Rotation moment invariants of 2D images with order ≤ 10

This file contains all 62 independent invariants of 2D images from geometric moments with respect to 2D rotation with order ≤ 10.

$$\Phi_1 = m_{20} + m_{02}$$

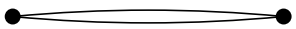
Generating graph:

1
1

$$\Phi_2 = m_{20}^2 + 2m_{11}^2 + m_{02}^2$$

Generating graph:

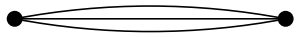
1 1
2 2



$$\Phi_3 = m_{30}^2 + 3m_{21}^2 + 3m_{12}^2 + m_{03}^2$$

Generating graph:

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{matrix}$$



$$\Phi_4 = m_{30}^2 + 2m_{30}m_{12} + m_{21}^2 + 2m_{21}m_{03} + m_{12}^2 + m_{03}^2$$

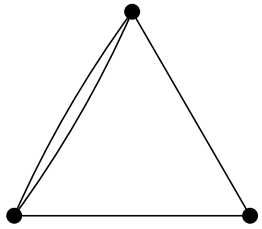
Generating graph:

$$\begin{matrix} 1 & 1 & 2 \\ 1 & 2 & 2 \end{matrix}$$

$$\begin{aligned} \Phi_5 \quad = \quad & m_{20}m_{30}^2 + 2m_{20}m_{21}^2 + m_{20}m_{12}^2 + 2m_{11}m_{30}m_{21} + 4m_{11}m_{21}m_{12} + 2m_{11}m_{12}m_{03} + m_{02}m_{21}^2 \\ & +2m_{02}m_{12}^2 + m_{02}m_{03}^2 \end{aligned}$$

Generating graph:

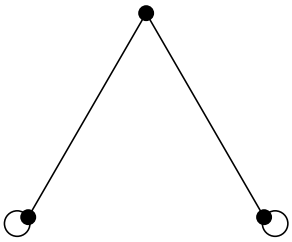
$$\begin{matrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 3 & 3 \end{matrix}$$



$$\begin{aligned}\Phi_6 &= m_{20}m_{30}^2 + 2m_{20}m_{30}m_{12} + m_{20}m_{12}^2 + 2m_{11}m_{30}m_{21} + 2m_{11}m_{30}m_{03} + 2m_{11}m_{21}m_{12} \\ &\quad +2m_{11}m_{12}m_{03} + m_{02}m_{21}^2 + 2m_{02}m_{21}m_{03} + m_{02}m_{03}^2\end{aligned}$$

Generating graph:

$$\begin{matrix}1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 3\end{matrix}$$



$$\Phi_7 = m_{40} + 2m_{22} + m_{04}$$

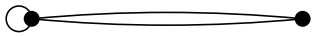
Generating graph:

$$\begin{matrix}1 & 1 \\ 1 & 1\end{matrix}$$

$$\Phi_8 = m_{20}m_{40} + m_{20}m_{22} + 2m_{11}m_{31} + 2m_{11}m_{13} + m_{02}m_{22} + m_{02}m_{04}$$

Generating graph:

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 2 \end{matrix}$$



$$\Phi_9 = m_{40}^2 + 4m_{31}^2 + 6m_{22}^2 + 4m_{13}^2 + m_{04}^2$$

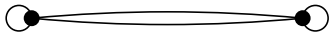
Generating graph:

$$\begin{matrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \end{matrix}$$

$$\Phi_{10} = m_{40}^2 + 2m_{40}m_{22} + 2m_{31}^2 + 4m_{31}m_{13} + 2m_{22}^2 + 2m_{22}m_{04} + 2m_{13}^2 + m_{04}^2$$

Generating graph:

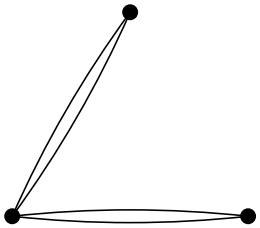
$$\begin{matrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 \end{matrix}$$



$$\Phi_{11} = m_{20}^2 m_{40} + 4m_{20}m_{11}m_{31} + 2m_{20}m_{02}m_{22} + 4m_{11}^2 m_{22} + 4m_{11}m_{02}m_{13} + m_{02}^2 m_{04}$$

Generating graph:

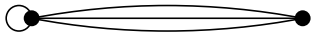
$$\begin{matrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 3 \end{matrix}$$



$$\Phi_{12} = m_{30}m_{50} + m_{30}m_{32} + 3m_{21}m_{41} + 3m_{21}m_{23} + 3m_{12}m_{32} + 3m_{12}m_{14} + m_{03}m_{23} + m_{03}m_{05}$$

Generating graph:

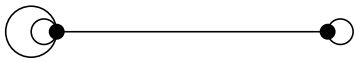
$$\begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \end{matrix}$$



$$\begin{aligned}\Phi_{13} &= m_{30}m_{50} + 2m_{30}m_{32} + m_{30}m_{14} + m_{21}m_{41} + 2m_{21}m_{23} + m_{21}m_{05} + m_{12}m_{50} + 2m_{12}m_{32} + m_{12}m_{14} \\ &\quad + m_{03}m_{41} + 2m_{03}m_{23} + m_{03}m_{05}\end{aligned}$$

Generating graph:

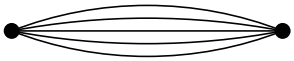
$$\begin{matrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{matrix}$$



$$\Phi_{14} = m_{50}^2 + 5m_{41}^2 + 10m_{32}^2 + 10m_{23}^2 + 5m_{14}^2 + m_{05}^2$$

Generating graph:

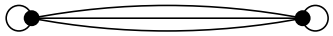
$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$



$$\Phi_{15} = m_{50}^2 + 2m_{50}m_{32} + 3m_{41}^2 + 6m_{41}m_{23} + 4m_{32}^2 + 6m_{32}m_{14} + 4m_{23}^2 + 2m_{23}m_{05} + 3m_{14}^2 + m_{05}^2$$

Generating graph:

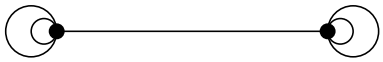
$$\begin{matrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 2 \end{matrix}$$



$$\begin{aligned}\Phi_{16} &= m_{50}^2 + 4m_{50}m_{32} + 2m_{50}m_{14} + m_{41}^2 + 4m_{41}m_{23} + 2m_{41}m_{05} + 4m_{32}^2 + 4m_{32}m_{14} + 4m_{23}^2 + 4m_{23}m_{05} \\ &\quad + m_{14}^2 + m_{05}^2\end{aligned}$$

Generating graph:

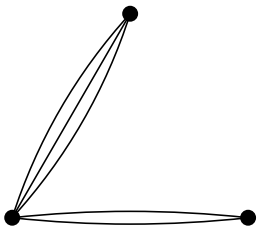
$$\begin{array}{ccccc} 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{17} &= m_{20}m_{30}m_{50} + 3m_{20}m_{21}m_{41} + 3m_{20}m_{12}m_{32} + m_{20}m_{03}m_{23} + 2m_{11}m_{30}m_{41} + 6m_{11}m_{21}m_{32} \\ &\quad + 6m_{11}m_{12}m_{23} + 2m_{11}m_{03}m_{14} + m_{02}m_{30}m_{32} + 3m_{02}m_{21}m_{23} + 3m_{02}m_{12}m_{14} + m_{02}m_{03}m_{05}\end{aligned}$$

Generating graph:

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 & 3 \end{array}$$



$$\Phi_{18} = m_{60} + 3m_{42} + 3m_{24} + m_{06}$$

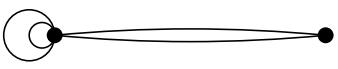
Generating graph:

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

$$\begin{aligned} \Phi_{19} \quad = \quad & m_{20}m_{60} + 2m_{20}m_{42} + m_{20}m_{24} + 2m_{11}m_{51} + 4m_{11}m_{33} + 2m_{11}m_{15} + m_{02}m_{42} + 2m_{02}m_{24} \\ & + m_{02}m_{06} \end{aligned}$$

Generating graph:

$$\begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{matrix}$$



$$\begin{aligned}\Phi_{20} &= m_{40}m_{60} + m_{40}m_{42} + 4m_{31}m_{51} + 4m_{31}m_{33} + 6m_{22}m_{42} + 6m_{22}m_{24} + 4m_{13}m_{33} + 4m_{13}m_{15} \\ &\quad + m_{04}m_{24} + m_{04}m_{06}\end{aligned}$$

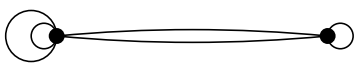
Generating graph:

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{21} &= m_{40}m_{60} + 2m_{40}m_{42} + m_{40}m_{24} + 2m_{31}m_{51} + 4m_{31}m_{33} + 2m_{31}m_{15} + m_{22}m_{60} + 3m_{22}m_{42} \\ &\quad + 3m_{22}m_{24} + m_{22}m_{06} + 2m_{13}m_{51} + 4m_{13}m_{33} + 2m_{13}m_{15} + m_{04}m_{42} + 2m_{04}m_{24} + m_{04}m_{06}\end{aligned}$$

Generating graph:

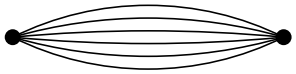
$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 2 \end{array}$$



$$\Phi_{22} = m_{60}^2 + 6m_{51}^2 + 15m_{42}^2 + 20m_{33}^2 + 15m_{24}^2 + 6m_{15}^2 + m_{06}^2$$

Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{23} &= m_{60}^2 + 2m_{60}m_{42} + 4m_{51}^2 + 8m_{51}m_{33} + 7m_{42}^2 + 12m_{42}m_{24} + 8m_{33}^2 + 8m_{33}m_{15} + 7m_{24}^2 \\ &\quad +2m_{24}m_{06} + 4m_{15}^2 + m_{06}^2\end{aligned}$$

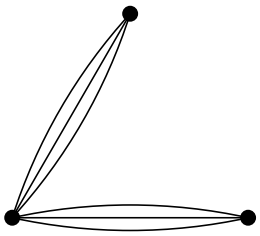
Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{24} &= m_{30}^2 m_{60} + 6m_{30}m_{21}m_{51} + 6m_{30}m_{12}m_{42} + 2m_{30}m_{03}m_{33} + 9m_{21}^2 m_{42} + 18m_{21}m_{12}m_{33} \\ &\quad +6m_{21}m_{03}m_{24} + 9m_{12}^2 m_{24} + 6m_{12}m_{03}m_{15} + m_{03}^2 m_{06}\end{aligned}$$

Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 & 3 & 3 \end{array}$$



$$\begin{aligned}\Phi_{25} &= m_{30}m_{70} + 2m_{30}m_{52} + m_{30}m_{34} + 3m_{21}m_{61} + 6m_{21}m_{43} + 3m_{21}m_{25} + 3m_{12}m_{52} + 6m_{12}m_{34} \\ &\quad +3m_{12}m_{16} + m_{03}m_{43} + 2m_{03}m_{25} + m_{03}m_{07}\end{aligned}$$

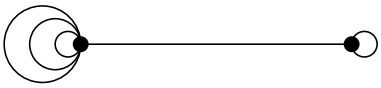
Generating graph:

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{26} &= m_{30}m_{70} + 3m_{30}m_{52} + 3m_{30}m_{34} + m_{30}m_{16} + m_{21}m_{61} + 3m_{21}m_{43} + 3m_{21}m_{25} + m_{21}m_{07} + m_{12}m_{70} \\ &\quad +3m_{12}m_{52} + 3m_{12}m_{34} + m_{12}m_{16} + m_{03}m_{61} + 3m_{03}m_{43} + 3m_{03}m_{25} + m_{03}m_{07}\end{aligned}$$

Generating graph:

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{27} &= m_{50}m_{70} + m_{50}m_{52} + 5m_{41}m_{61} + 5m_{41}m_{43} + 10m_{32}m_{52} + 10m_{32}m_{34} + 10m_{23}m_{43} \\ &\quad +10m_{23}m_{25} + 5m_{14}m_{34} + 5m_{14}m_{16} + m_{05}m_{25} + m_{05}m_{07}\end{aligned}$$

Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{28} &= m_{50}m_{70} + 2m_{50}m_{52} + m_{50}m_{34} + 3m_{41}m_{61} + 6m_{41}m_{43} + 3m_{41}m_{25} + m_{32}m_{70} + 5m_{32}m_{52} \\ &\quad +7m_{32}m_{34} + 3m_{32}m_{16} + 3m_{23}m_{61} + 7m_{23}m_{43} + 5m_{23}m_{25} + m_{23}m_{07} + 3m_{14}m_{52} + 6m_{14}m_{34} \\ &\quad +3m_{14}m_{16} + m_{05}m_{43} + 2m_{05}m_{25} + m_{05}m_{07}\end{aligned}$$

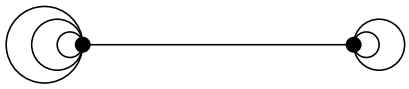
Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{29} \;=\; & m_{50}m_{70} + 3m_{50}m_{52} + 3m_{50}m_{34} + m_{50}m_{16} + m_{41}m_{61} + 3m_{41}m_{43} + 3m_{41}m_{25} + m_{41}m_{07} \\ & +2m_{32}m_{70} + 6m_{32}m_{52} + 6m_{32}m_{34} + 2m_{32}m_{16} + 2m_{23}m_{61} + 6m_{23}m_{43} + 6m_{23}m_{25} + 2m_{23}m_{07} \\ & +m_{14}m_{70} + 3m_{14}m_{52} + 3m_{14}m_{34} + m_{14}m_{16} + m_{05}m_{61} + 3m_{05}m_{43} + 3m_{05}m_{25} + m_{05}m_{07}\end{aligned}$$

Generating graph:

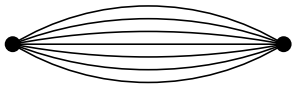
$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 & 2 & 2 \end{array}$$



$$\Phi_{30} = m_{70}^2 + 7m_{61}^2 + 21m_{52}^2 + 35m_{43}^2 + 35m_{34}^2 + 21m_{25}^2 + 7m_{16}^2 + m_{07}^2$$

Generating graph:

$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{31} &= m_{70}^2 + 2m_{70}m_{52} + 5m_{61}^2 + 10m_{61}m_{43} + 11m_{52}^2 + 20m_{52}m_{34} + 15m_{43}^2 + 20m_{43}m_{25} + 15m_{34}^2 \\ &\quad +10m_{34}m_{16} + 11m_{25}^2 + 2m_{25}m_{07} + 5m_{16}^2 + m_{07}^2\end{aligned}$$

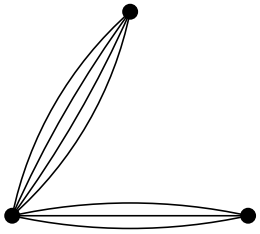
Generating graph:

$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{32} &= m_{30}m_{40}m_{70} + 4m_{30}m_{31}m_{61} + 6m_{30}m_{22}m_{52} + 4m_{30}m_{13}m_{43} + m_{30}m_{04}m_{34} + 3m_{21}m_{40}m_{61} \\ &\quad +12m_{21}m_{31}m_{52} + 18m_{21}m_{22}m_{43} + 12m_{21}m_{13}m_{34} + 3m_{21}m_{04}m_{25} + 3m_{12}m_{40}m_{52} \\ &\quad +12m_{12}m_{31}m_{43} + 18m_{12}m_{22}m_{34} + 12m_{12}m_{13}m_{25} + 3m_{12}m_{04}m_{16} + m_{03}m_{40}m_{43} \\ &\quad +4m_{03}m_{31}m_{34} + 6m_{03}m_{22}m_{25} + 4m_{03}m_{13}m_{16} + m_{03}m_{04}m_{07}\end{aligned}$$

Generating graph:

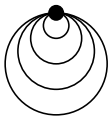
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 3 & 3 & 3 \end{array}$$



$$\Phi_{33} = m_{80} + 4m_{62} + 6m_{44} + 4m_{26} + m_{08}$$

Generating graph:

$$\begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix}$$



$$\begin{aligned} \Phi_{34} \;=\; & m_{20}m_{80} + 3m_{20}m_{62} + 3m_{20}m_{44} + m_{20}m_{26} + 2m_{11}m_{71} + 6m_{11}m_{53} + 6m_{11}m_{35} + 2m_{11}m_{17} \\ & + m_{02}m_{62} + 3m_{02}m_{44} + 3m_{02}m_{26} + m_{02}m_{08} \end{aligned}$$

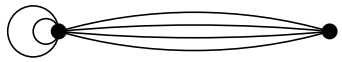
Generating graph:

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 \end{matrix}$$

$$\begin{aligned}\Phi_{35} &= m_{40}m_{80} + 2m_{40}m_{62} + m_{40}m_{44} + 4m_{31}m_{71} + 8m_{31}m_{53} + 4m_{31}m_{35} + 6m_{22}m_{62} + 12m_{22}m_{44} \\ &\quad +6m_{22}m_{26} + 4m_{13}m_{53} + 8m_{13}m_{35} + 4m_{13}m_{17} + m_{04}m_{44} + 2m_{04}m_{26} + m_{04}m_{08}\end{aligned}$$

Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{36} &= m_{40}m_{80} + 3m_{40}m_{62} + 3m_{40}m_{44} + m_{40}m_{26} + 2m_{31}m_{71} + 6m_{31}m_{53} + 6m_{31}m_{35} + 2m_{31}m_{17} \\ &\quad +m_{22}m_{80} + 4m_{22}m_{62} + 6m_{22}m_{44} + 4m_{22}m_{26} + m_{22}m_{08} + 2m_{13}m_{71} + 6m_{13}m_{53} + 6m_{13}m_{35} \\ &\quad +2m_{13}m_{17} + m_{04}m_{62} + 3m_{04}m_{44} + 3m_{04}m_{26} + m_{04}m_{08}\end{aligned}$$

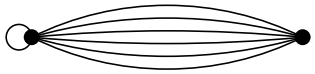
Generating graph:

$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 2 \end{array}$$

$$\begin{aligned}\Phi_{37} &= m_{60}m_{80} + m_{60}m_{62} + 6m_{51}m_{71} + 6m_{51}m_{53} + 15m_{42}m_{62} + 15m_{42}m_{44} + 20m_{33}m_{53} \\ &\quad +20m_{33}m_{35} + 15m_{24}m_{44} + 15m_{24}m_{26} + 6m_{15}m_{35} + 6m_{15}m_{17} + m_{06}m_{26} + m_{06}m_{08}\end{aligned}$$

Generating graph:

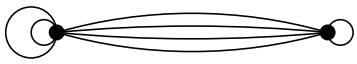
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{38} &= m_{60}m_{80} + 2m_{60}m_{62} + m_{60}m_{44} + 4m_{51}m_{71} + 8m_{51}m_{53} + 4m_{51}m_{35} + m_{42}m_{80} + 8m_{42}m_{62} \\ &\quad +13m_{42}m_{44} + 6m_{42}m_{26} + 4m_{33}m_{71} + 12m_{33}m_{53} + 12m_{33}m_{35} + 4m_{33}m_{17} + 6m_{24}m_{62} \\ &\quad +13m_{24}m_{44} + 8m_{24}m_{26} + m_{24}m_{08} + 4m_{15}m_{53} + 8m_{15}m_{35} + 4m_{15}m_{17} + m_{06}m_{44} + 2m_{06}m_{26} \\ &\quad +m_{06}m_{08}\end{aligned}$$

Generating graph:

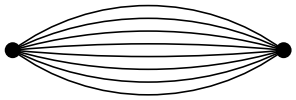
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\Phi_{39} = m_{80}^2 + 8m_{71}^2 + 28m_{62}^2 + 56m_{53}^2 + 70m_{44}^2 + 56m_{35}^2 + 28m_{26}^2 + 8m_{17}^2 + m_{08}^2$$

Generating graph:

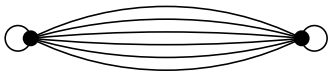
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{40} &= m_{80}^2 + 2m_{80}m_{62} + 6m_{71}^2 + 12m_{71}m_{53} + 16m_{62}^2 + 30m_{62}m_{44} + 26m_{53}^2 + 40m_{53}m_{35} + 30m_{44}^2 \\ &\quad +30m_{44}m_{26} + 26m_{35}^2 + 12m_{35}m_{17} + 16m_{26}^2 + 2m_{26}m_{08} + 6m_{17}^2 + m_{08}^2\end{aligned}$$

Generating graph:

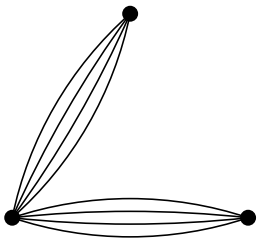
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{41} &= m_{40}^2 m_{80} + 8m_{40}m_{31}m_{71} + 12m_{40}m_{22}m_{62} + 8m_{40}m_{13}m_{53} + 2m_{40}m_{04}m_{44} + 16m_{31}^2 m_{62} \\ &\quad +48m_{31}m_{22}m_{53} + 32m_{31}m_{13}m_{44} + 8m_{31}m_{04}m_{35} + 36m_{22}^2 m_{44} + 48m_{22}m_{13}m_{35} \\ &\quad +12m_{22}m_{04}m_{26} + 16m_{13}^2 m_{26} + 8m_{13}m_{04}m_{17} + m_{04}^2 m_{08}\end{aligned}$$

Generating graph:

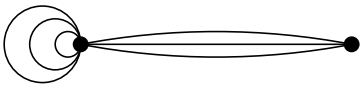
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 \end{array}$$



$$\begin{aligned}\Phi_{42} &= m_{30}m_{90} + 3m_{30}m_{72} + 3m_{30}m_{54} + m_{30}m_{36} + 3m_{21}m_{81} + 9m_{21}m_{63} + 9m_{21}m_{45} + 3m_{21}m_{27} \\ &\quad +3m_{12}m_{72} + 9m_{12}m_{54} + 9m_{12}m_{36} + 3m_{12}m_{18} + m_{03}m_{63} + 3m_{03}m_{45} + 3m_{03}m_{27} + m_{03}m_{09}\end{aligned}$$

Generating graph:

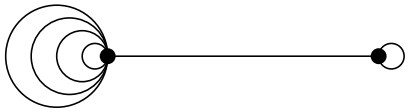
$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{43} &= m_{30}m_{90} + 4m_{30}m_{72} + 6m_{30}m_{54} + 4m_{30}m_{36} + m_{30}m_{18} + m_{21}m_{81} + 4m_{21}m_{63} + 6m_{21}m_{45} \\ &\quad +4m_{21}m_{27} + m_{21}m_{09} + m_{12}m_{90} + 4m_{12}m_{72} + 6m_{12}m_{54} + 4m_{12}m_{36} + m_{12}m_{18} + m_{03}m_{81} \\ &\quad +4m_{03}m_{63} + 6m_{03}m_{45} + 4m_{03}m_{27} + m_{03}m_{09}\end{aligned}$$

Generating graph:

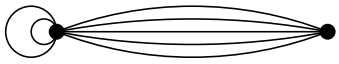
$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{44} &= m_{50}m_{90} + 2m_{50}m_{72} + m_{50}m_{54} + 5m_{41}m_{81} + 10m_{41}m_{63} + 5m_{41}m_{45} + 10m_{32}m_{72} + 20m_{32}m_{54} \\ &\quad +10m_{32}m_{36} + 10m_{23}m_{63} + 20m_{23}m_{45} + 10m_{23}m_{27} + 5m_{14}m_{54} + 10m_{14}m_{36} + 5m_{14}m_{18} \\ &\quad +m_{05}m_{45} + 2m_{05}m_{27} + m_{05}m_{09}\end{aligned}$$

Generating graph:

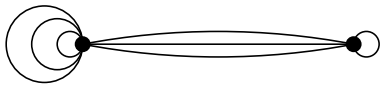
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{45} &= m_{50}m_{90} + 3m_{50}m_{72} + 3m_{50}m_{54} + m_{50}m_{36} + 3m_{41}m_{81} + 9m_{41}m_{63} + 9m_{41}m_{45} + 3m_{41}m_{27} \\ &\quad +m_{32}m_{90} + 6m_{32}m_{72} + 12m_{32}m_{54} + 10m_{32}m_{36} + 3m_{32}m_{18} + 3m_{23}m_{81} + 10m_{23}m_{63} \\ &\quad +12m_{23}m_{45} + 6m_{23}m_{27} + m_{23}m_{09} + 3m_{14}m_{72} + 9m_{14}m_{54} + 9m_{14}m_{36} + 3m_{14}m_{18} + m_{05}m_{63} \\ &\quad +3m_{05}m_{45} + 3m_{05}m_{27} + m_{05}m_{09}\end{aligned}$$

Generating graph:

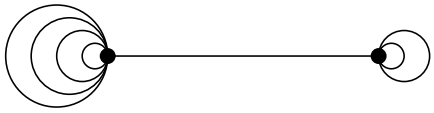
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{46} &= m_{50}m_{90} + 4m_{50}m_{72} + 6m_{50}m_{54} + 4m_{50}m_{36} + m_{50}m_{18} + m_{41}m_{81} + 4m_{41}m_{63} + 6m_{41}m_{45} \\ &\quad +4m_{41}m_{27} + m_{41}m_{09} + 2m_{32}m_{90} + 8m_{32}m_{72} + 12m_{32}m_{54} + 8m_{32}m_{36} + 2m_{32}m_{18} + 2m_{23}m_{81} \\ &\quad +8m_{23}m_{63} + 12m_{23}m_{45} + 8m_{23}m_{27} + 2m_{23}m_{09} + m_{14}m_{90} + 4m_{14}m_{72} + 6m_{14}m_{54} + 4m_{14}m_{36} \\ &\quad +m_{14}m_{18} + m_{05}m_{81} + 4m_{05}m_{63} + 6m_{05}m_{45} + 4m_{05}m_{27} + m_{05}m_{09}\end{aligned}$$

Generating graph:

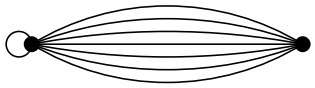
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{47} &= m_{70}m_{90} + m_{70}m_{72} + 7m_{61}m_{81} + 7m_{61}m_{63} + 21m_{52}m_{72} + 21m_{52}m_{54} + 35m_{43}m_{63} \\ &\quad +35m_{43}m_{45} + 35m_{34}m_{54} + 35m_{34}m_{36} + 21m_{25}m_{45} + 21m_{25}m_{27} + 7m_{16}m_{36} + 7m_{16}m_{18} \\ &\quad +m_{07}m_{27} + m_{07}m_{09}\end{aligned}$$

Generating graph:

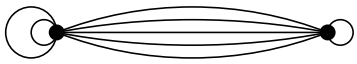
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{48} \;=\; & m_{70}m_{90} + 2m_{70}m_{72} + m_{70}m_{54} + 5m_{61}m_{81} + 10m_{61}m_{63} + 5m_{61}m_{45} + m_{52}m_{90} + 12m_{52}m_{72} \\ & +21m_{52}m_{54} + 10m_{52}m_{36} + 5m_{43}m_{81} + 20m_{43}m_{63} + 25m_{43}m_{45} + 10m_{43}m_{27} + 10m_{34}m_{72} \\ & +25m_{34}m_{54} + 20m_{34}m_{36} + 5m_{34}m_{18} + 10m_{25}m_{63} + 21m_{25}m_{45} + 12m_{25}m_{27} + m_{25}m_{09} \\ & +5m_{16}m_{54} + 10m_{16}m_{36} + 5m_{16}m_{18} + m_{07}m_{45} + 2m_{07}m_{27} + m_{07}m_{09}\end{aligned}$$

Generating graph:

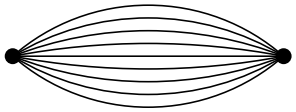
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\Phi_{49} = m_{90}^2 + 9m_{81}^2 + 36m_{72}^2 + 84m_{63}^2 + 126m_{54}^2 + 126m_{45}^2 + 84m_{36}^2 + 36m_{27}^2 + 9m_{18}^2 + m_{09}^2$$

Generating graph:

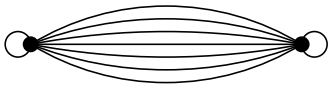
$$\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{50} &= m_{90}^2 + 2m_{90}m_{72} + 7m_{81}^2 + 14m_{81}m_{63} + 22m_{72}^2 + 42m_{72}m_{54} + 42m_{63}^2 + 70m_{63}m_{45} + 56m_{54}^2 \\ &\quad +70m_{54}m_{36} + 56m_{45}^2 + 42m_{45}m_{27} + 42m_{36}^2 + 14m_{36}m_{18} + 22m_{27}^2 + 2m_{27}m_{09} + 7m_{18}^2 + m_{09}^2\end{aligned}$$

Generating graph:

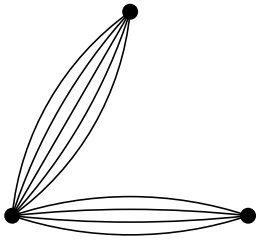
1	1	1	1	1	1	1	1	2
1	2	2	2	2	2	2	2	2



$$\begin{aligned}\Phi_{51} &= m_{40}m_{50}m_{90} + 5m_{40}m_{41}m_{81} + 10m_{40}m_{32}m_{72} + 10m_{40}m_{23}m_{63} + 5m_{40}m_{14}m_{54} + m_{40}m_{05}m_{45} \\ &\quad +4m_{31}m_{50}m_{81} + 20m_{31}m_{41}m_{72} + 40m_{31}m_{32}m_{63} + 40m_{31}m_{23}m_{54} + 20m_{31}m_{14}m_{45} \\ &\quad +4m_{31}m_{05}m_{36} + 6m_{22}m_{50}m_{72} + 30m_{22}m_{41}m_{63} + 60m_{22}m_{32}m_{54} + 60m_{22}m_{23}m_{45} \\ &\quad +30m_{22}m_{14}m_{36} + 6m_{22}m_{05}m_{27} + 4m_{13}m_{50}m_{63} + 20m_{13}m_{41}m_{54} + 40m_{13}m_{32}m_{45} \\ &\quad +40m_{13}m_{23}m_{36} + 20m_{13}m_{14}m_{27} + 4m_{13}m_{05}m_{18} + m_{04}m_{50}m_{54} + 5m_{04}m_{41}m_{45} \\ &\quad +10m_{04}m_{32}m_{36} + 10m_{04}m_{23}m_{27} + 5m_{04}m_{14}m_{18} + m_{04}m_{05}m_{09}\end{aligned}$$

Generating graph:

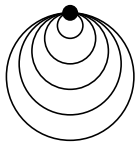
1	1	1	1	1	1	1	1	1
2	2	2	2	2	3	3	3	3



$$\Phi_{52} = m_{10,0} + 5m_{82} + 10m_{64} + 10m_{46} + 5m_{28} + m_{0,10}$$

Generating graph:

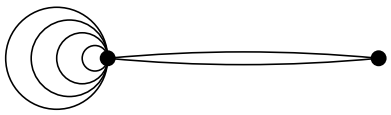
$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{array}$$



$$\begin{aligned} \Phi_{53} \;=\; & m_{20}m_{10,0} + 4m_{20}m_{82} + 6m_{20}m_{64} + 4m_{20}m_{46} + m_{20}m_{28} + 2m_{11}m_{91} + 8m_{11}m_{73} + 12m_{11}m_{55} \\ & +8m_{11}m_{37} + 2m_{11}m_{19} + m_{02}m_{82} + 4m_{02}m_{64} + 6m_{02}m_{46} + 4m_{02}m_{28} + m_{02}m_{0,10} \end{aligned}$$

Generating graph:

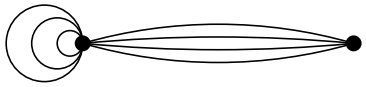
$$\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{54} &= m_{40}m_{10,0} + 3m_{40}m_{82} + 3m_{40}m_{64} + m_{40}m_{46} + 4m_{31}m_{91} + 12m_{31}m_{73} + 12m_{31}m_{55} + 4m_{31}m_{37} \\ &\quad +6m_{22}m_{82} + 18m_{22}m_{64} + 18m_{22}m_{46} + 6m_{22}m_{28} + 4m_{13}m_{73} + 12m_{13}m_{55} + 12m_{13}m_{37} \\ &\quad +4m_{13}m_{19} + m_{04}m_{64} + 3m_{04}m_{46} + 3m_{04}m_{28} + m_{04}m_{0,10}\end{aligned}$$

Generating graph:

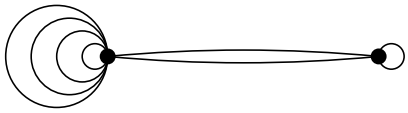
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{55} &= m_{40}m_{10,0} + 4m_{40}m_{82} + 6m_{40}m_{64} + 4m_{40}m_{46} + m_{40}m_{28} + 2m_{31}m_{91} + 8m_{31}m_{73} + 12m_{31}m_{55} \\ &\quad +8m_{31}m_{37} + 2m_{31}m_{19} + m_{22}m_{10,0} + 5m_{22}m_{82} + 10m_{22}m_{64} + 10m_{22}m_{46} + 5m_{22}m_{28} \\ &\quad +m_{22}m_{0,10} + 2m_{13}m_{91} + 8m_{13}m_{73} + 12m_{13}m_{55} + 8m_{13}m_{37} + 2m_{13}m_{19} + m_{04}m_{82} + 4m_{04}m_{64} \\ &\quad +6m_{04}m_{46} + 4m_{04}m_{28} + m_{04}m_{0,10}\end{aligned}$$

Generating graph:

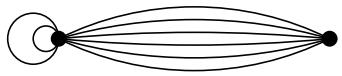
$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{56} &= m_{60}m_{10,0} + 2m_{60}m_{82} + m_{60}m_{64} + 6m_{51}m_{91} + 12m_{51}m_{73} + 6m_{51}m_{55} + 15m_{42}m_{82} \\ &\quad +30m_{42}m_{64} + 15m_{42}m_{46} + 20m_{33}m_{73} + 40m_{33}m_{55} + 20m_{33}m_{37} + 15m_{24}m_{64} \\ &\quad +30m_{24}m_{46} + 15m_{24}m_{28} + 6m_{15}m_{55} + 12m_{15}m_{37} + 6m_{15}m_{19} + m_{06}m_{46} + 2m_{06}m_{28} \\ &\quad +m_{06}m_{0,10}\end{aligned}$$

Generating graph:

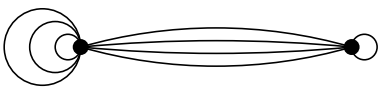
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{57} &= m_{60}m_{10,0} + 3m_{60}m_{82} + 3m_{60}m_{64} + m_{60}m_{46} + 4m_{51}m_{91} + 12m_{51}m_{73} + 12m_{51}m_{55} + 4m_{51}m_{37} \\ &\quad + m_{42}m_{10,0} + 9m_{42}m_{82} + 21m_{42}m_{64} + 19m_{42}m_{46} + 6m_{42}m_{28} + 4m_{33}m_{91} + 16m_{33}m_{73} \\ &\quad + 24m_{33}m_{55} + 16m_{33}m_{37} + 4m_{33}m_{19} + 6m_{24}m_{82} + 19m_{24}m_{64} + 21m_{24}m_{46} + 9m_{24}m_{28} \\ &\quad + m_{24}m_{0,10} + 4m_{15}m_{73} + 12m_{15}m_{55} + 12m_{15}m_{37} + 4m_{15}m_{19} + m_{06}m_{64} + 3m_{06}m_{46} \\ &\quad + 3m_{06}m_{28} + m_{06}m_{0,10}\end{aligned}$$

Generating graph:

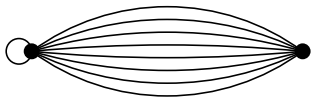
$$\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{58} &= m_{80}m_{10,0} + m_{80}m_{82} + 8m_{71}m_{91} + 8m_{71}m_{73} + 28m_{62}m_{82} + 28m_{62}m_{64} + 56m_{53}m_{73} \\ &\quad + 56m_{53}m_{55} + 70m_{44}m_{64} + 70m_{44}m_{46} + 56m_{35}m_{55} + 56m_{35}m_{37} + 28m_{26}m_{46} \\ &\quad + 28m_{26}m_{28} + 8m_{17}m_{37} + 8m_{17}m_{19} + m_{08}m_{28} + m_{08}m_{0,10}\end{aligned}$$

Generating graph:

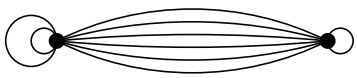
$$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{matrix}$$



$$\begin{aligned} \Phi_{59} \;=\; & m_{80}m_{10,0} + 2m_{80}m_{82} + m_{80}m_{64} + 6m_{71}m_{91} + 12m_{71}m_{73} + 6m_{71}m_{55} + m_{62}m_{10,0} + 17m_{62}m_{82} \\ & +31m_{62}m_{64} + 15m_{62}m_{46} + 6m_{53}m_{91} + 32m_{53}m_{73} + 46m_{53}m_{55} + 20m_{53}m_{37} + 15m_{44}m_{82} \\ & +45m_{44}m_{64} + 45m_{44}m_{46} + 15m_{44}m_{28} + 20m_{35}m_{73} + 46m_{35}m_{55} + 32m_{35}m_{37} + 6m_{35}m_{19} \\ & +15m_{26}m_{64} + 31m_{26}m_{46} + 17m_{26}m_{28} + m_{26}m_{0,10} + 6m_{17}m_{55} + 12m_{17}m_{37} + 6m_{17}m_{19} \\ & +m_{08}m_{46} + 2m_{08}m_{28} + m_{08}m_{0,10} \end{aligned}$$

Generating graph:

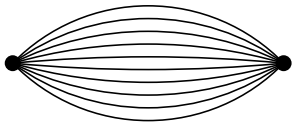
$$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{matrix}$$



$$\begin{aligned} \Phi_{60} \;=\; & m_{10,0}^2 + 10m_{91}^2 + 45m_{82}^2 + 120m_{73}^2 + 210m_{64}^2 + 252m_{55}^2 + 210m_{46}^2 + 120m_{37}^2 + 45m_{28}^2 \\ & +10m_{19}^2 + m_{0,10}^2 \end{aligned}$$

Generating graph:

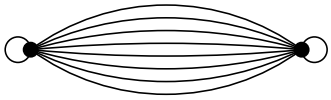
$$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{matrix}$$



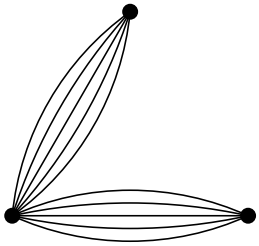
$$\begin{aligned}\Phi_{61} &= m_{10,0}^2 + 2m_{10,0}m_{82} + 8m_{91}^2 + 16m_{91}m_{73} + 29m_{82}^2 + 56m_{82}m_{64} + 64m_{73}^2 + 112m_{73}m_{55} \\ &\quad +98m_{64}^2 + 140m_{64}m_{46} + 112m_{55}^2 + 112m_{55}m_{37} + 98m_{46}^2 + 56m_{46}m_{28} + 64m_{37}^2 \\ &\quad +16m_{37}m_{19} + 29m_{28}^2 + 2m_{28}m_{0,10} + 8m_{19}^2 + m_{0,10}^2\end{aligned}$$

Generating graph:

$$\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$



$$\begin{aligned}\Phi_{62} &= m_{50}^2 m_{10,0} + 10m_{50}m_{41}m_{91} + 20m_{50}m_{32}m_{82} + 20m_{50}m_{23}m_{73} + 10m_{50}m_{14}m_{64} + 2m_{50}m_{05}m_{55} \\ &\quad +25m_{41}^2 m_{82} + 100m_{41}m_{32}m_{73} + 100m_{41}m_{23}m_{64} + 50m_{41}m_{14}m_{55} + 10m_{41}m_{05}m_{46} \\ &\quad +100m_{32}^2 m_{64} + 200m_{32}m_{23}m_{55} + 100m_{32}m_{14}m_{46} + 20m_{32}m_{05}m_{37} + 100m_{23}^2 m_{46} \\ &\quad +100m_{23}m_{14}m_{37} + 20m_{23}m_{05}m_{28} + 25m_{14}^2 m_{28} + 10m_{14}m_{05}m_{19} + m_{05}^2 m_{0,10}\end{aligned}$$

Generating graph:

$$\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 3 & 3 \end{array}$$