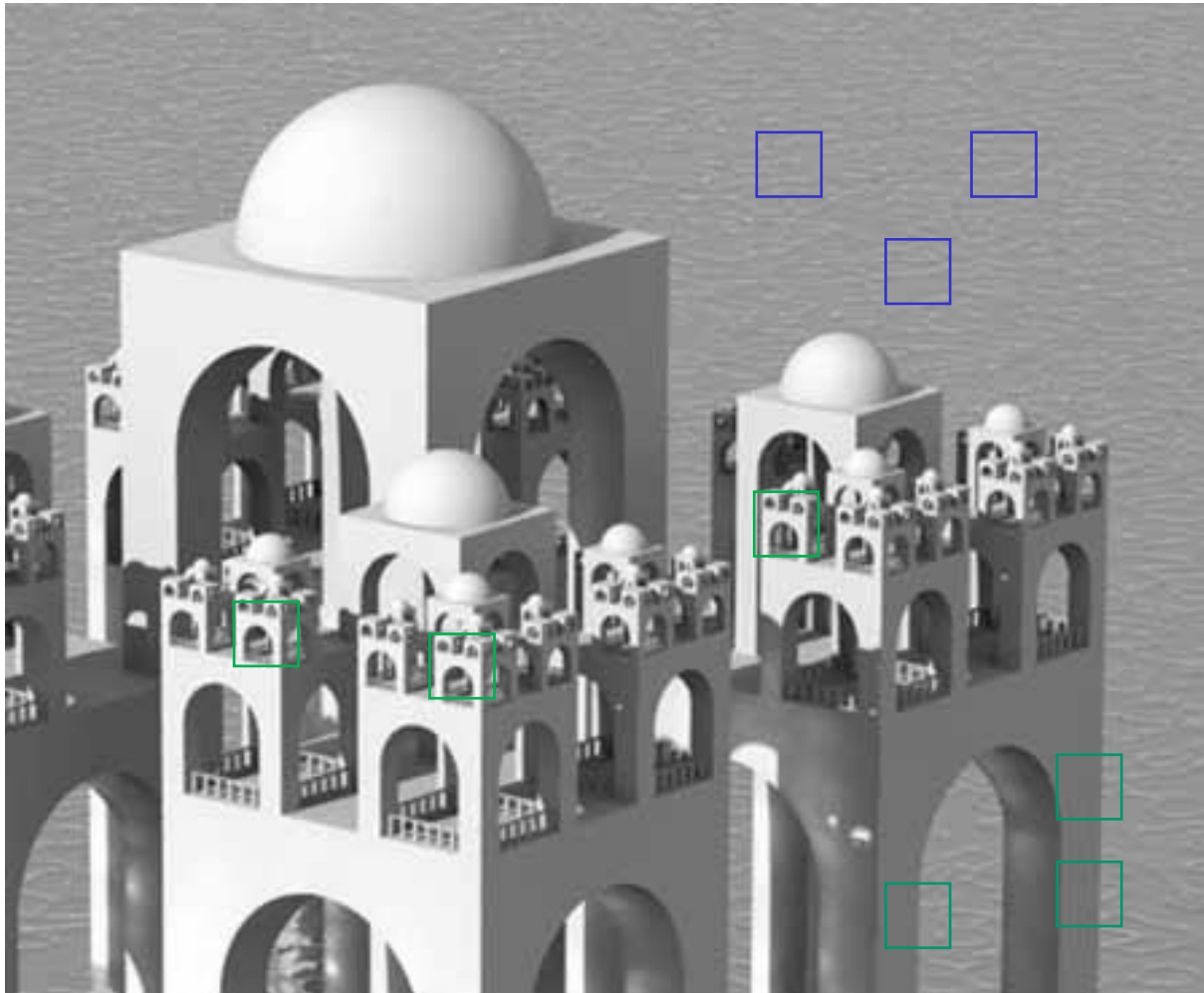
$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} \underbrace{G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|)}_{\text{Spatial}} \underbrace{G_{\sigma_r}(\|I_p - I_q\|)}_{\text{Range}} I_q$$



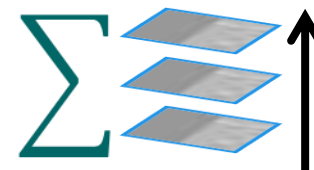
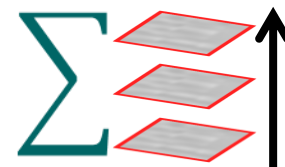
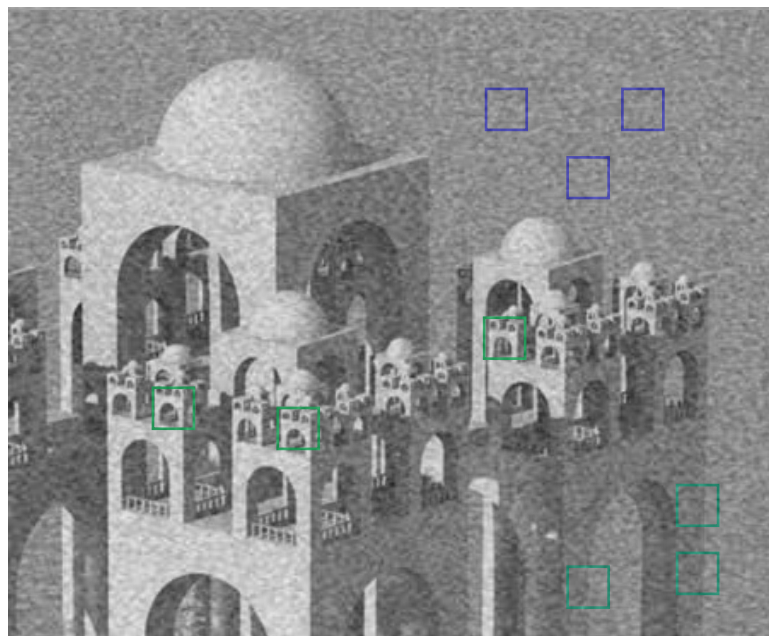
Bilateral Filtering



Využití redundance



Single – image „průměrování“



Různé!!!

From Bilateral Filter to NLM

$$\hat{x}(i)_{BL} = \frac{1}{C_i} \sum_j y(j) e^{-\frac{|y(i)-y(j)|^2}{2\sigma^2}} e^{-\frac{\|i-j\|^2}{2\rho^2}}$$

intensity weight
spatial weight $\rightarrow \infty$

$$\hat{x}(i)_{NLM_{1x1}} = \frac{1}{C_i} \sum_j y(j) e^{-\frac{|y(i)-y(j)|^2}{2\sigma^2}}$$

From Bilateral Filter to NLM

$$\hat{x}(i)_{NLM_{1x1}} = \frac{1}{C_i} \sum_j y(j) e^{-\frac{|y(i)-y(j)|^2}{2\sigma^2}}$$

↓ Patch similarity

$$\hat{x}(i)_{NLM} = \frac{1}{C_i} \sum_j y(j) e^{-\frac{GSSD(y(N_i)-y(N_j))}{2\sigma^2}}$$

Non Local Means (NLM)

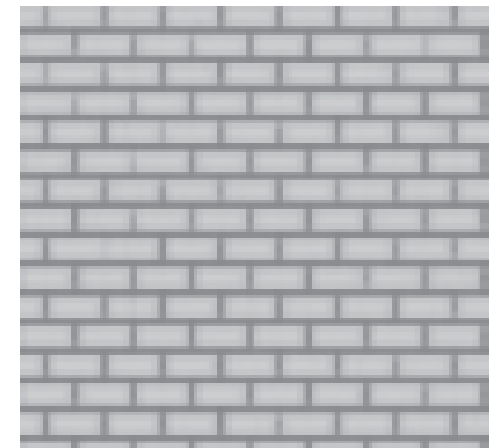
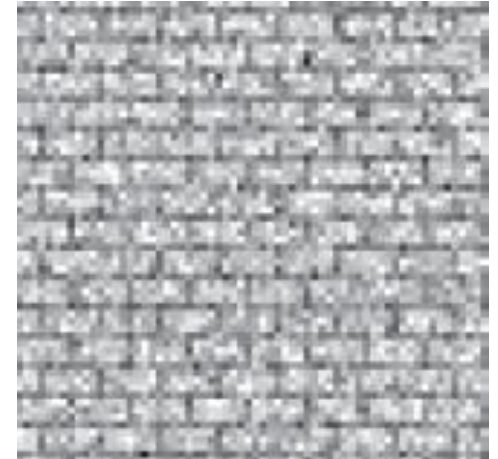
Baudes *et al.* (2005)

Use a weighted average based on similarity

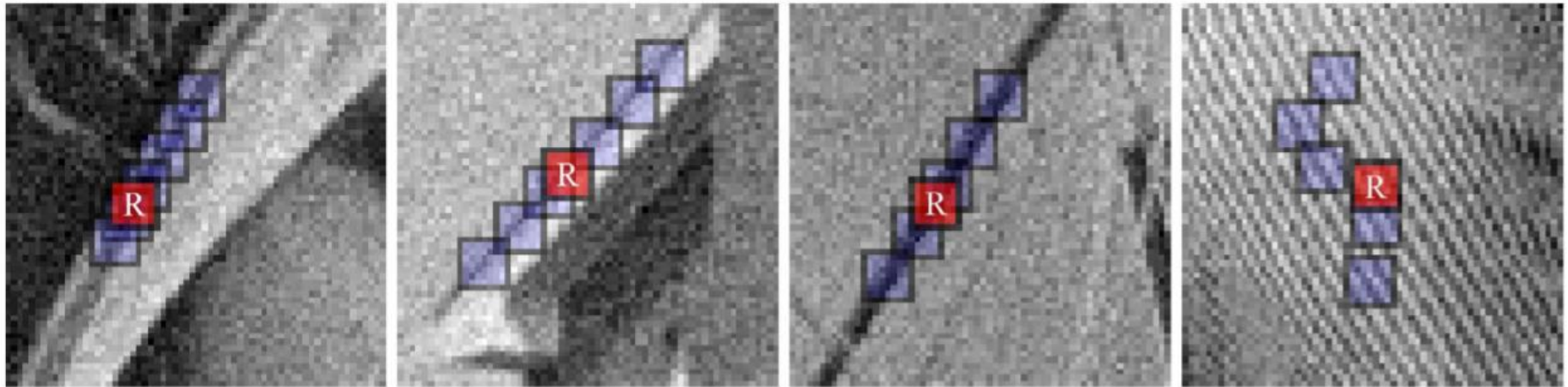
$$NLu(\mathbf{x}) = \frac{1}{C(\mathbf{x})} \int_{\Omega} e^{-\frac{(G_{\rho} * |u(\mathbf{x} + \cdot) - u(\mathbf{y} + \cdot)|^2)(0)}{h^2}} u(\mathbf{y}) d\mathbf{y},$$

$$(G_{\rho} * |u(\mathbf{x} + \cdot) - u(\mathbf{y} + \cdot)|^2)(0) = \int_{\mathbb{R}^2} G_{\rho}(\mathbf{t}) |u(\mathbf{x} + \mathbf{t}) - u(\mathbf{y} + \mathbf{t})|^2 dt.$$

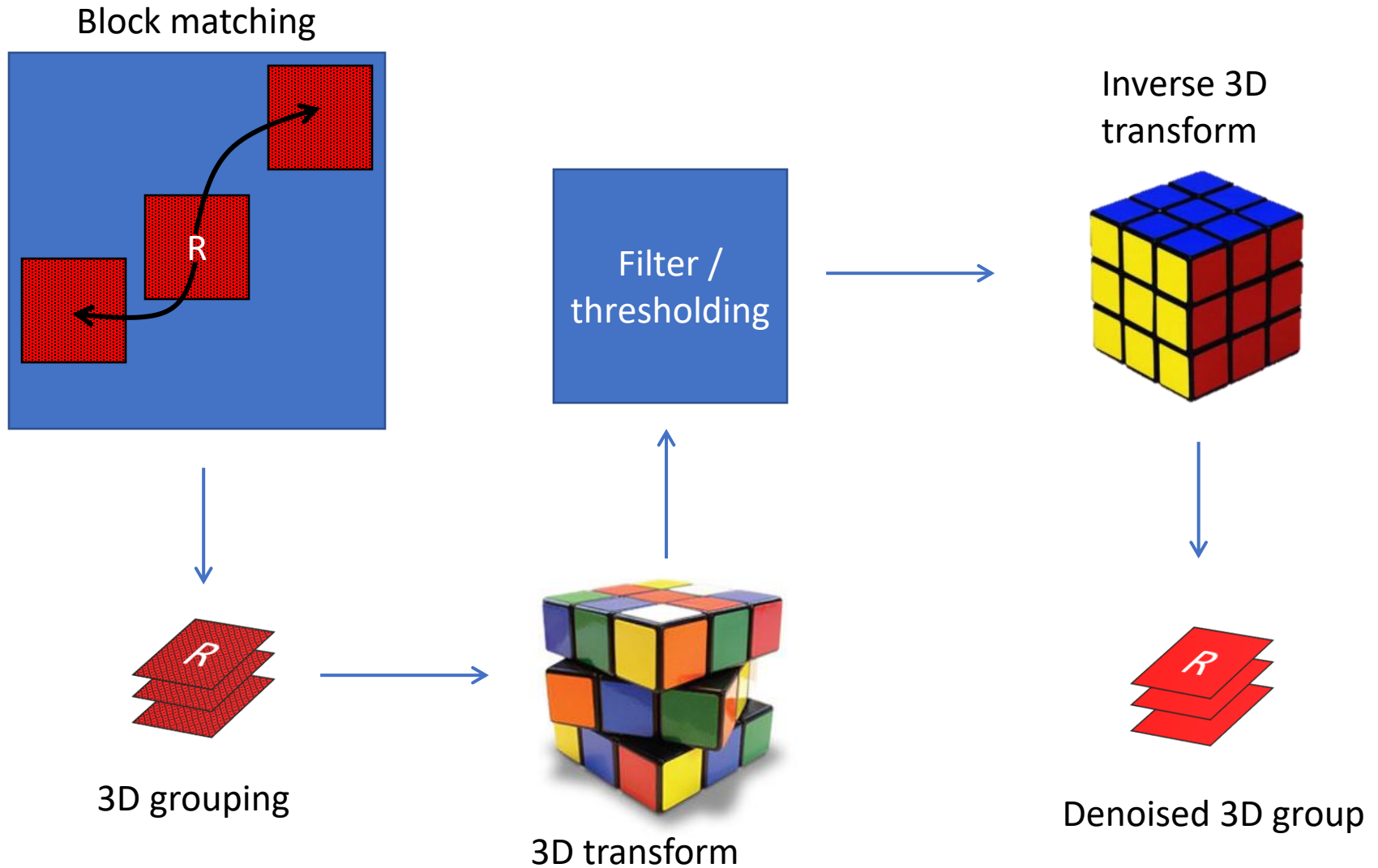
NLM



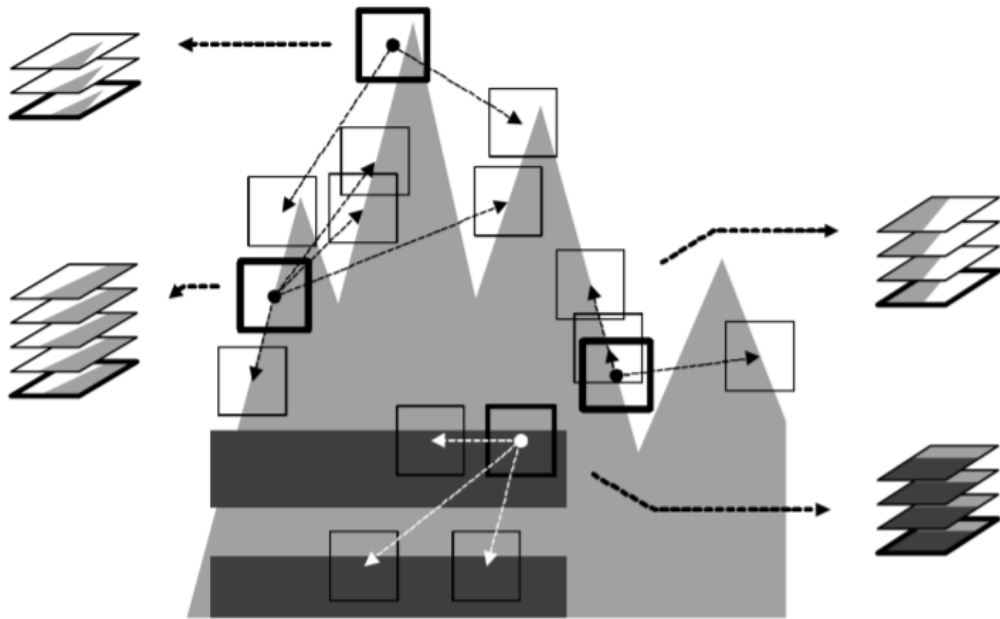
Block-Matching and 3D filter (BM3D)



Block-Matching and 3D filter (BM3D)

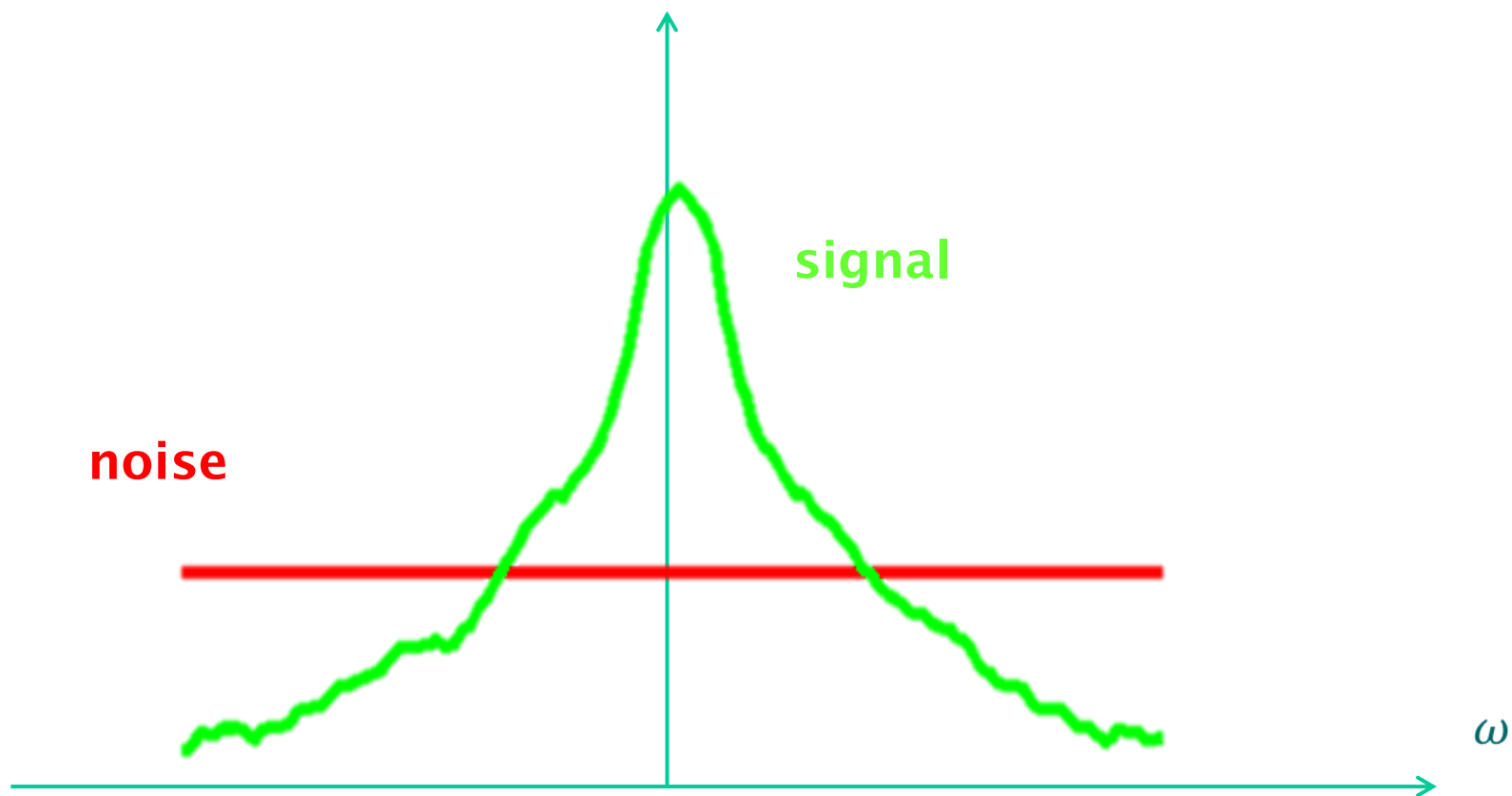


Block-Matching and 3D filter (BM3D)



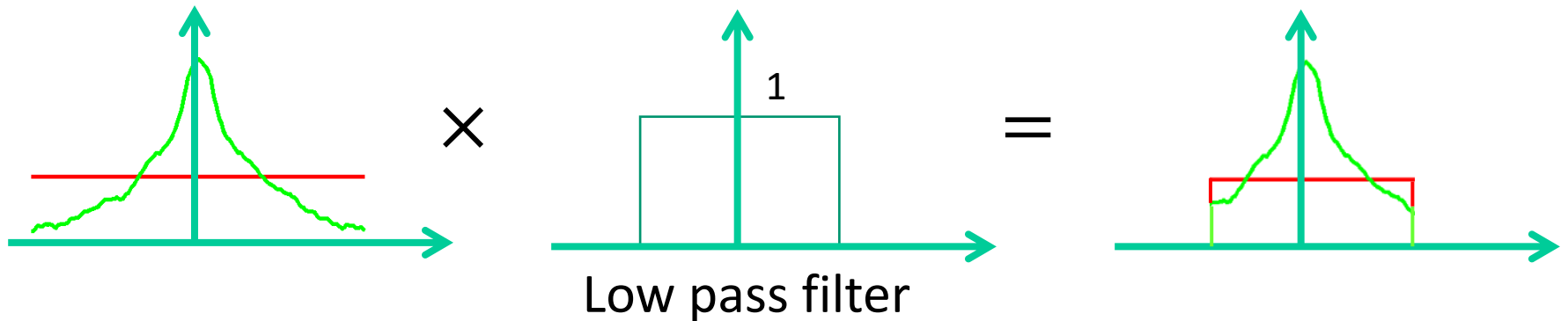
- **Element-wise averaging**
 - **Identical blocks**
 - **Multiple blocks**
- **3D transform (e.g., DWT, DFT, DCT)**

Filtrace ve frekvenční oblasti

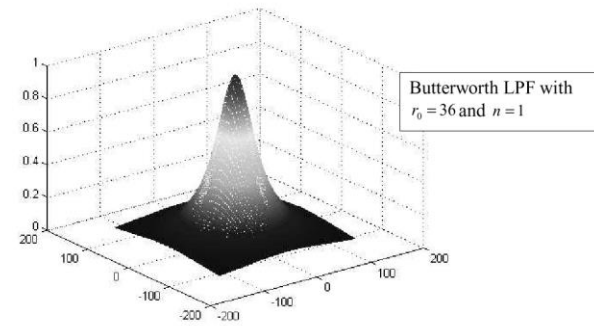


Low-Pass Filtrování

Zachovat co nejvíc signálu



Hladký low-pass filtr ve frekvenční oblasti



LPF image, $r_0 = 13$



LPF image, $r_0 = 10$

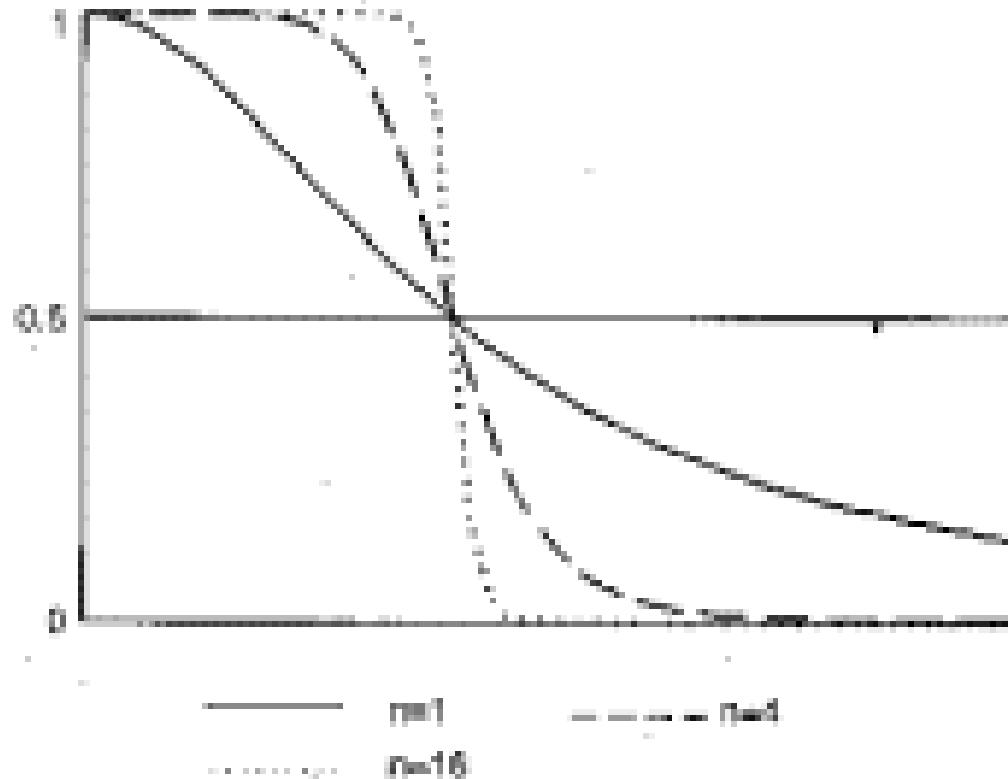
Butterworth filter

low pass

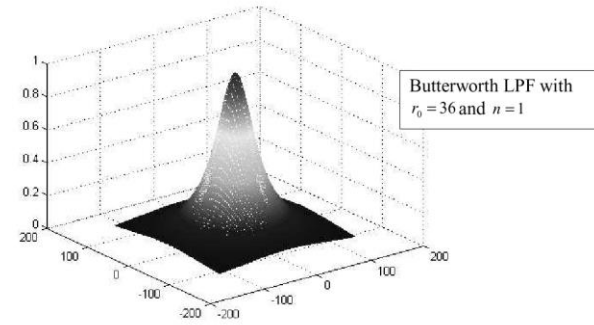
$n = 1, 4, 16$

$$H(u, v) = \frac{1}{1 + \left[\frac{D(u, v)}{D_0} \right]^{2n}}$$

$$D(u, v) = \sqrt{u^2 + v^2}$$



Butterworth filtr



LPF image, $r_0 = 13$



LPF image, $r_0 = 10$

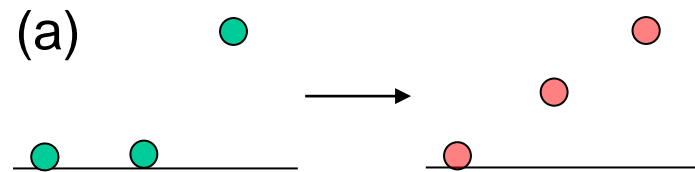
Nelineární filtry

Medián

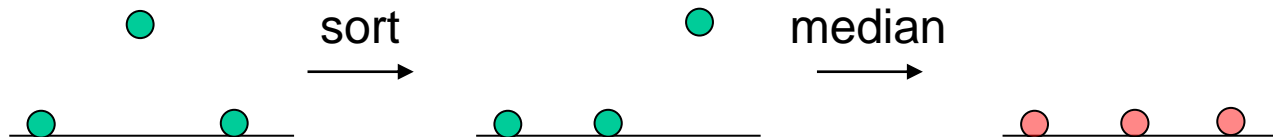


Medián Filtrace

Průměrování

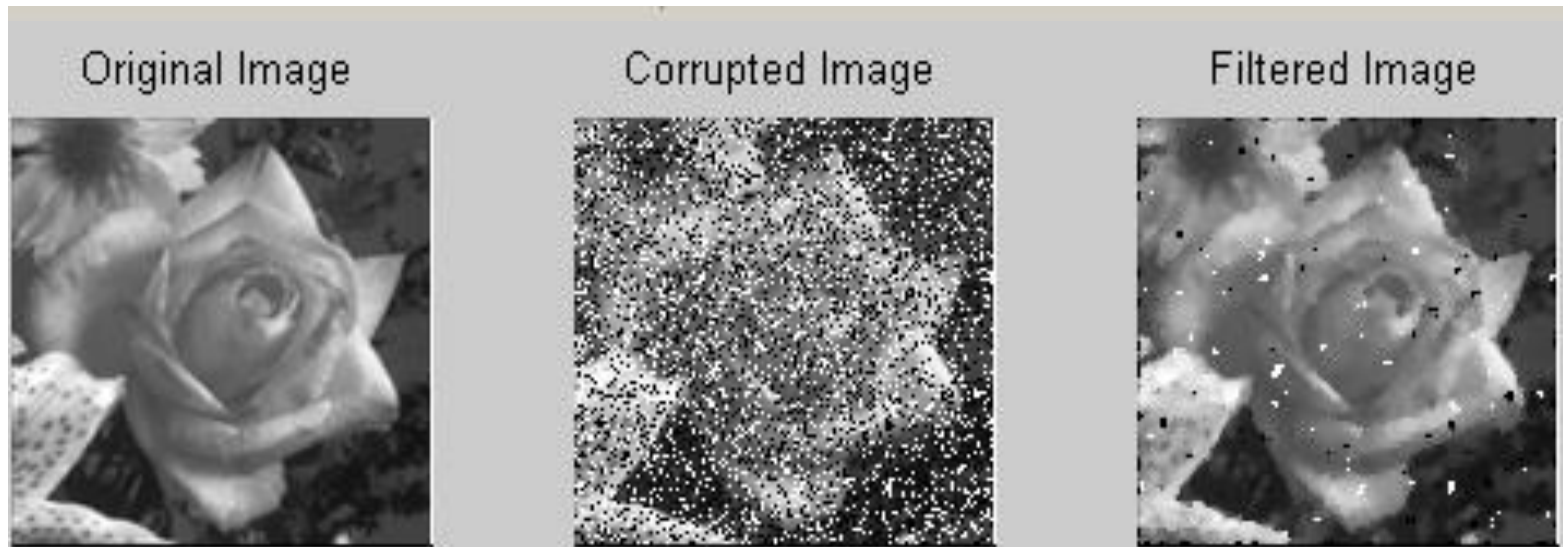


- Medián
nelineární



Nelineární filtry

Medián



Nelineární filtry

Medián

noisy



Gaussian filter



Median filter



Nelineární filtry

Medián



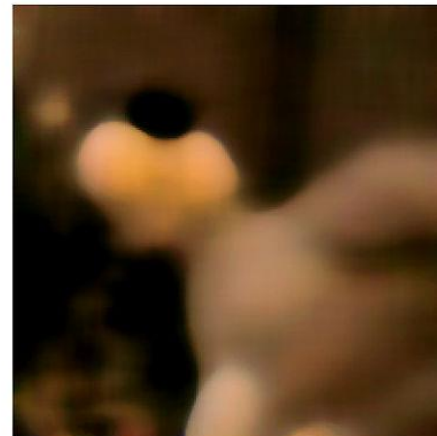
original image



1px median filter



3px median filter



10px median filter

Salt and pepper noise

Gaussian

Median

3x3



5x5



7x7



Gaussian noise

Gaussian

Median



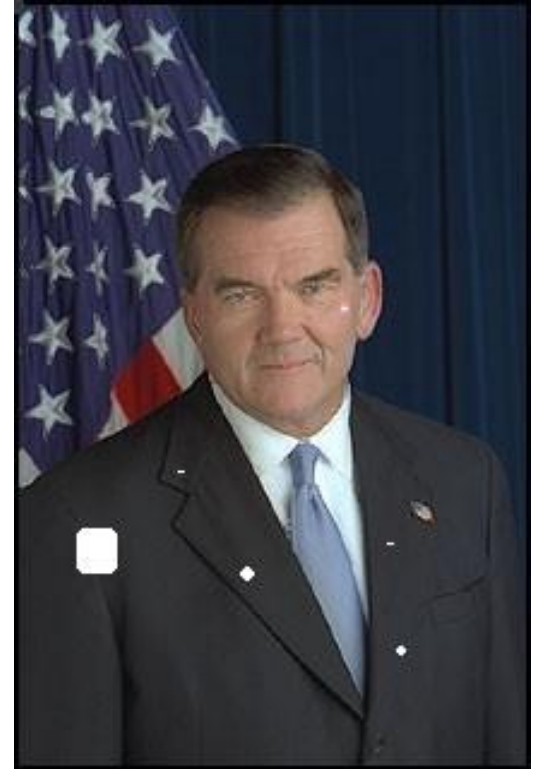
Medián filtrace



Tom Ridge left the Pennsylvania governorship last October, when U.S. President George W. Bush appointed him to head the newly created Office of Homeland Security.



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Median filter (5x5)

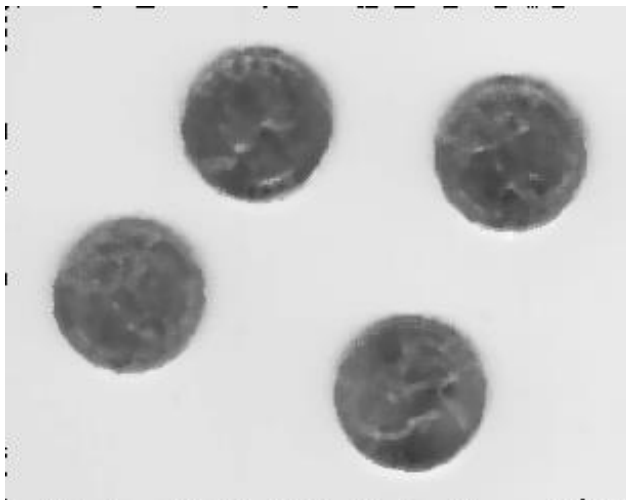
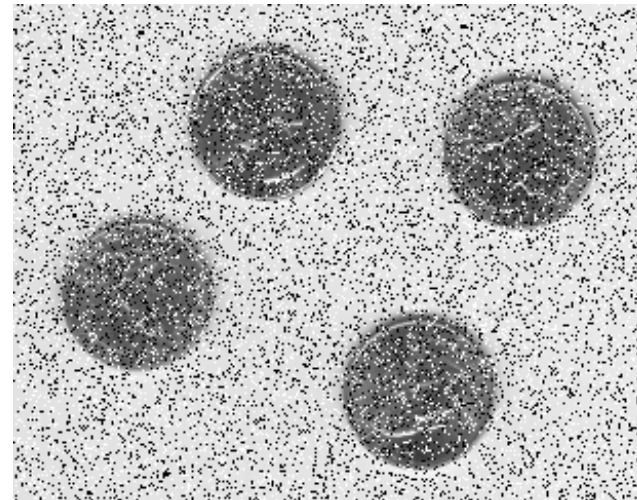
Nelineární filtry

Medián

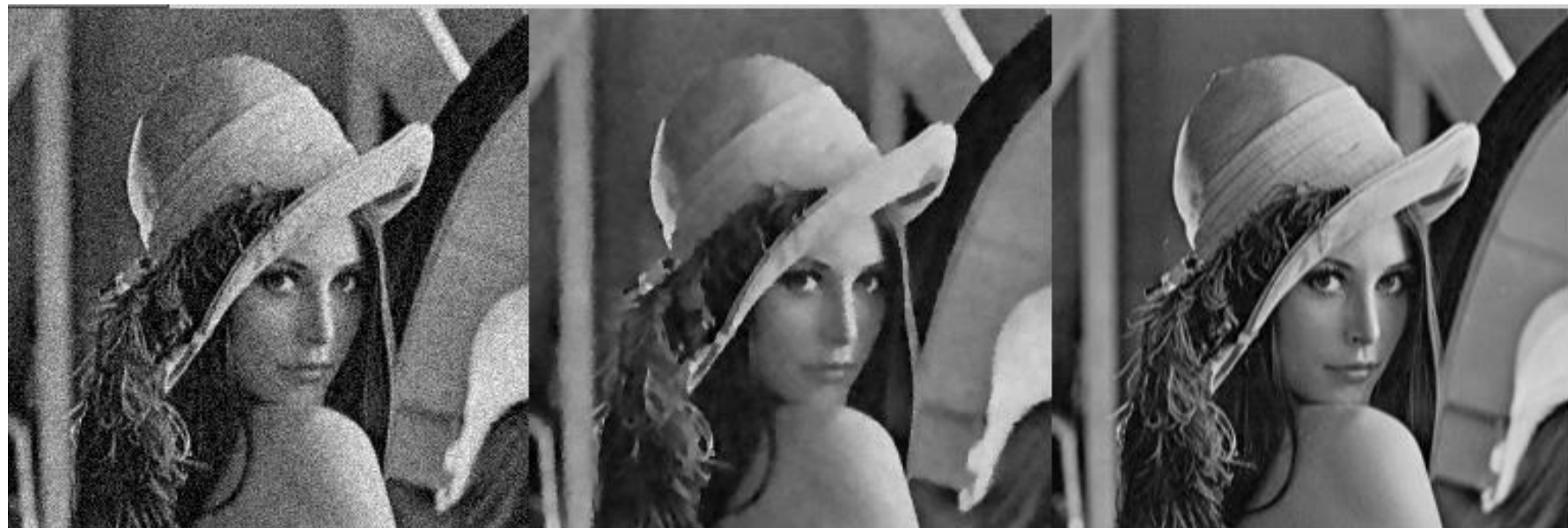
Rychlost

Tvar

Mediánový filtr, detekce šumu



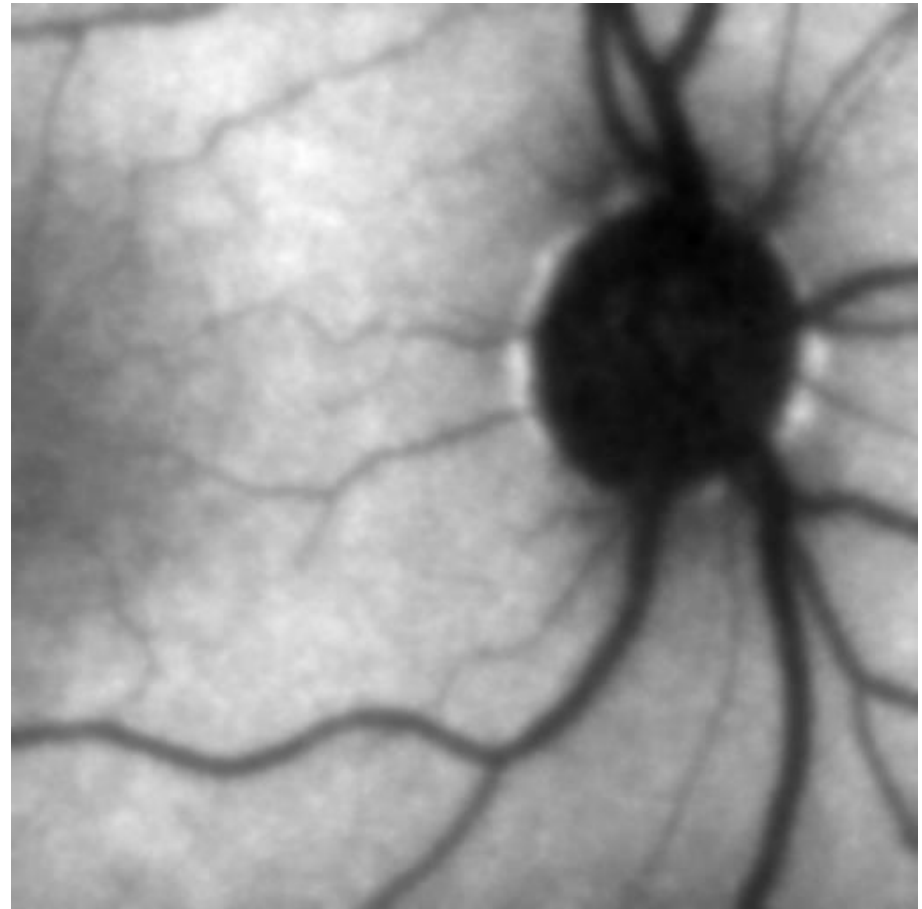
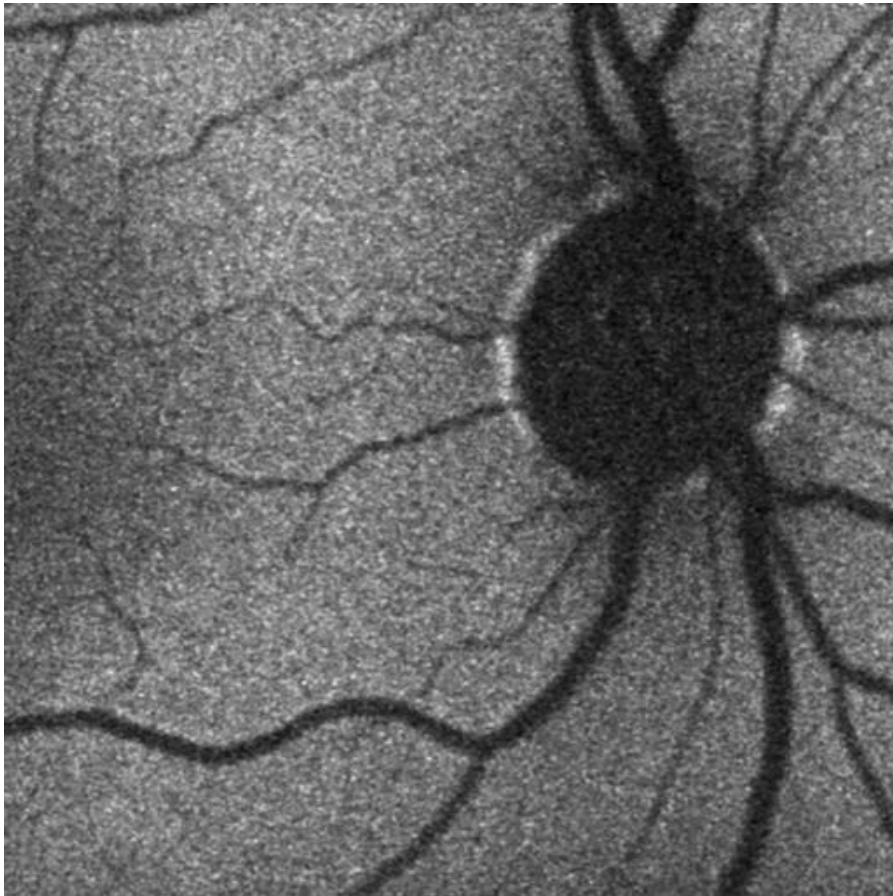
Minimalizace funkcionálu



Infračervený obrázek - splajny



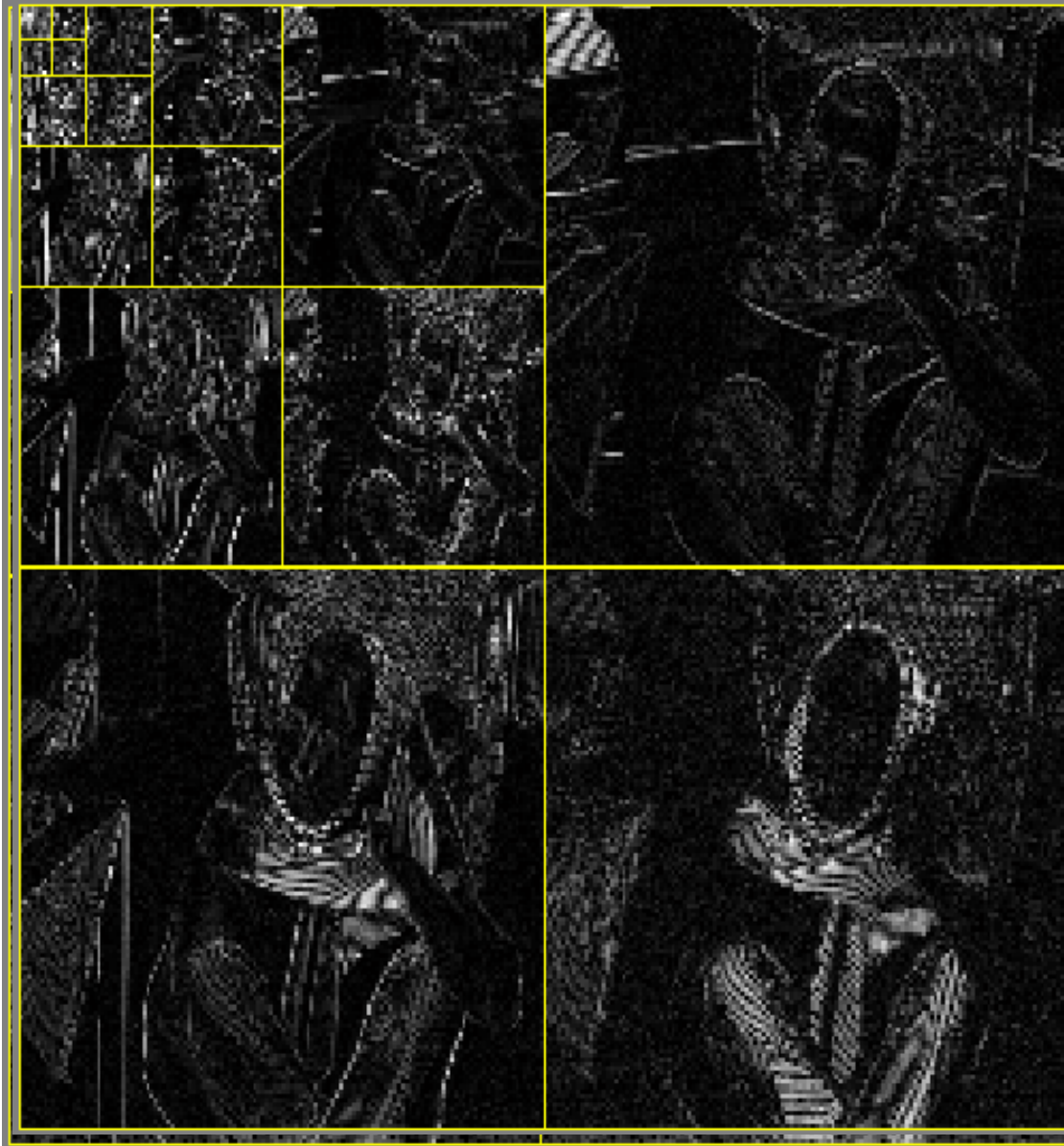
Autofluorescenční snímky oka



Anizotropní difuze



Wavelety

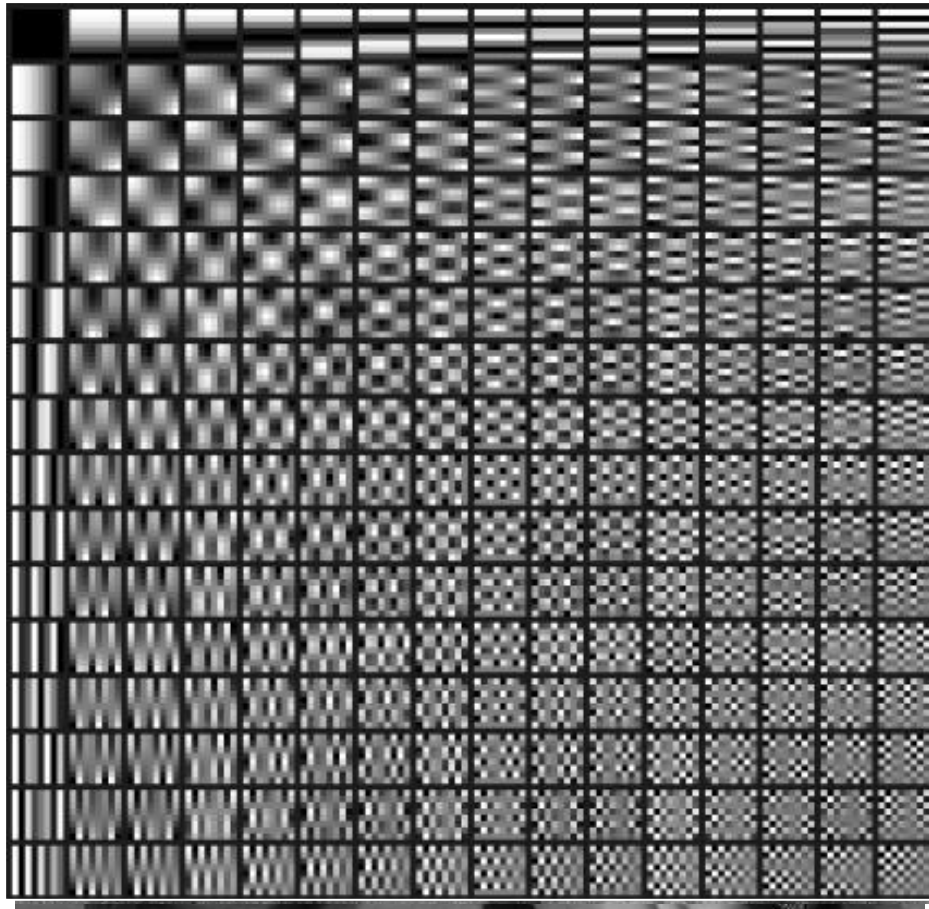


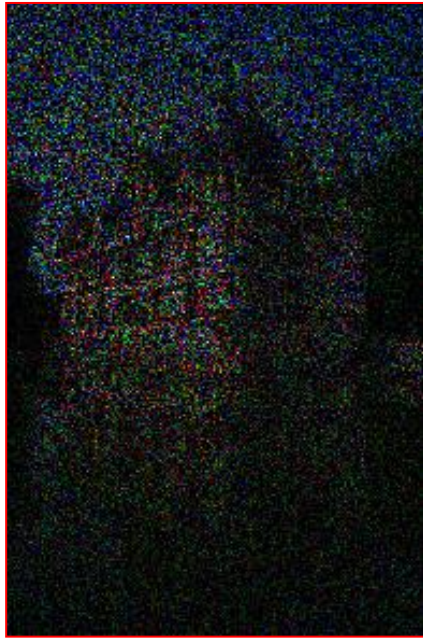
Wavelety



Aktuální směr výzkumu

- **Řídké reprezentace obrázků**
- **Slovníky**
- **CNN**







Noisy input
sigma = 25

BM3D

DN-CNN-S

DN-ResNet-13

Pre-Trained Image Processing Transformer CVPR 21



GT



Noisy ($\sigma=50$)



CBM3D [16]



Hanting Chen^{1,2}, Yunhe Wang^{2*}, Tianyu Guo^{1,2}, Chang Xu³, Yiping Deng⁴,
Zhenhua Liu^{2,5,6}, Siwei Ma^{5,6}, Chunjing Xu², Chao Xu¹, Wen Gao^{5,6}

Inpainting

Since 1699, when French explorers landed at the great bend of the Mississippi River and celebrated the first Mardi Gras in North America, New Orleans has brewed a fascinating melange of cultures. It was French, then Spanish, then French again, then sold to the United States. Through all these years, and even into the 1900s, others arrived from everywhere: Acadians (Cajuns), Africans, indige-



Inpainting - Adobe, Midjourney, ...

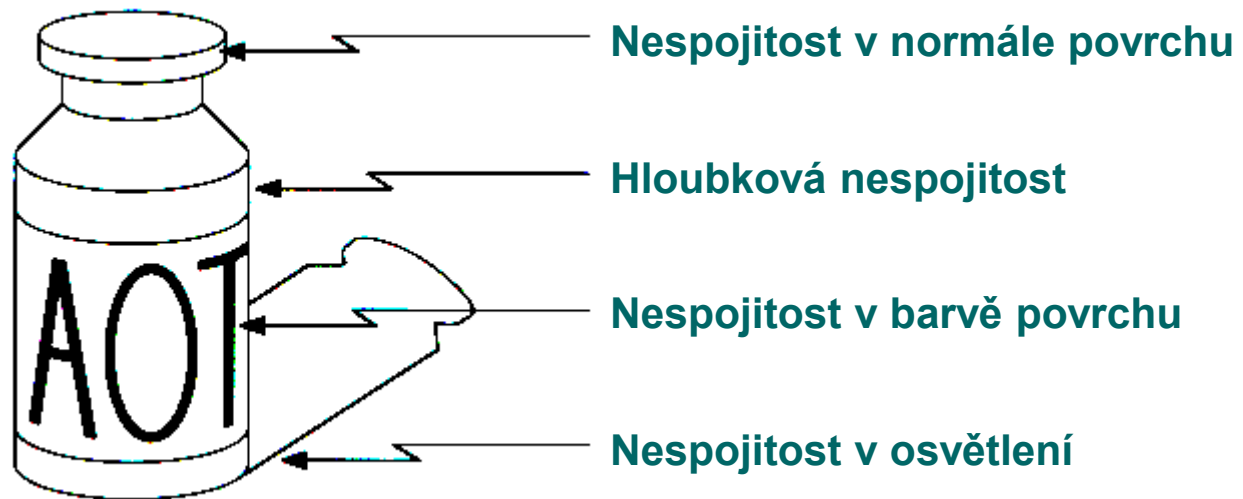


Detekce a zvýraznění hran

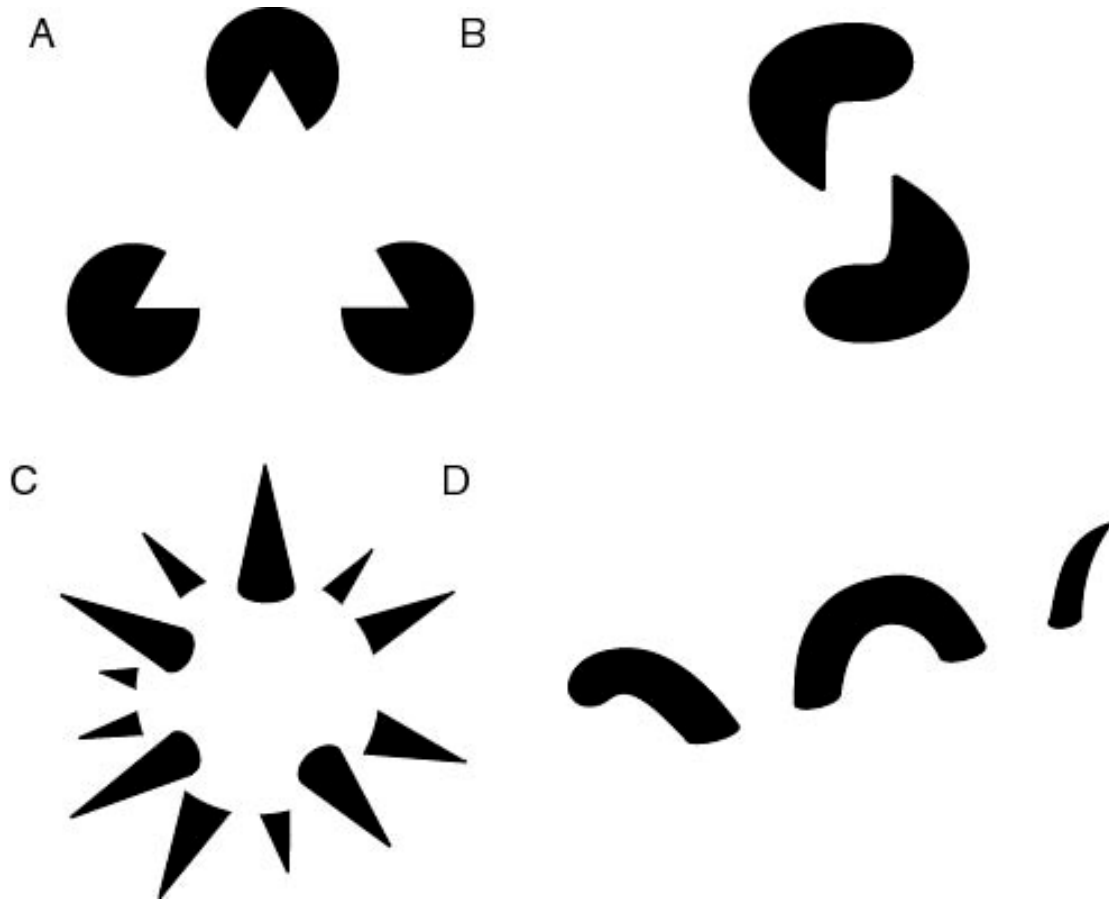
Jednoduché metody zaostření obrazu

Detekce hran

Detekce hran



Detekce hran



Detekce hran

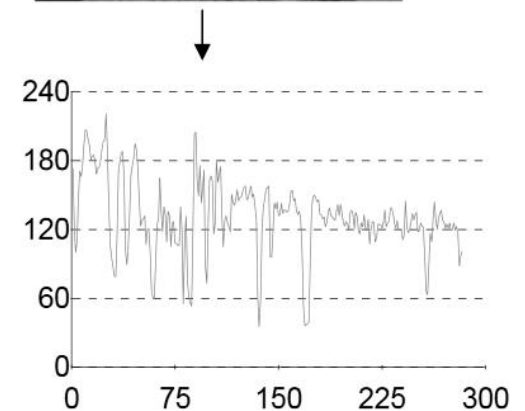
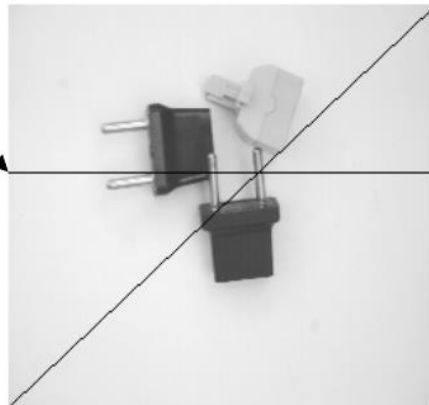
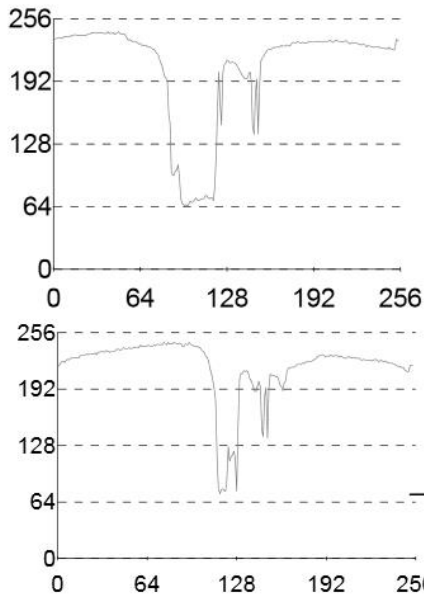
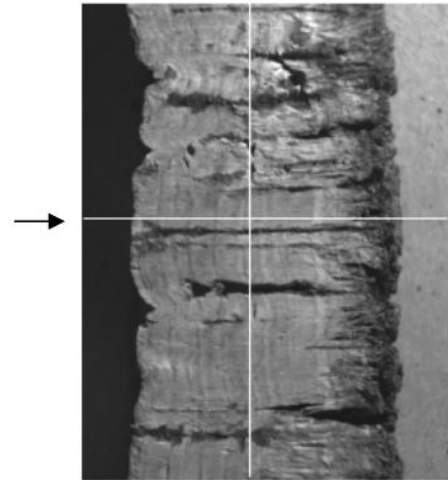
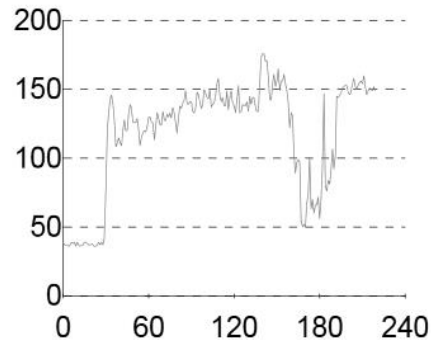
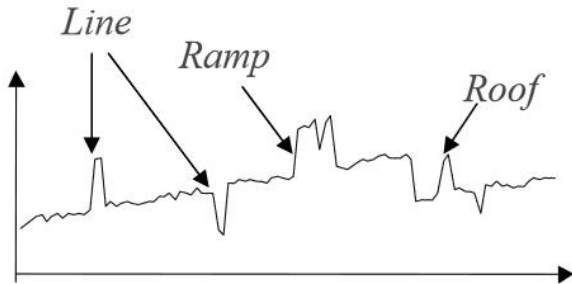
Hrany – hranice objektů

segmentace, registrace, rozpoznávání

Co je hrana?

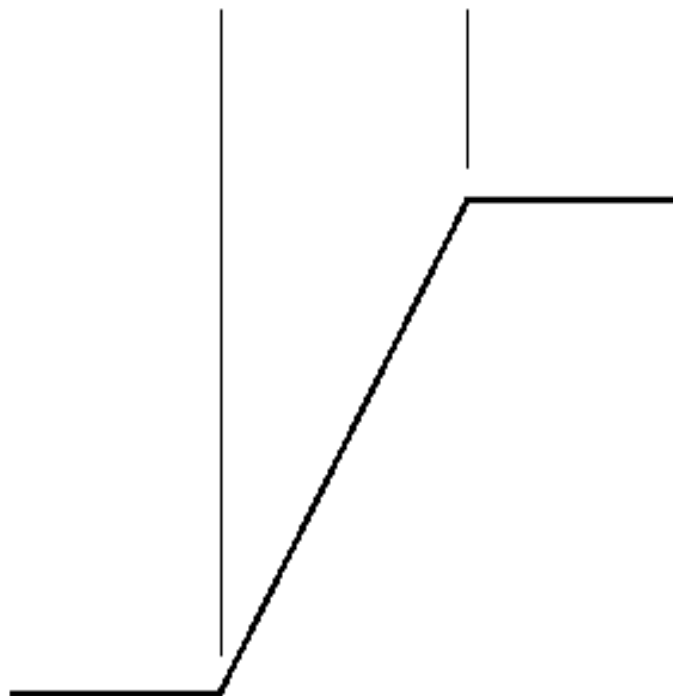
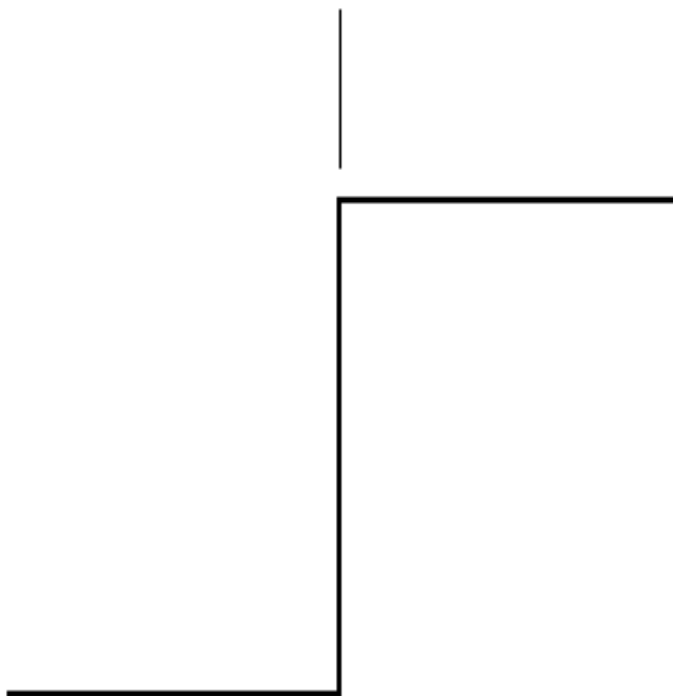
**Hrana – singularita, náhlá změna ->
gradient**

Hrana

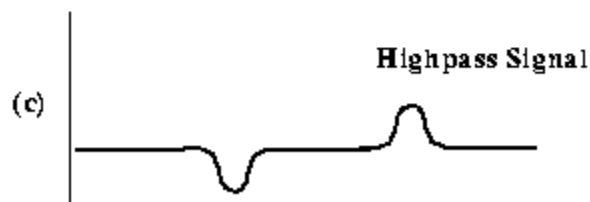
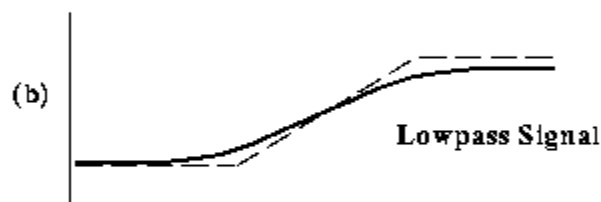
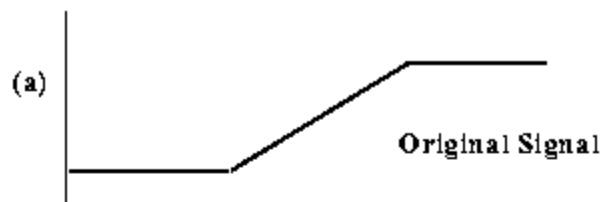


Courtesy of ²¹²A.Campilho

Ideální a „ramp“ hrana



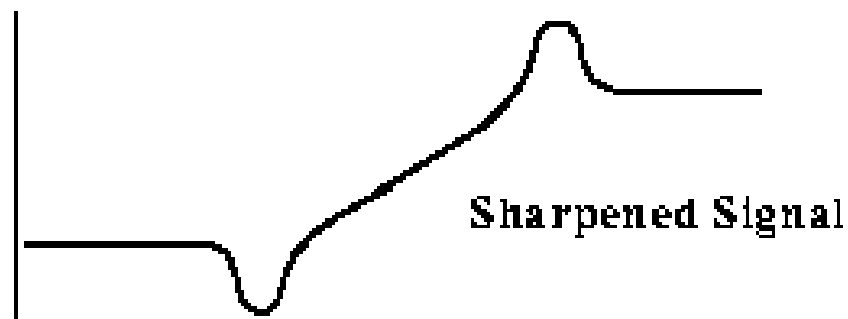
Jednoduché metody zaostření obrazu



Unsharp masking

$$g(x, y) = f(x, y) - f_{smooth}(x, y)$$

$$f_{sharp}(x, y) = f(x, y) + k * g(x, y)$$



Neostré maskování - Laplace

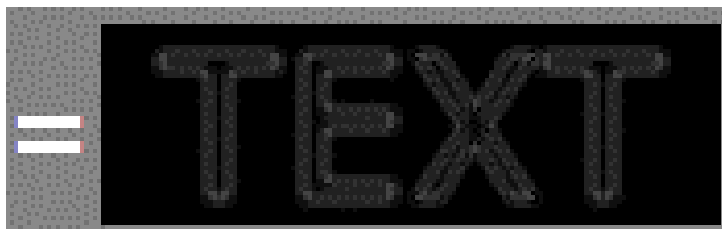
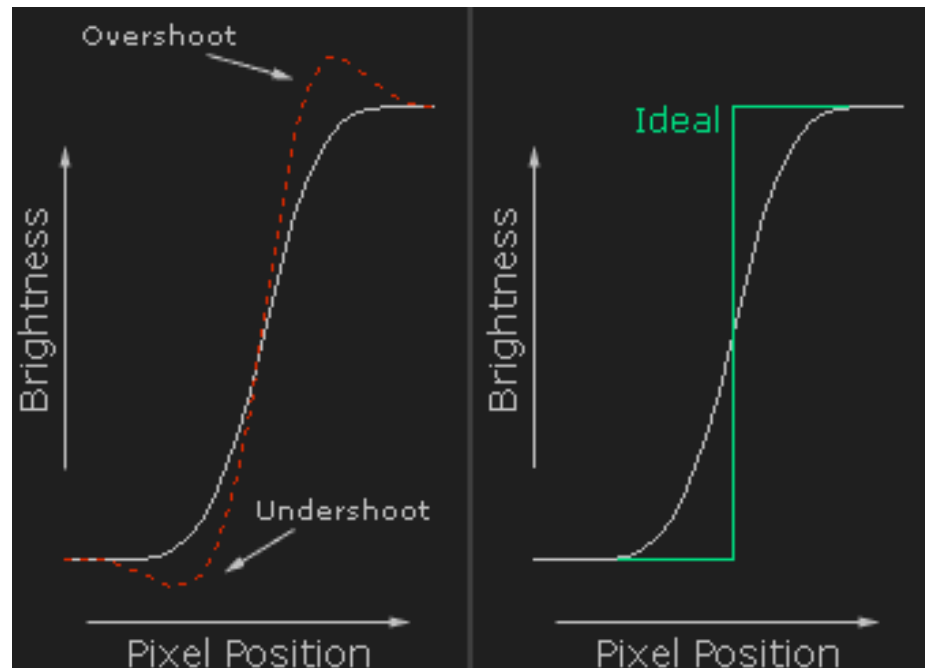
$$A = \frac{1}{5} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} - \left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} \right)$$

$$B = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$B-A = \frac{1}{5} \begin{vmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{vmatrix}$$

Jednoduché metody zaostření obrazu

Unsharp masking (neostré maskování)





original



smoothed (5x5 Gaussian)



smoothed – original

Neostré maskování





Přehnané zaostření obrazu - Laplace



Laplacian

(lineární operátor)

$$\nabla^2 f = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

4, 8 okolí

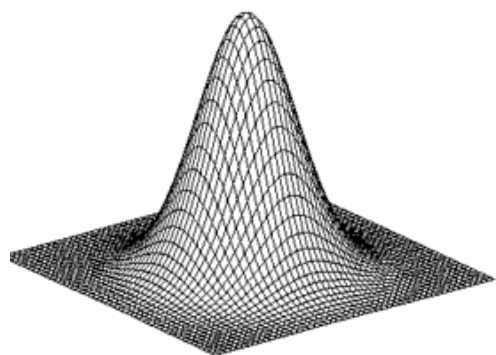
| | | | | | |
|----|----|----|----|----|----|
| 0 | -1 | 0 | -1 | -1 | -1 |
| -1 | 4 | -1 | -1 | 8 | -1 |
| 0 | -1 | 0 | -1 | -1 | -1 |

Laplacian

Zvýšený význam středního pixelu

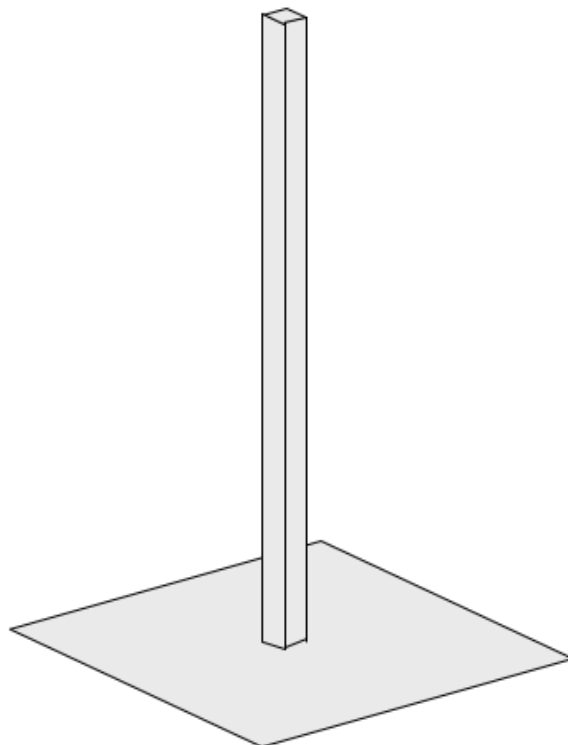
$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$



Gaussian

—

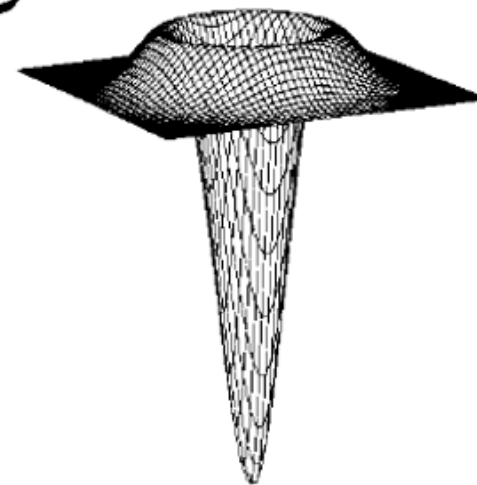


delta function

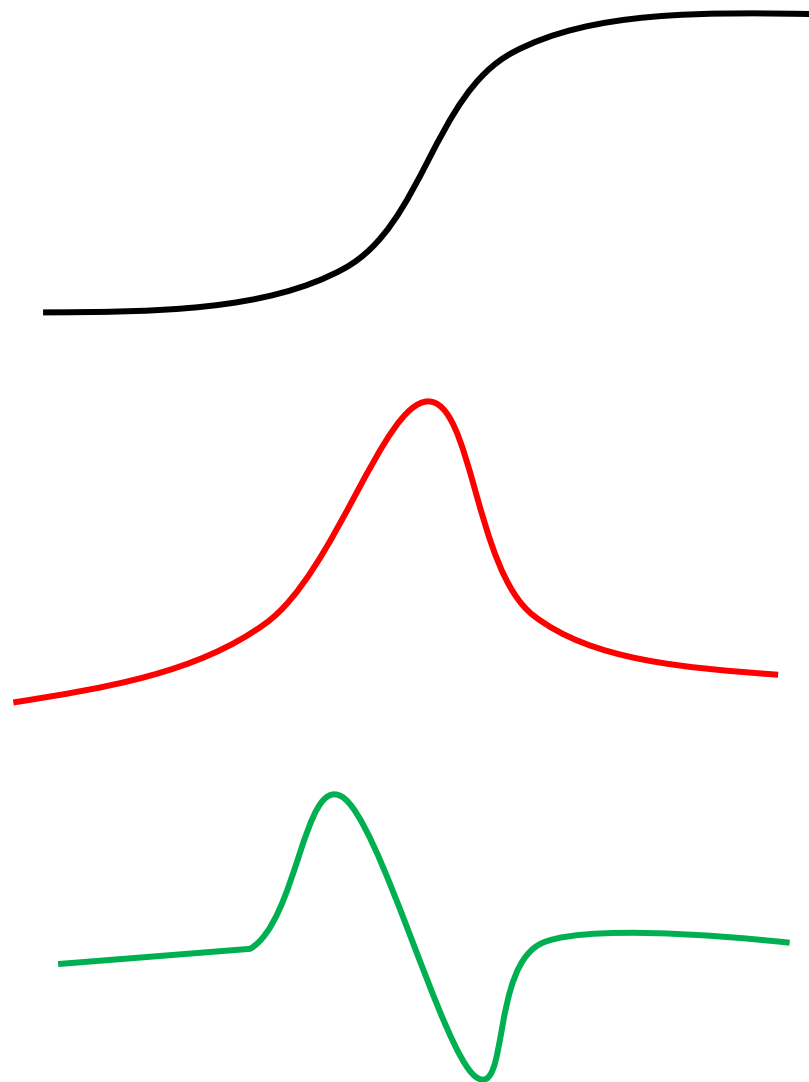
\approx



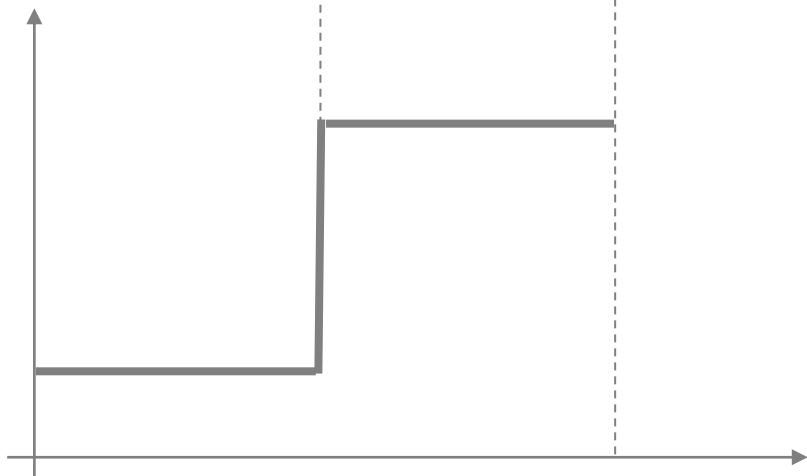
Fourier Transform



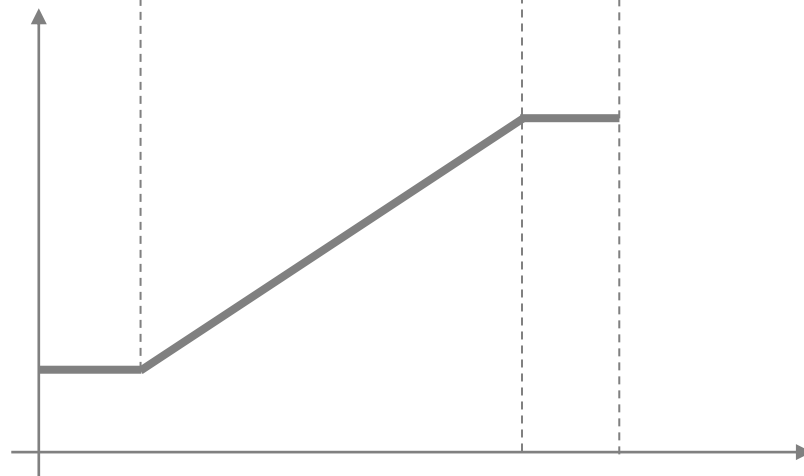
Detekce hran



Ideal

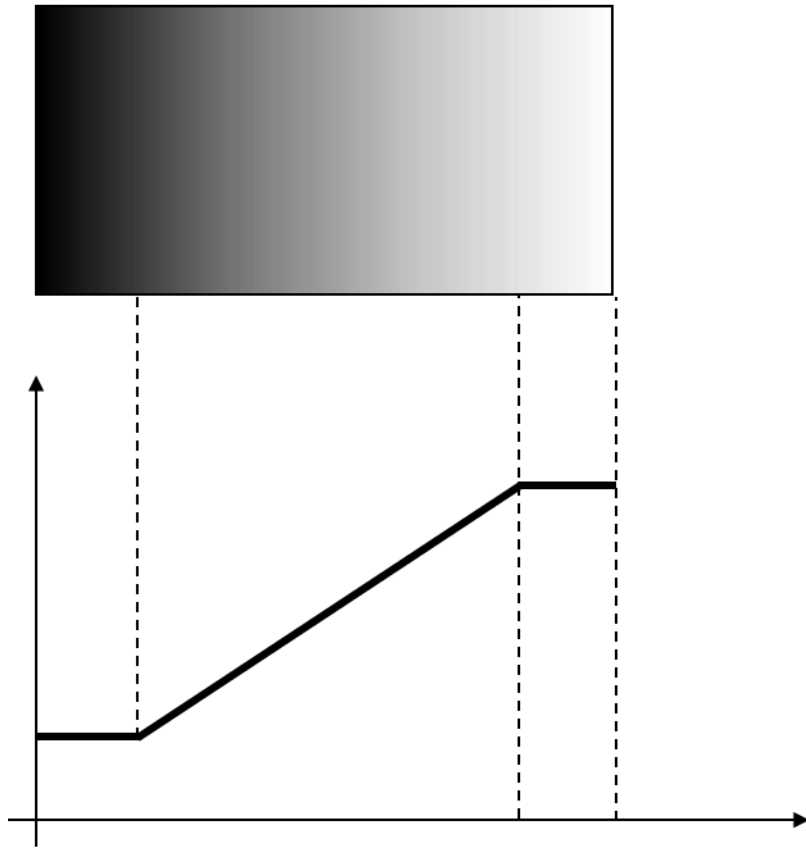


Ramp

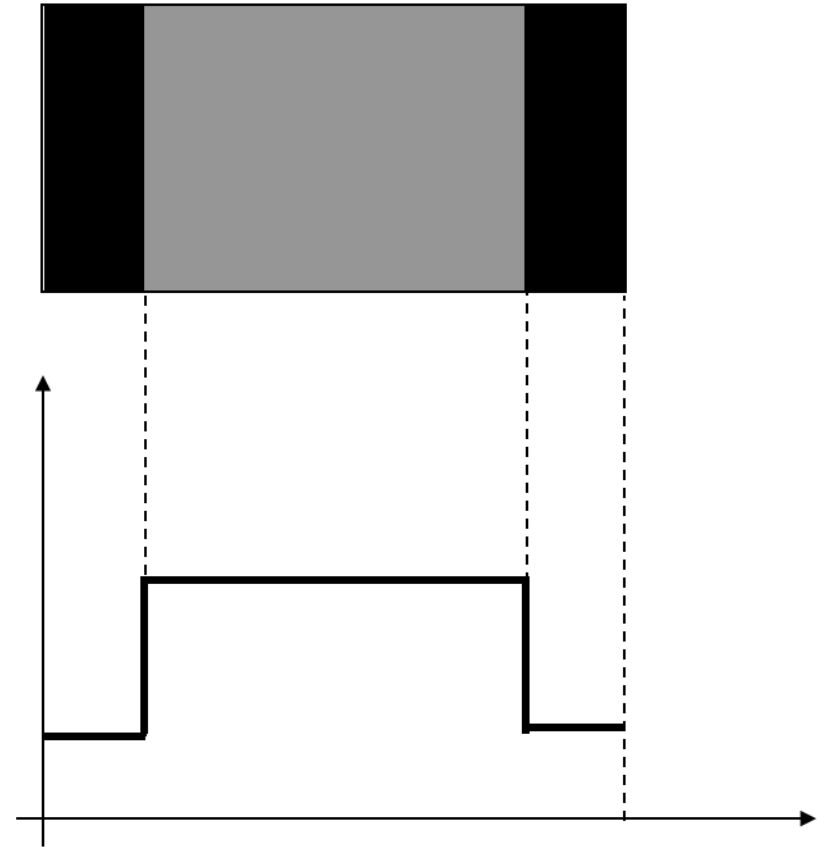


Ideální a „ramp“ hrana

Original

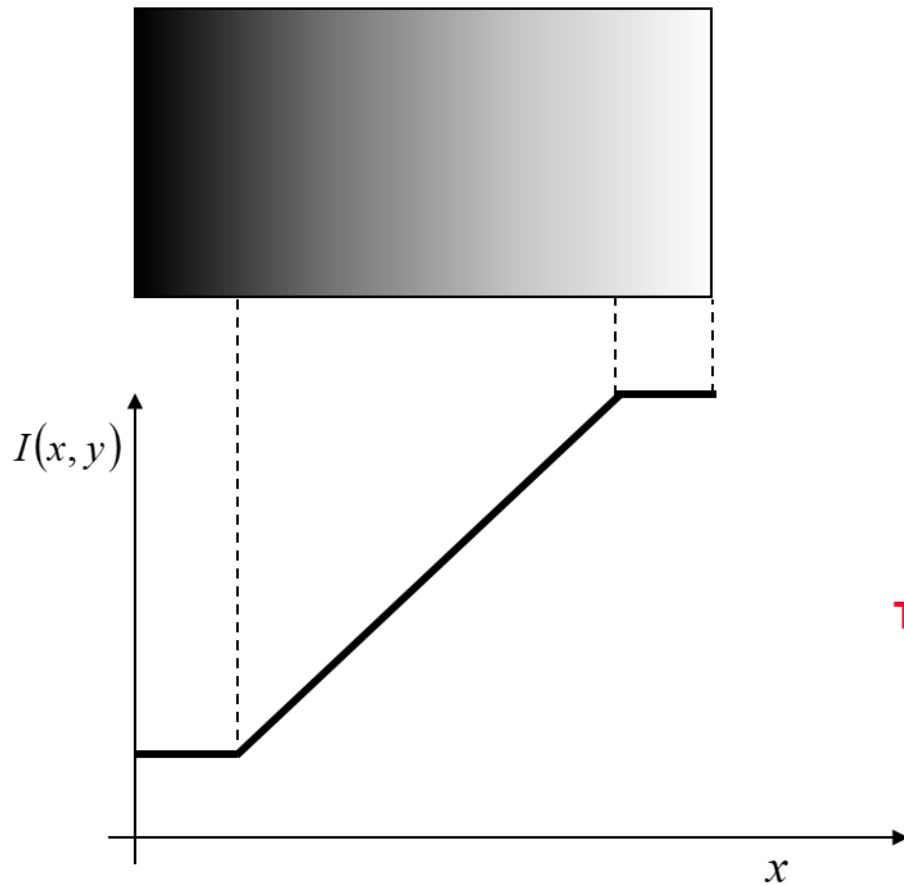


First Derivative

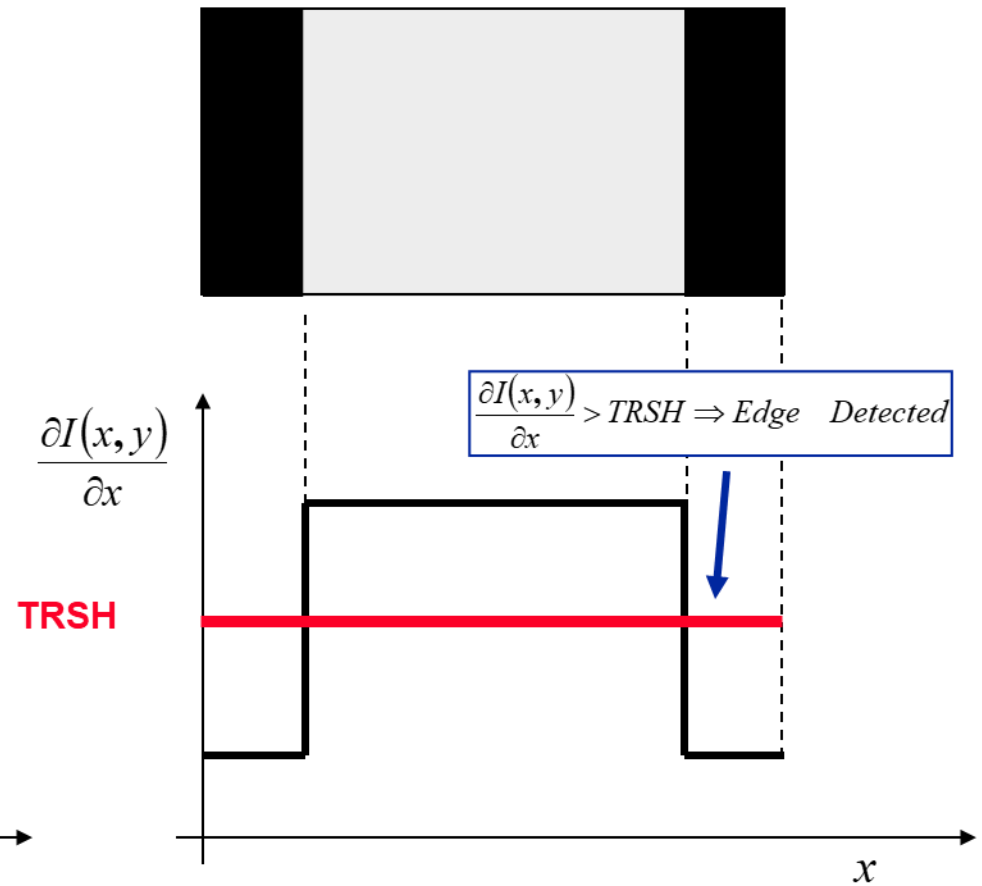


Ideální a „ramp“ hrana

Original

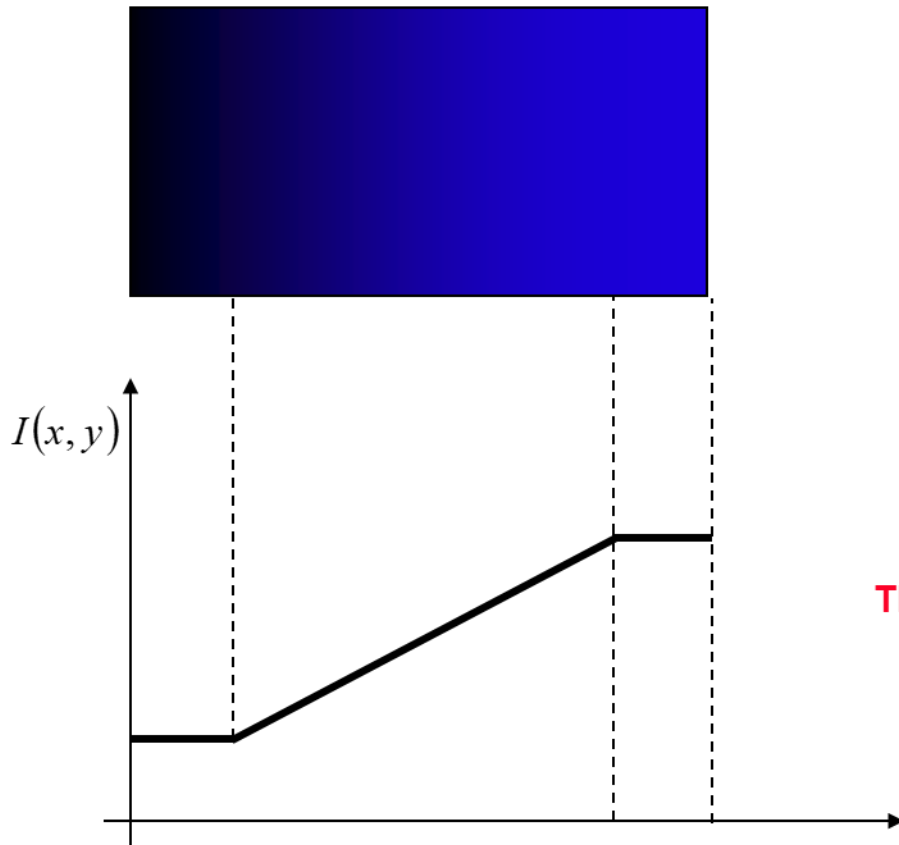


First Derivative

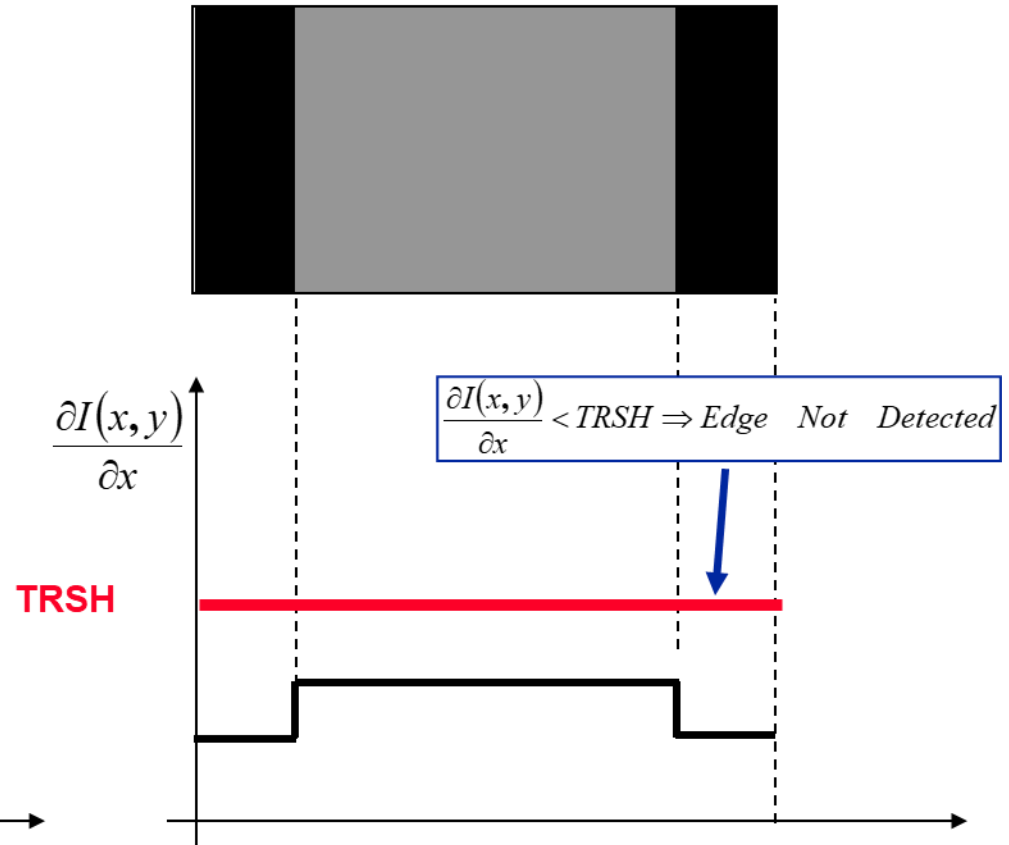


Ideální a „ramp“ hrana

Original



First Derivative



Detekce hran

Detektory založené na 1. derivaci obrazu

Detektory založené na 2. derivaci obrazu

Detektory které nepracují s derivacemi

Detekce hran ve Fourierově oblasti

Diferencování a konvoluce

Pro 2D funkci, $f(x,y)$:

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon} \right)$$

Můžeme aproximovat

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}, y) - f(x_n, y)}{\Delta x}$$

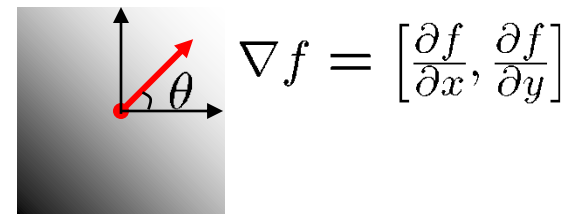
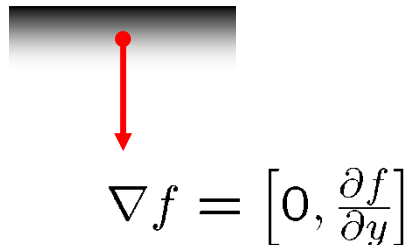
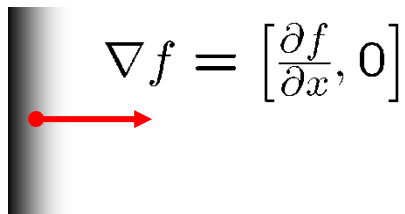
Konvoluce

| | |
|----|---|
| -1 | 1 |
|----|---|

Gradient obrázku

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

Gradient ukazuje ve směru nejrychlejší změny intenzity



Směr gradientu je dán

$$\theta = \tan^{-1} \left(\frac{\partial f / \partial y}{\partial f / \partial x} \right)$$

Síla hrany je dána velikostí gradientu

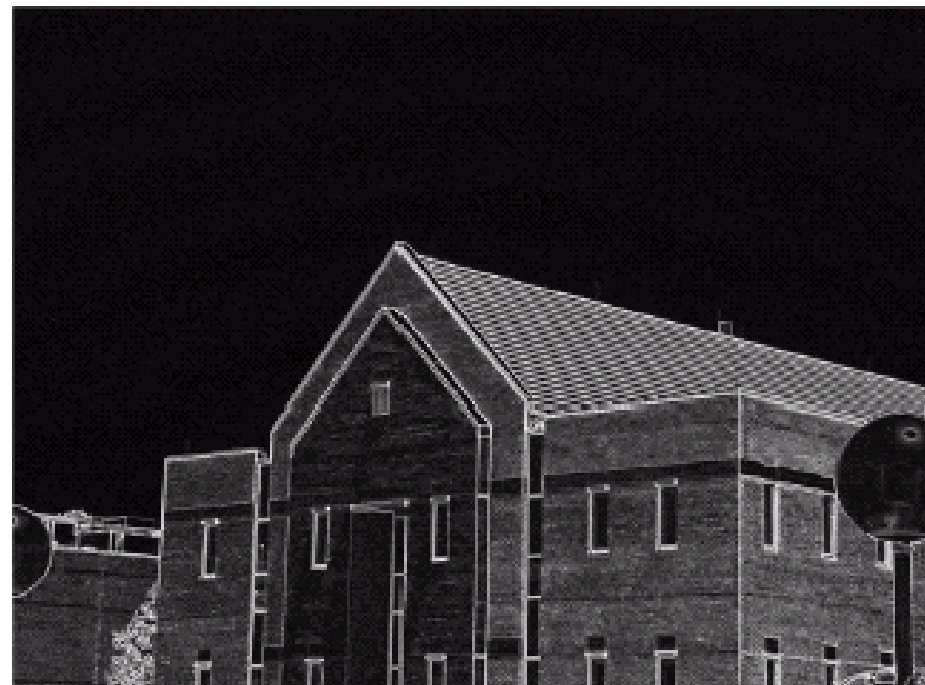
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

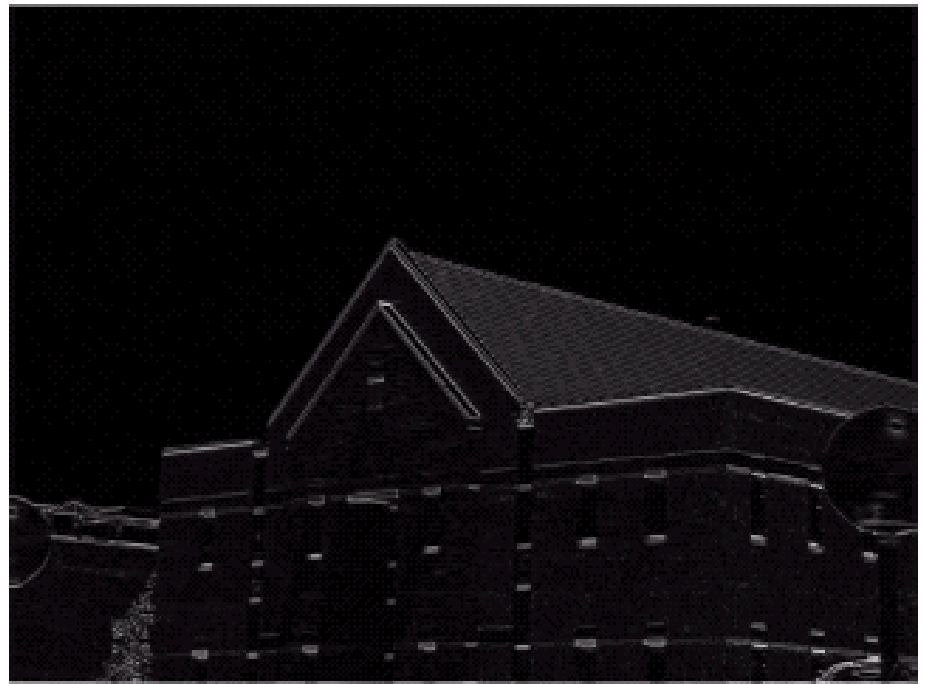


Jednoduché detektory založené na 1. derivaci

- Roberts
- Prewitt
- Sobel

$$\frac{\partial f}{\partial x}[x, y] \approx f[x + 1, y] - f[x, y]$$





Jednoduché detektory založené na 1. derivaci

Roberts Operator

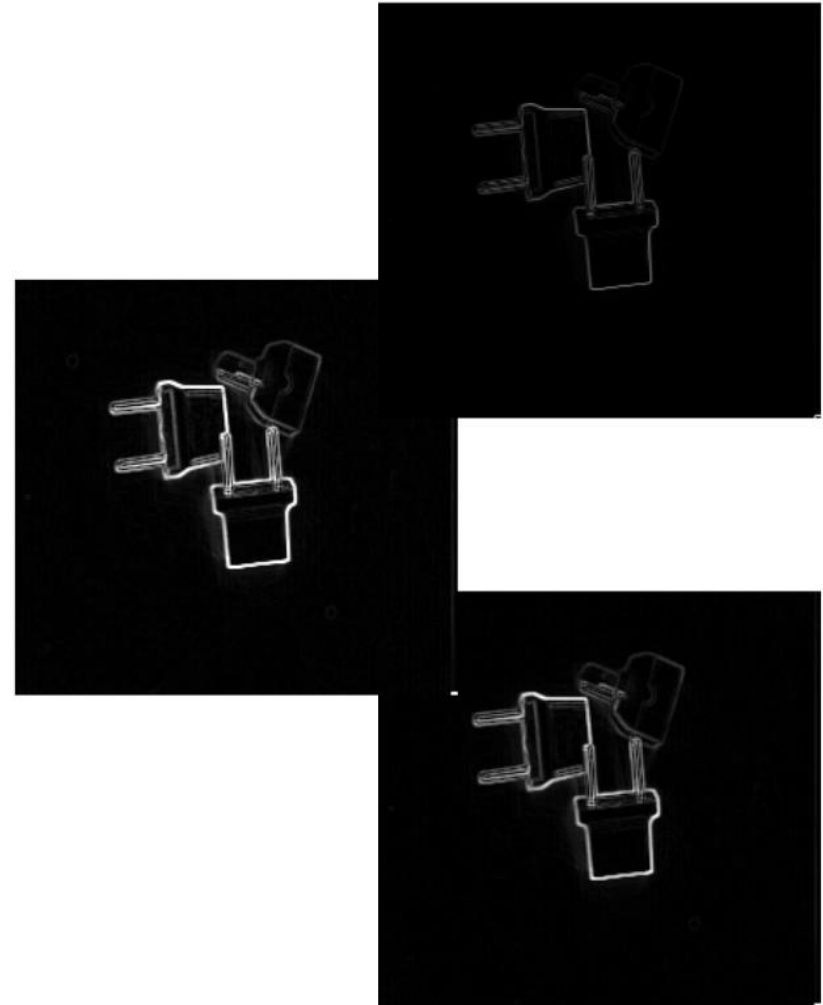
$$G_x \approx \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad G_y \approx \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Sobel Operator

$$G_x \approx \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad G_y \approx \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Prewitt Operator

$$G_x \approx \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad G_y \approx \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$



Courtesy of A.Campilho

Comparing Edge Operators

Gradient: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

Good Localization
Noise Sensitive
Poor Detection

Roberts (2 x 2):

| | |
|----|---|
| 0 | 1 |
| -1 | 0 |

| | |
|---|----|
| 1 | 0 |
| 0 | -1 |

Sobel (3 x 3):

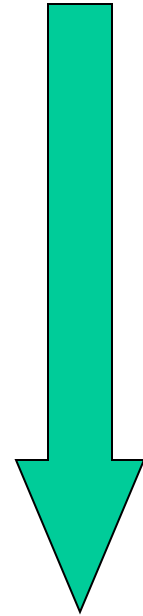
| | | |
|----|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

| | | |
|----|----|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | 1 |

Sobel (5 x 5):

| | | | | |
|----|----|---|---|---|
| -1 | -2 | 0 | 2 | 1 |
| -2 | -3 | 0 | 3 | 2 |
| -3 | -5 | 0 | 5 | 3 |
| -2 | -3 | 0 | 3 | 2 |
| -1 | -2 | 0 | 2 | 1 |

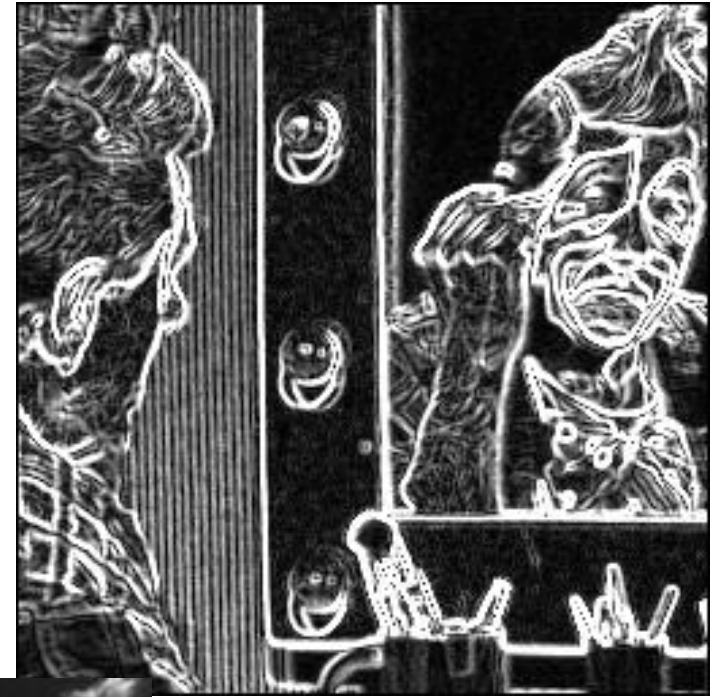
| | | | | |
|----|----|----|----|----|
| 1 | 2 | 3 | 2 | 1 |
| 2 | 3 | 5 | 3 | 2 |
| 0 | 0 | 0 | 0 | 0 |
| -2 | -3 | -5 | -3 | -2 |
| -1 | -2 | -3 | -2 | -1 |



Poor Localization
Less Noise Sensitive
Good Detection



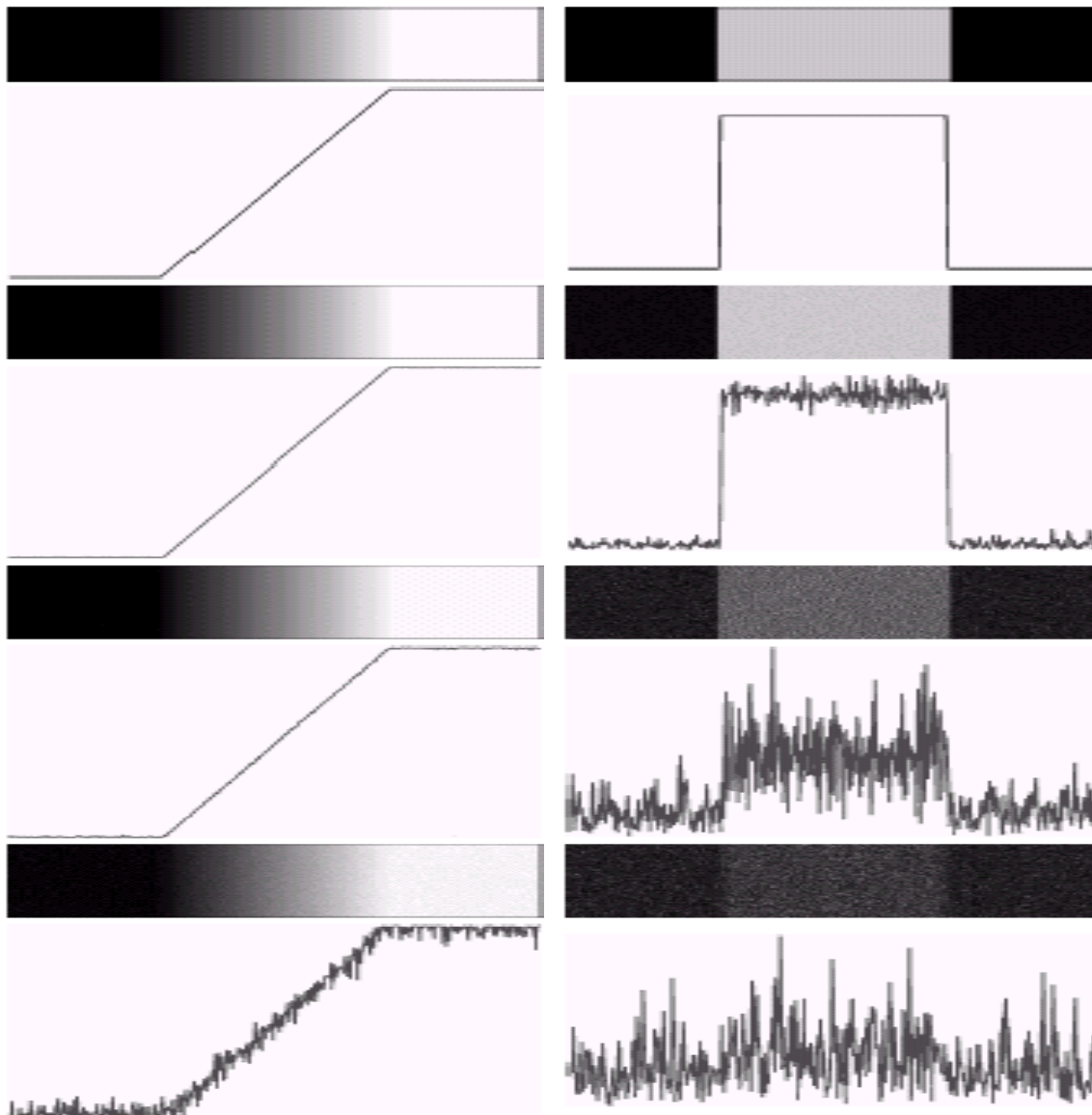
- Roberts

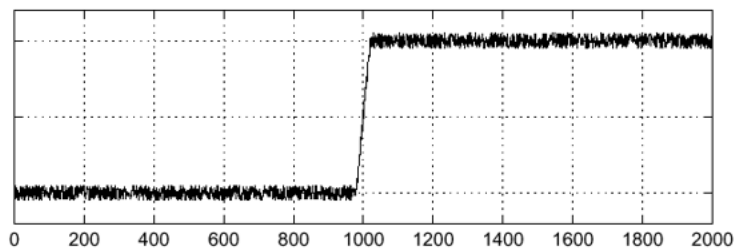
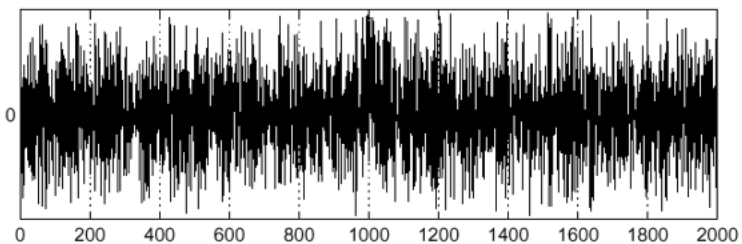


Sobel

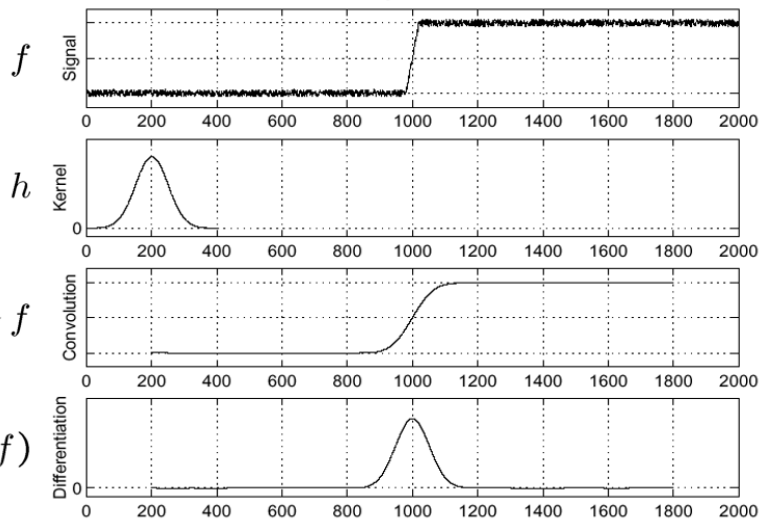


Hrany a šum

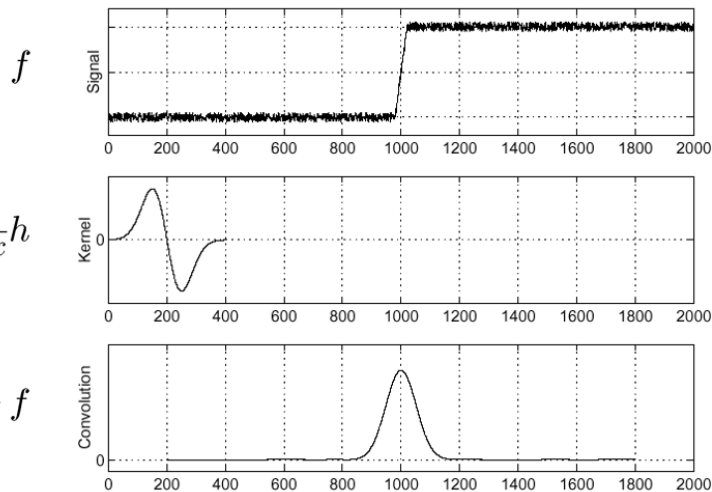


$f(x)$  $\frac{d}{dx}f(x)$ 

Sigma = 50



Sigma = 50



Canny

- málo, dobře, jednou
- vyhlazení
- první derivace
- detekce hřbetů
- non-maximal suppression



Canny Edge Detector

(J. Canny'1986)

Original image

Smoothing by Gaussian convolution

Differential operators along x and y axis

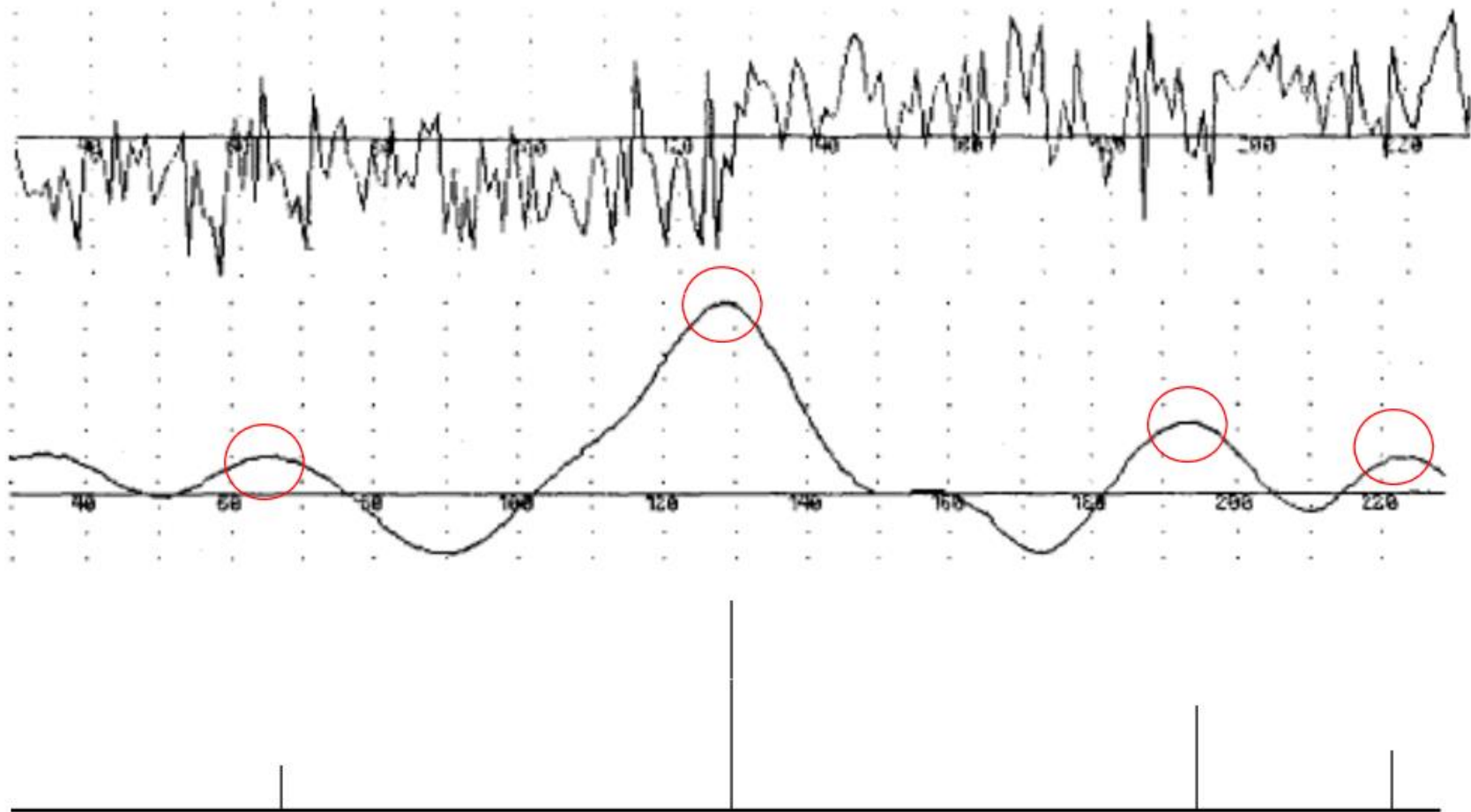
Non-maximum suppression

Hysteresis thresholding locates edge strings

Edge map



Non-maximum suppression



Hysteresis thresholding



original image



high threshold
(strong edges)



low threshold
(weak edges)



hysteresis threshold



Canny Edge Detector



norm of the gradient



after thresholding



after thinning



Effect of σ (Gaussian kernel size)



original



Canny with $\sigma = 1$



Canny with $\sigma = 2$



Scale

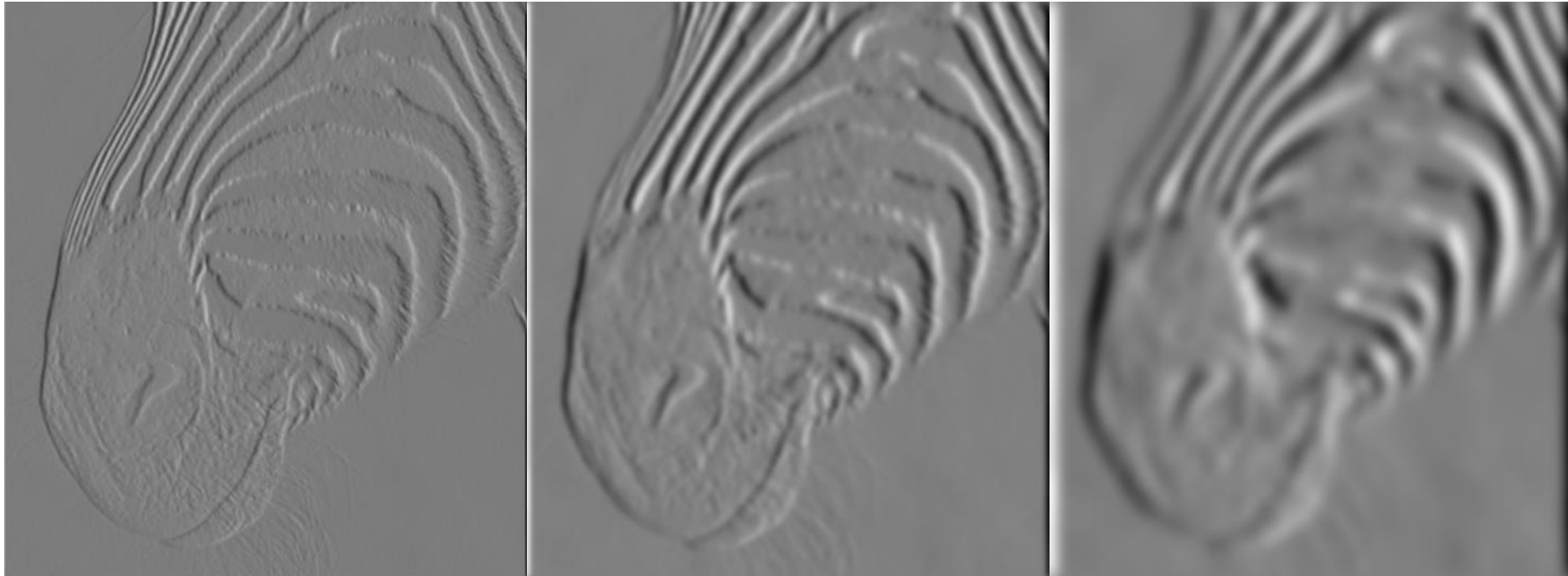
Smoothing

Eliminates noise edges.

Makes edges smoother.

Removes fine detail.

Tradeoff between smoothing and localization



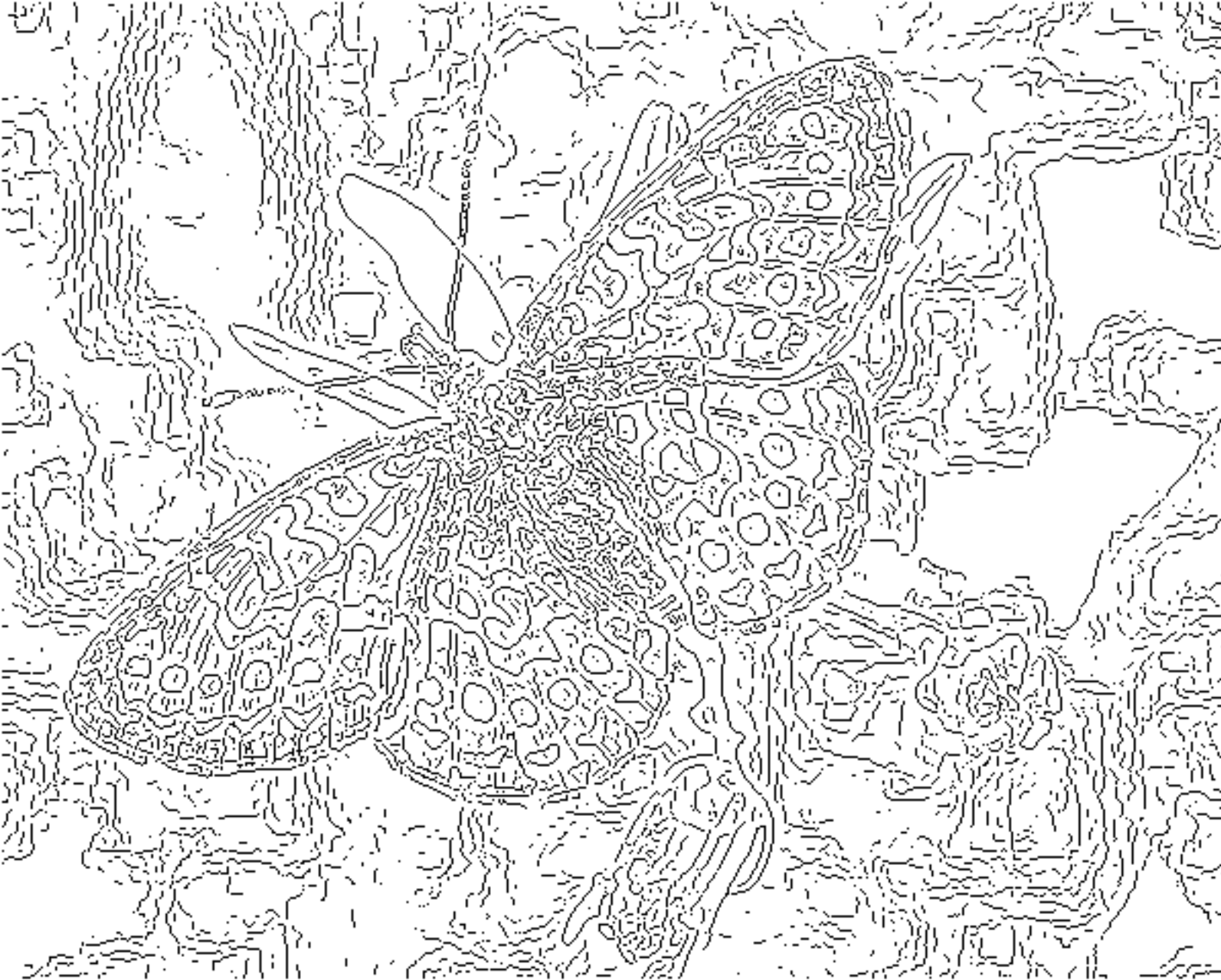
1 pixel

3 pixels

7 pixels

**Smoothed derivative removes noise, but blurs edge.
Also finds edges at different “scales”.**





fine sca
high
thresho



**coarse
scale,
high
threshold**



coarse

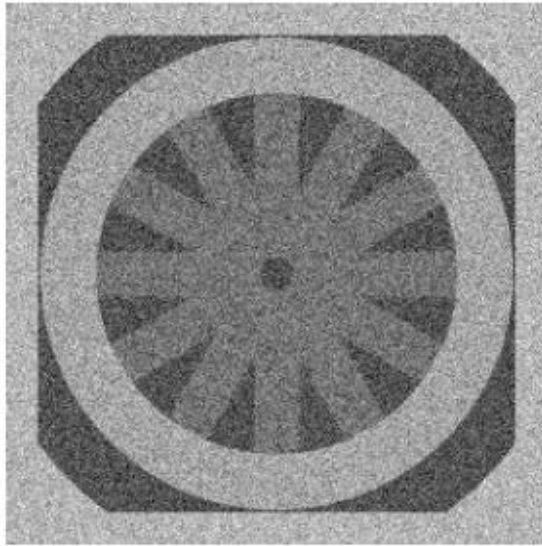
scale

low

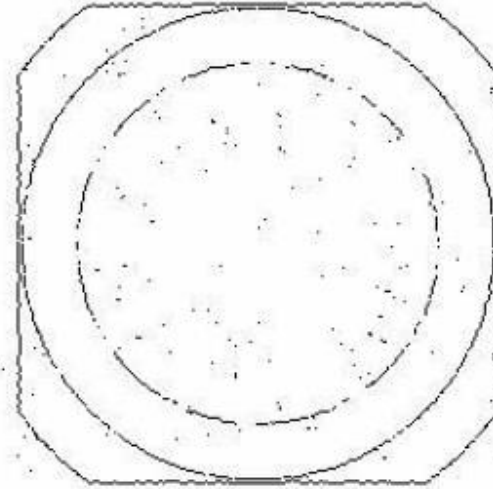
threshold

Porovnání

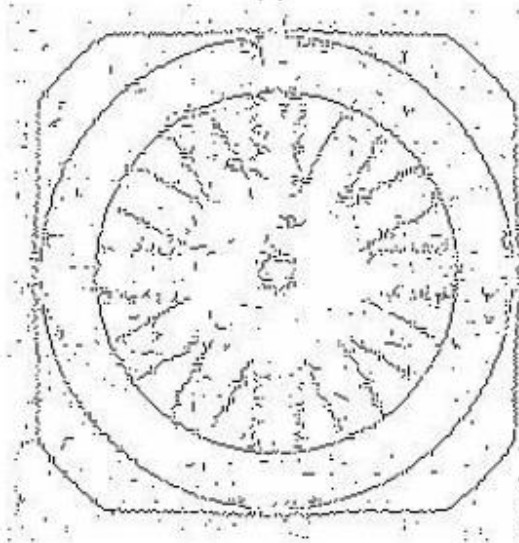
obr a šum



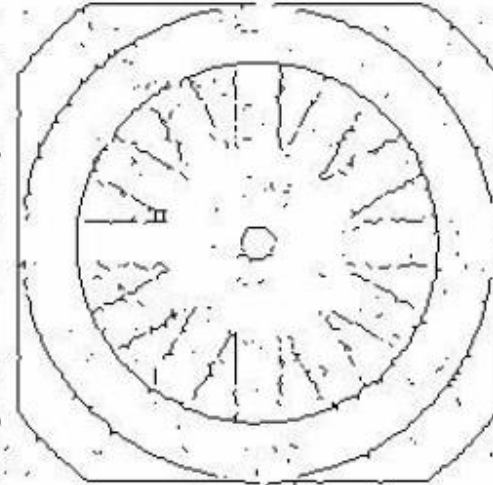
Sobel



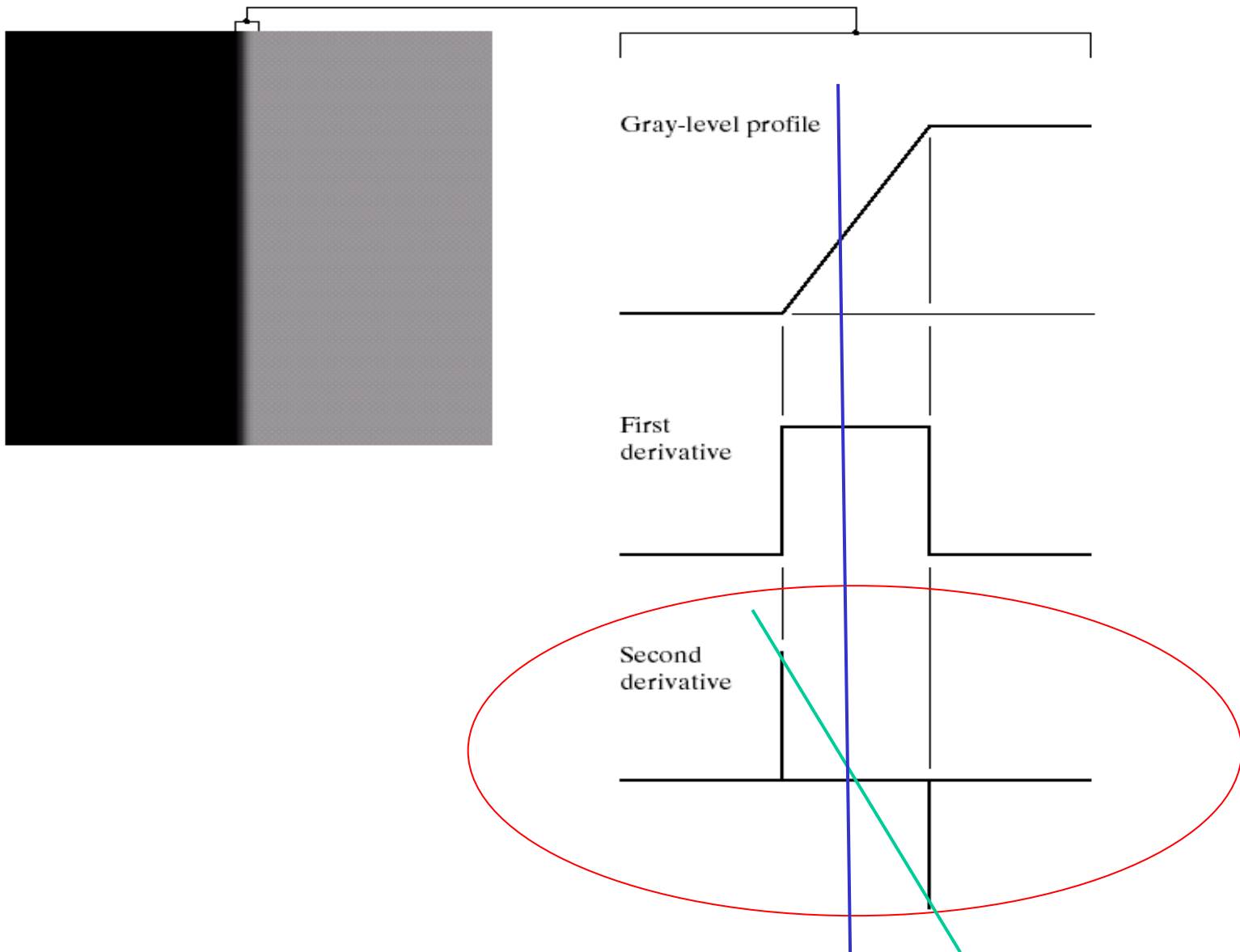
Canny



wavelety

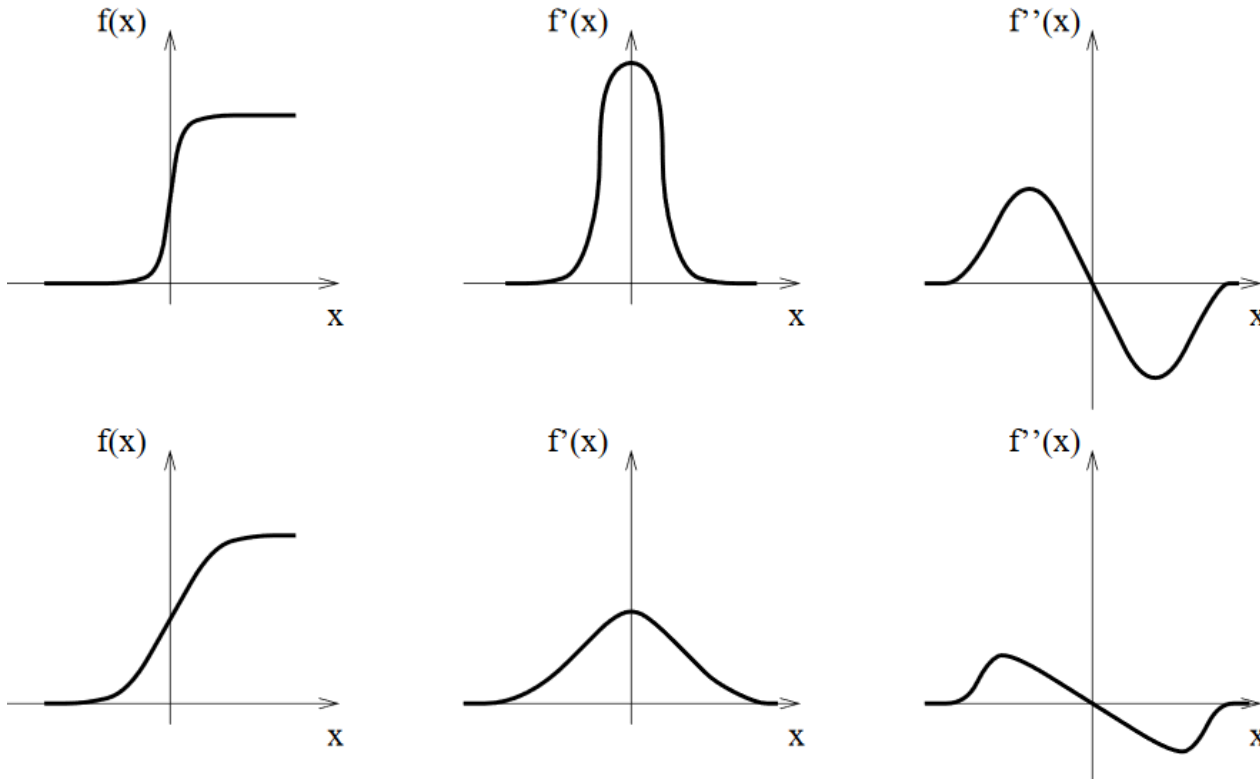


Ideální a „ramp“ hrana



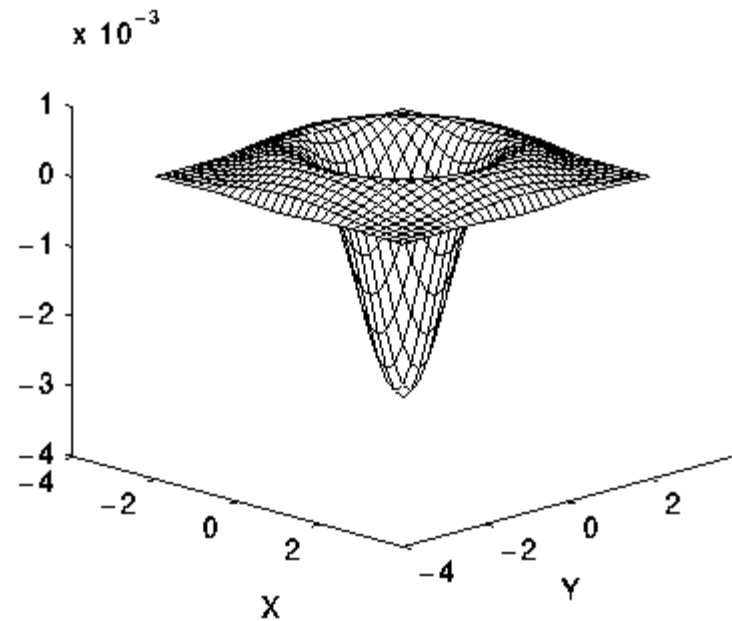
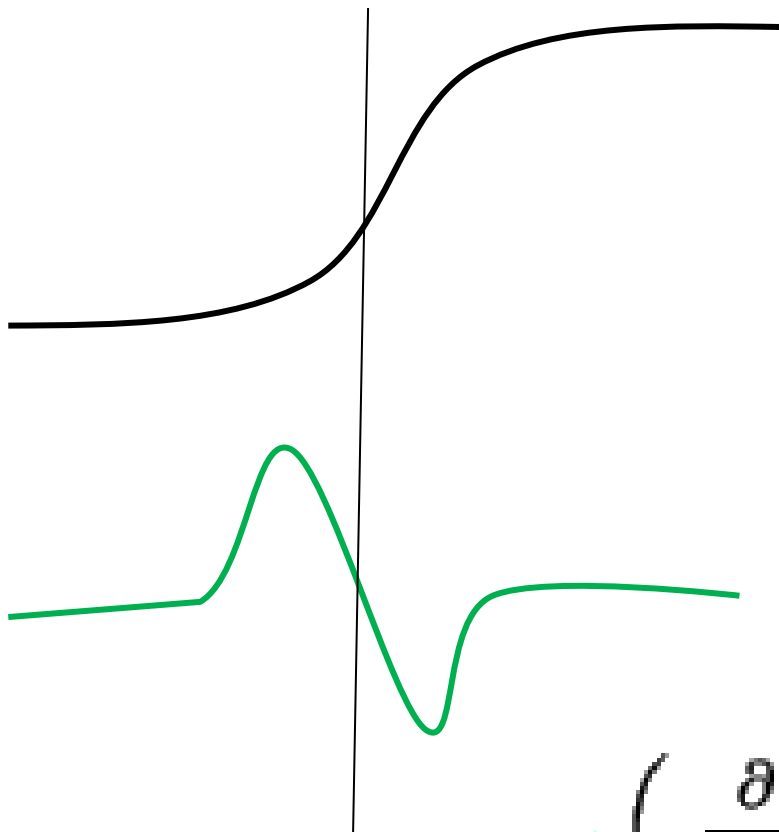
Detektory založené na 2. derivaci

D. Marr, E. Hildreth



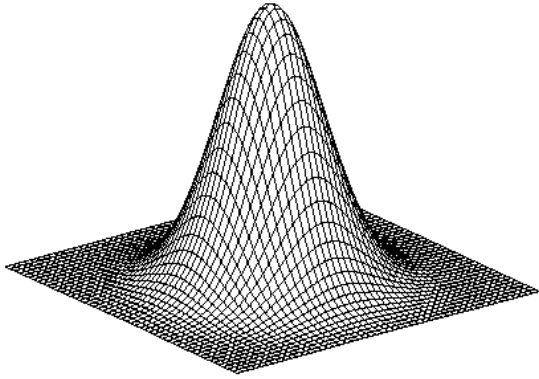
Detektory založené na 2. derivaci

D. Marr, E. Hildreth (1980) -- LoG



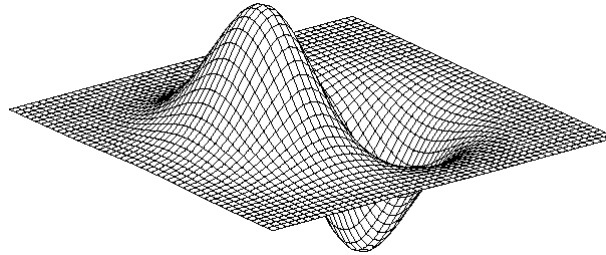
$$\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} \right)$$

2D edge detection filters



Gaussian

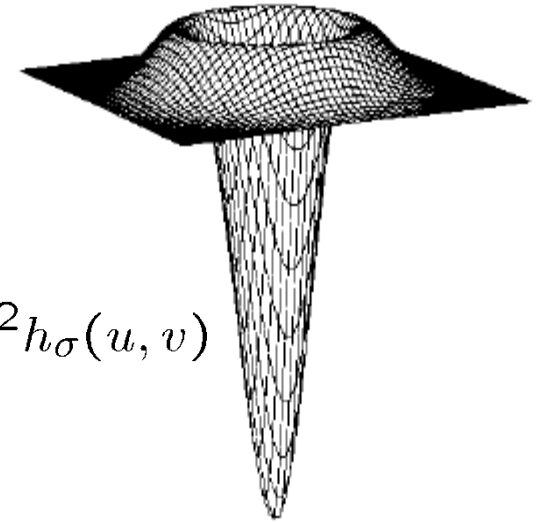
$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

Laplacian of Gaussian



$$\nabla^2 h_{\sigma}(u, v)$$

∇^2 ; the Laplacian operator:

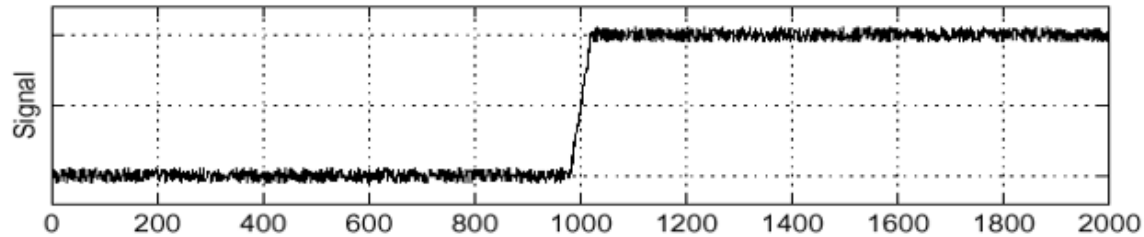
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2}{\partial x^2}(h * f) = \left(\frac{\partial^2}{\partial x^2} h \right) * f$$

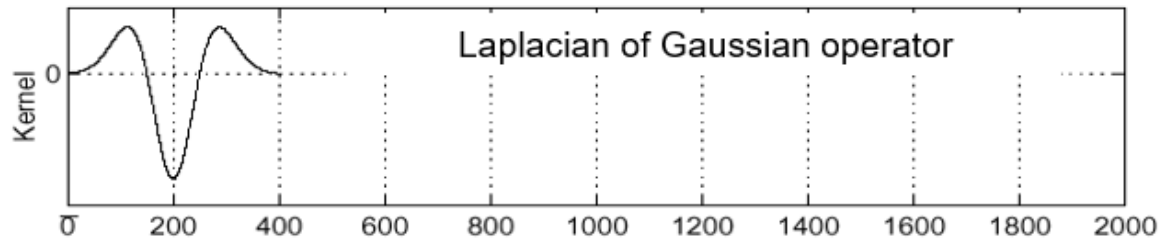
Laplacian of Gaussian

Sigma = 50

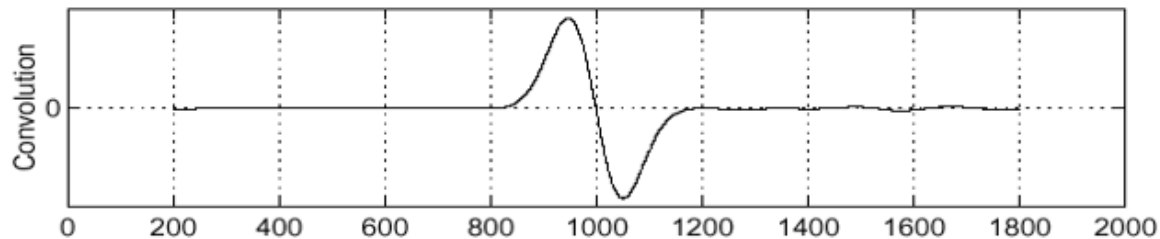
f



$\frac{\partial^2}{\partial x^2} h$



$\left(\frac{\partial^2}{\partial x^2} h \right) * f$



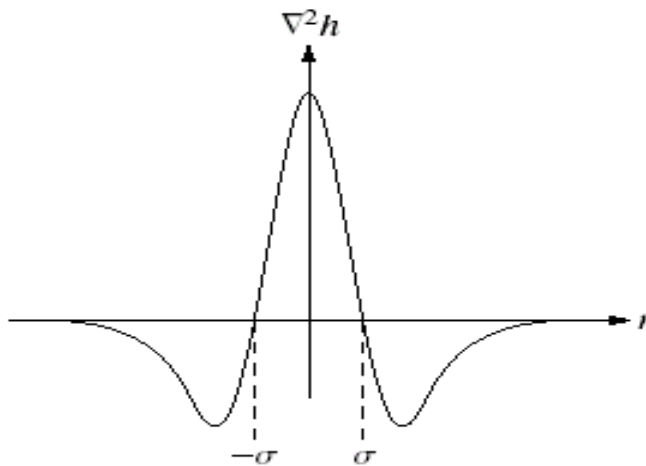
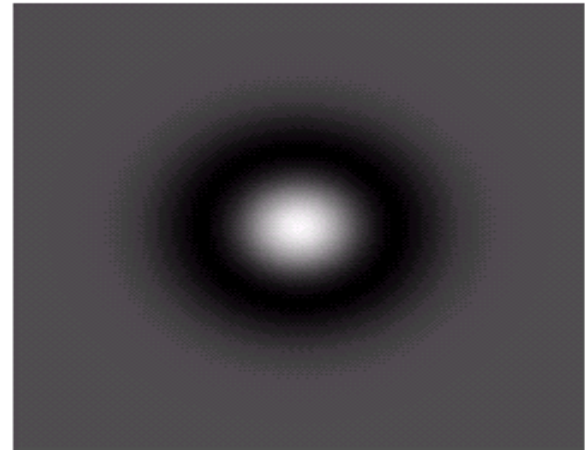
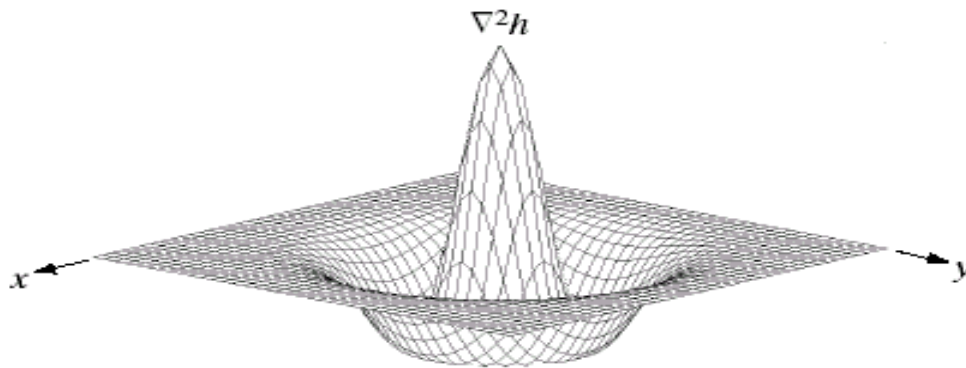
Laplacian Gaussianu

Šum, průchody 0

$$h(r) = -e^{-\frac{r^2}{2\sigma^2}}$$

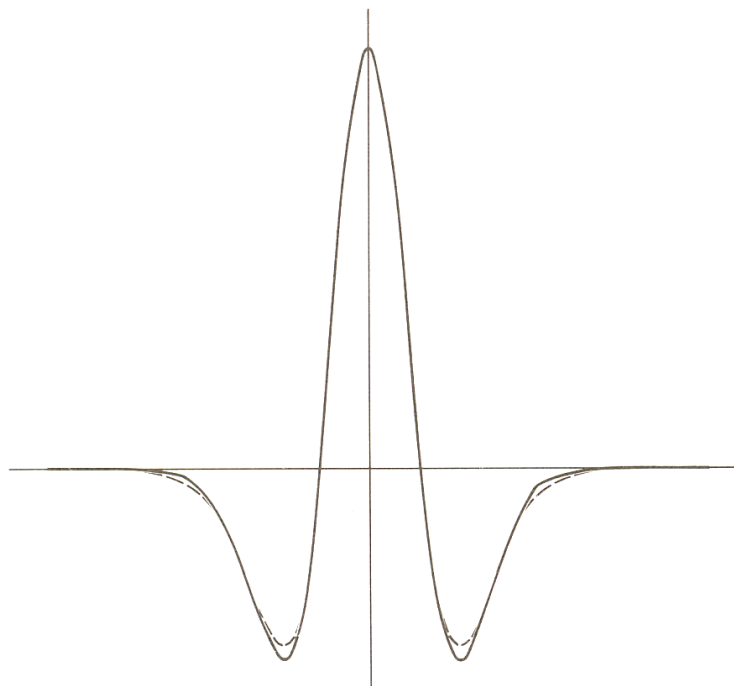
$$\nabla^2 h(r) = -\left[\frac{r^2 - \sigma^2}{\sigma^4} \right] e^{-\frac{r^2}{2\sigma^2}}$$

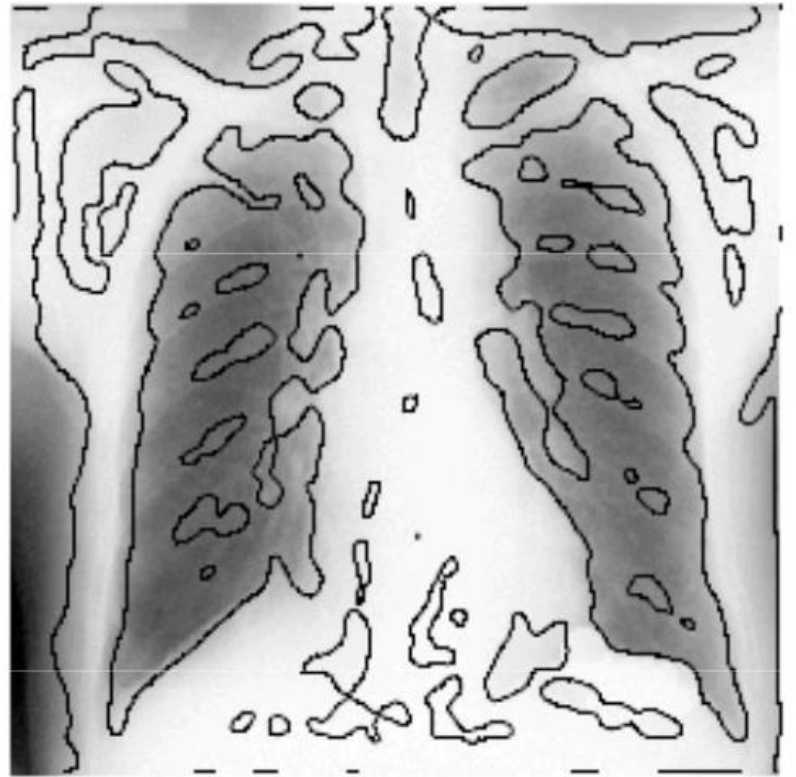
Mexický klobouk



| | | | | |
|----|----|----|----|----|
| 0 | 0 | -1 | 0 | 0 |
| 0 | -1 | -2 | -1 | 0 |
| -1 | -2 | 16 | -2 | -1 |
| 0 | -1 | -2 | -1 | 0 |
| 0 | 0 | -1 | 0 | 0 |

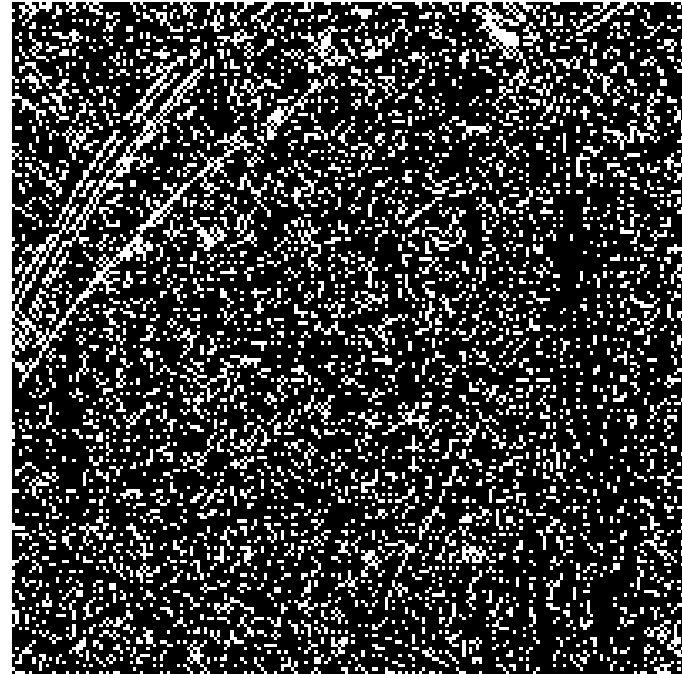
DoG







original



zero-crossings

Detektory založené na 2. derivaci

D. Marr, E. Hildreth (1980) -- LoG



sigma = 0.5



1



1.5

Zero crossing množiny ΔG^*f



sigma = 2

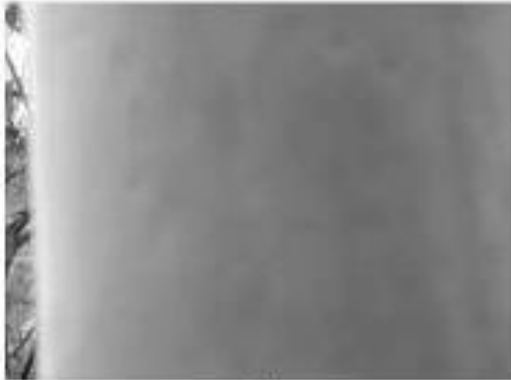


3



4

Importance of Scale



A



B



C



D

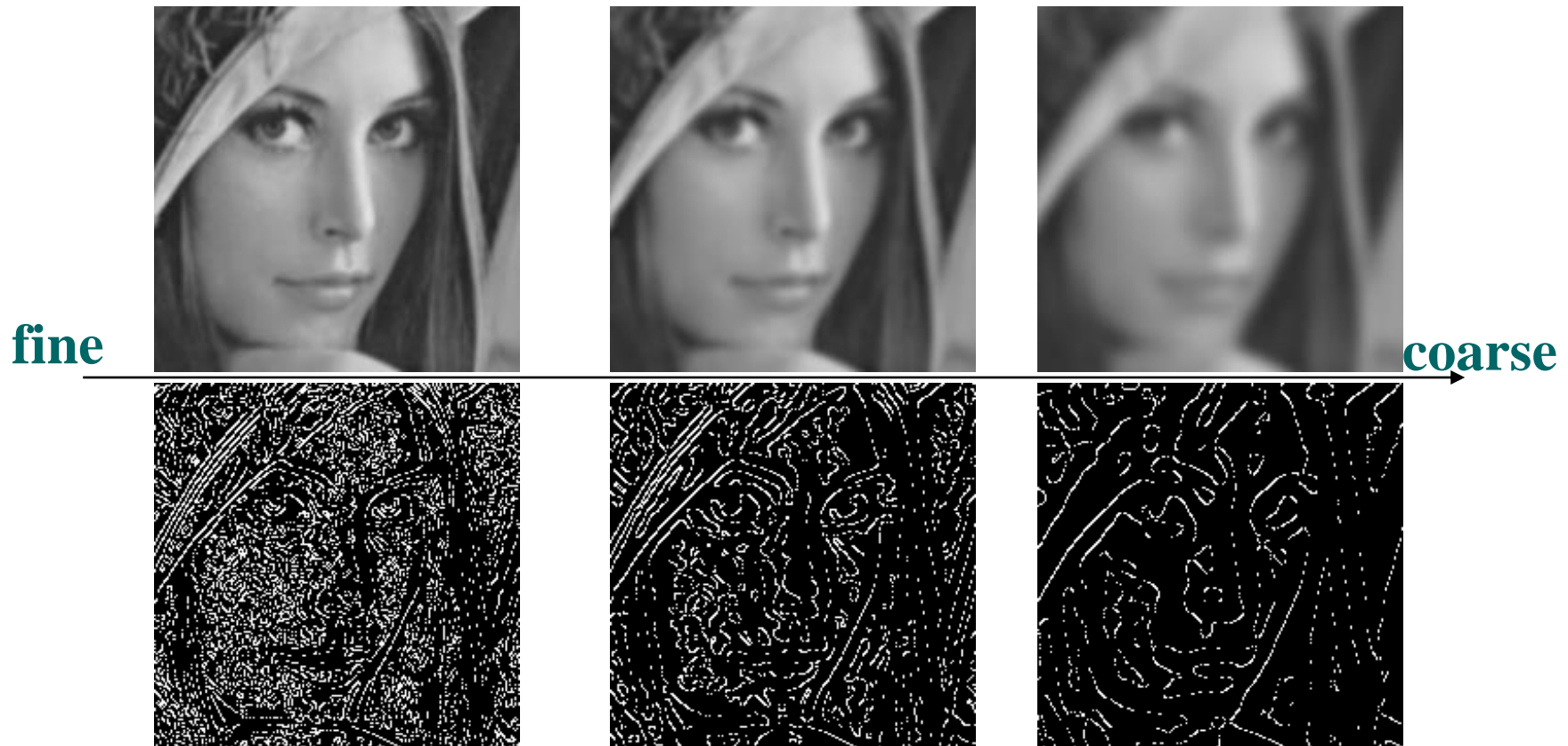


E



F

Scale-Space Edge Detection Examples

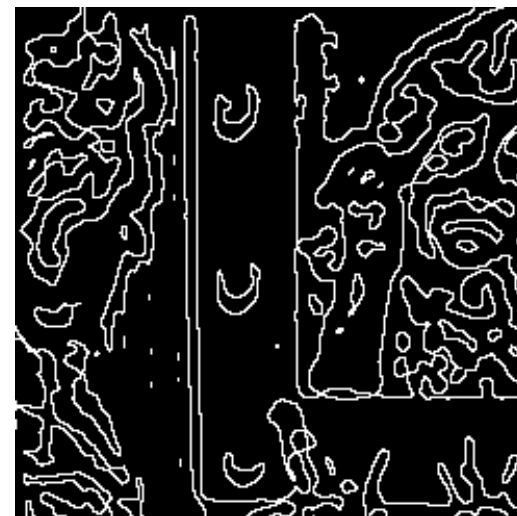


Porovnání Canny a Marr

originál

Canny

Marr

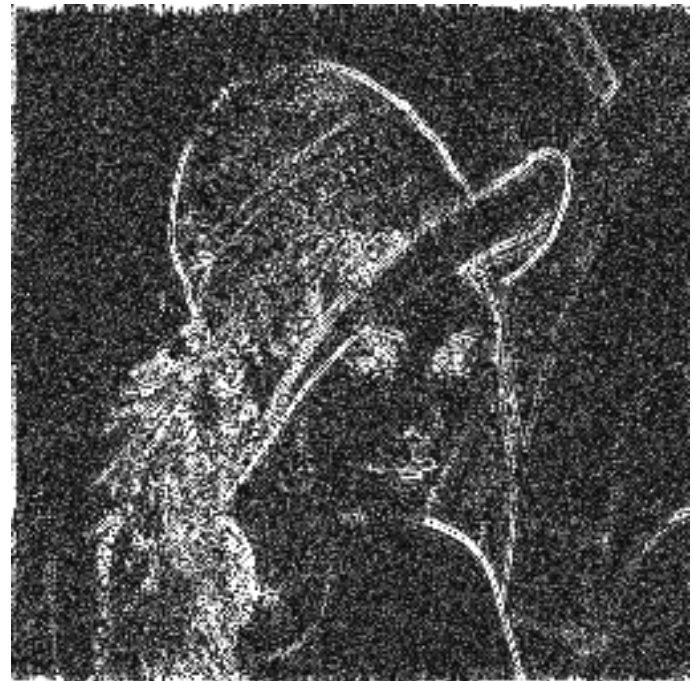


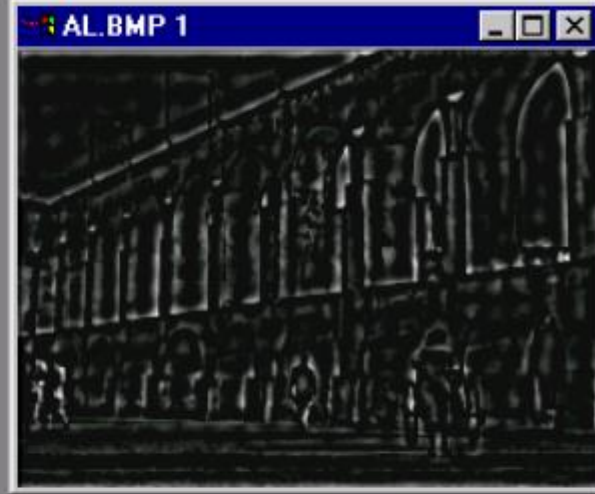
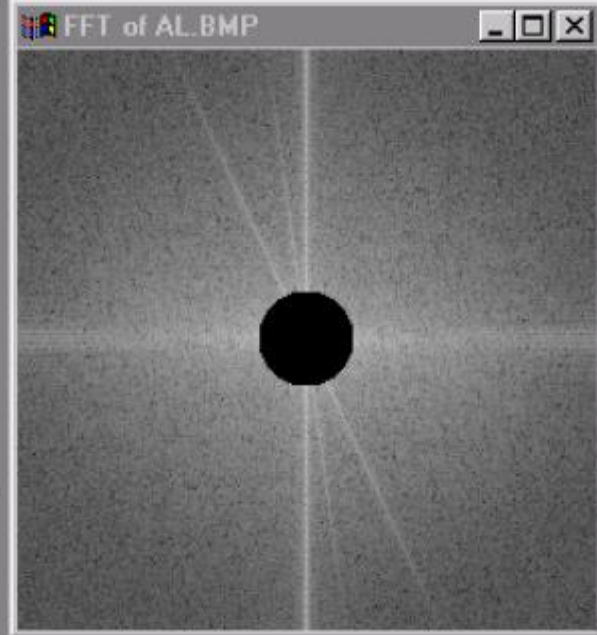
Whitening

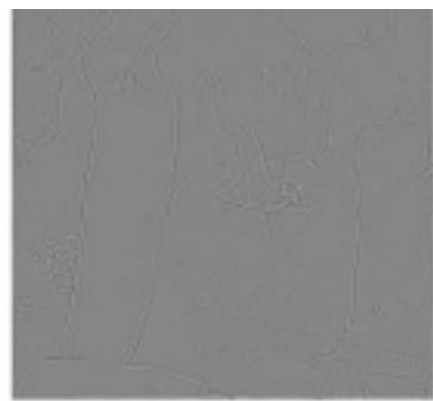
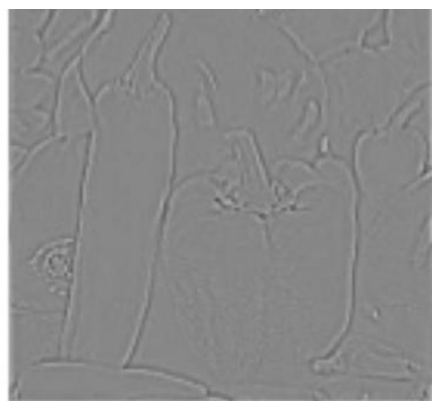
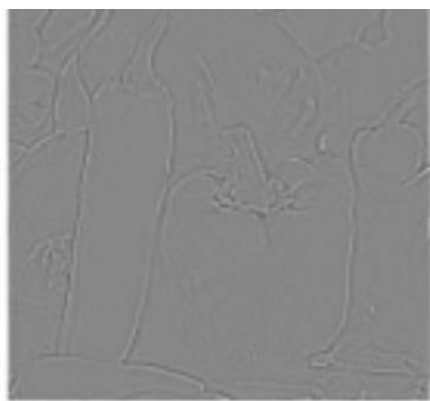
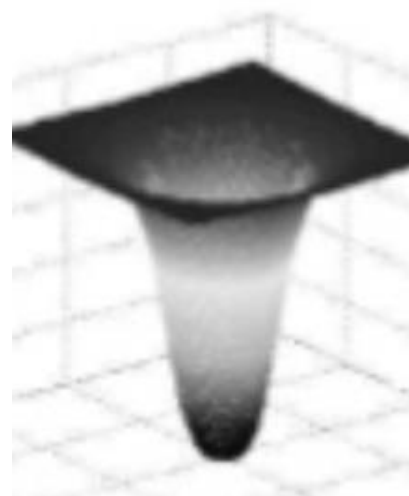
original image



“whitened” image







Citra asli

$r_0 = 47$

$r_0 = 36$

$r_0 = 81$



(a) original image



(b) ground truth



(c) HED: output



(d) HED: side output 2



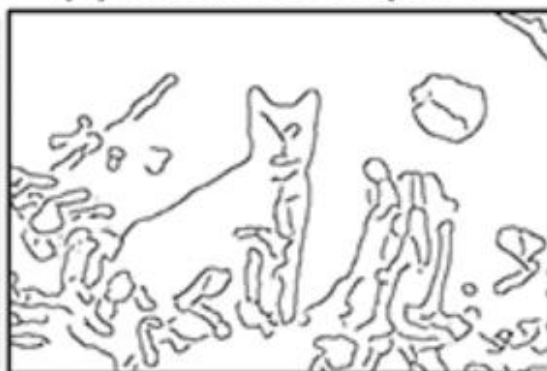
(e) HED: side output 3



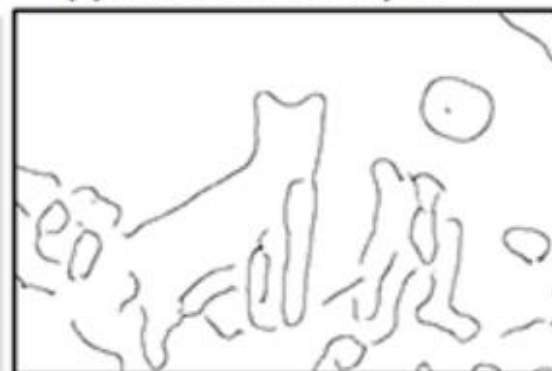
(f) HED: side output 4



(g) Canny: $\sigma = 2$



(h) Canny: $\sigma = 4$



(i) Canny: $\sigma = 8$